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Applications of Fuzzy Logic in Decision Making and Management Science

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Contents

An Introduction to Fuzzy Logic in Real Time Application Paradigm

Arnab Basu and Chandrashekhar Lall Chaudhury

Abstract Since the inception of the digital world, computers have been able to comprehend the binary values of either '0' or '1'. Performing huge arrays of complex calculations on the basis of the binary values was an essential development but in practical and tangible situations, this is not always the case. The limitations faced by a computer when classifying many ambiguous questions like whether the temperature of water is warm enough or whether a person is beautiful or not etc. lead to the birth of the concept of 'fuzzy logic', an approach proposed to give high accuracy in a predictive system. Such fuzzy logic approach imitates how the human brain analyses and resolves issues. In contrast to the conventional Boolean approach to logic, fuzzy logic allows computers to respond to varying degrees of truth. The goal of fuzzy logic is to simulate human decision-making by employing natural language words rather than strictly mathematical ones. Fuzzy logic has applications in cutting-edge research, technology, and commercial domains. Engineers (electrical, mechanical, civil, chemical, aerospace, agricultural, biomedical, computer, environmental, geological, industrial, and mechatronics), mathematicians, computer software developers and researchers, natural scientists (biology, chemistry, earth science, and physics), medical researchers, and social scientists (economics, management, political science, and psychoanalysis) all find fuzzy logic to be of great use in research and development. In this chapter, we look into the internal mechanics, limitations and the various applications of fuzzy logic.

Keywords Fuzzy set · Fuzzy logic · Decision making · Fuzzy logic control systems

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1 Introduction

In the realm of modern computing and control systems, the demand for more adaptive and flexible decision-making processes has given rise to the integration of fuzzy logic. Fuzzy logic is a mathematical framework introduced by Lotfi Zadeh in the 1960s. It offers a change from orthodox binary logic by taking into consideration the various degrees of truth between absolute true and absolute false values. This unique characteristic makes fuzzy logic particularly well-suited for scenarios where degrees of uncertainty and imprecision are inherent, thereby allowing a more practical and human-like approach to decision-making [1–4].

The word ‘fuzzy’ means ambiguous or in simpler terms, unclear. Thus, fuzzy logic operates on the principle of “fuzziness”, which allows it to model complex and ambiguous relationships within a system. Unlike the classical binary logic, where propositions are either true or false i.e. 0 or 1, fuzzy logic allows the provision for varying degrees of truth, enabling a more disintegrated portrayal of information [2, 6].

As said earlier—one of the key strengths of fuzzy logic lies in its ability to handle linguistic variables and imprecise data, hence providing a more natural way to express knowledge and reasoning. Thus, in situations where human intuition plays a significant role, such as in control systems, decision support, and artificial intelligence applications, it plays a major role. Whether in adaptive control, pattern recognition, or decision support systems, fuzzy logic’s capacity to model complex relationships has proven instrumental in achieving more robust and intelligent system behaviour. The integration of fuzzy logic in real-time systems is driven by its effectiveness in dealing with uncertain and dynamic environments. Real-time applications often require rapid responses to changing conditions, and fuzzy logic excels in capturing the inherent uncertainties associated with such environments. This chapter delves into the foundational principles of fuzzy logic and explores its application in the real-time domain where quick and adaptive decision-making is of utmost importance [5–12].

2 Understanding Fuzzy Logic

Fuzzy logic is a mathematical framework that deals with reasoning that is approximate rather than fixed and exact. Traditional binary logic relies on crisp values (0 or 1), whereas fuzzy logic allows for a continuum or a range of possibilities between true and false. This flexibility makes it well-suited for applications where ambiguity and vagueness are inherent, as is often the case in real-time scenarios. The key distinction of fuzzy logic is its recognition of shades of truth, introducing the concept of membership functions and fuzzy sets. Instead of following the binary classification of true or false, fuzzy logic considers the notion that elements can belong to a set of varying degrees. These membership functions define the degree of membership of an element in a fuzzy set, enabling a more sophisticated representation of information [13, 14].

In practical terms, fuzzy logic has demonstrated its efficacy in capturing and formalizing human-like decision-making processes. By incorporating linguistic variables and rule-based systems, fuzzy logic can emulate and imitate human reasoning, making it an indispensable tool in applications where human intuition is essential. This includes fields such as expert systems, where the ability to handle imprecise data and mimic human decision processes is crucial [12].

Let's consider a simple example of a fuzzy logic system to control the speed of a fan based on the temperature in a room. The goal is to design a fuzzy logic controller that adjusts the fan speed based on the current temperature.

The linguistic variables involved in the example would be [15–21]:

Input Variable: Temperature.

- *Linguistic terms*: Cold, Cool, Comfort, Hot.
- *Universe of discourse*: (0–100) °C.

Output Variable: Fan Speed.

- *Linguistic terms*: Slow, Medium, Fast.
- *Universe of discourse*: 0–100%

Now, let us define some fuzzy rules:

1. If Temperature is Cold, then Fan Speed is Slow.
2. If Temperature is Cool, then Fan Speed is Medium.
3. If Temperature is Comfort, then Fan Speed is Medium.
4. If Temperature is Hot, then Fan Speed is Fast (Fig. 1).

Now, let's define the membership functions for each linguistic term. These functions define how much a given temperature belongs to each linguistic term. For simplicity, let's use triangular membership functions:

- Cold:

  $\mu\ \mathrm{Cold}(x) = \mathrm{triangular}(0, 0, 20)$

- Cool:

  $\mu\ \mathrm{Cool}(x) = \mathrm{triangular}(10, 20, 30)$

- Comfort:

  $\mu\ \mathrm{Comfort}(x) = \mathrm{triangular}(25, 35, 45)$

- Hot:

  $\mu\ \mathrm{Hot}(x) = \mathrm{triangular}(40, 60, 80)$

- Slow:

  $\mu\ \mathrm{Slow}(y) = \mathrm{triangular}(0, 0, 50)$

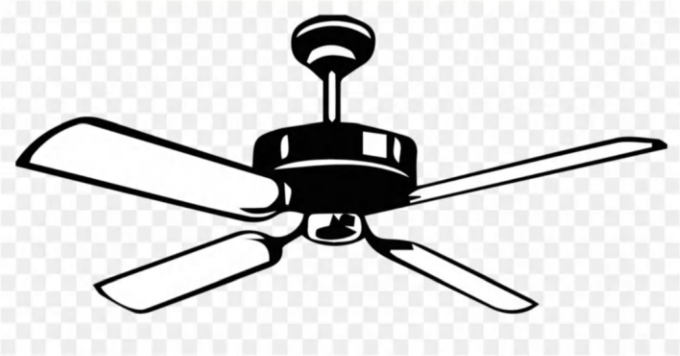

Fig. 1 The different linguistic variables for the measure of temperature—Cold, cool, comfort and warm

- Medium:

  μ Medium(y) = triangular(20, 50, 80)

- Fast:

  μ Fast(y) = triangular(50, 100, 100)

Now, let's say the current temperature is 30 degrees Celsius. We can evaluate the degree of membership for each linguistic term in the input variable:

- μ Cold(30) = 0
- μ Cool(30) = 0.5
- μ Comfort(30) = 0.5
- μ Hot(30) = 0

Now, using the fuzzy rules and the fuzzy inference process, we can determine the degree of membership for each linguistic term in the output variable (Fan Speed). Finally, we can use the centroid defuzzification method to find the crisp output value, which represents the fan speed.

This is a simple example, and real-world fuzzy logic systems are of course created involving significantly more complex rules, variables, and membership functions. Fuzzy logic is particularly useful when dealing with systems that have uncertain or imprecise inputs and outputs.

3 Key Components of Fuzzy Logic

3.1 Fuzzy Sets

Fuzzy sets form the foundation of fuzzy logic, representing a generalization of classical sets. While classical sets strictly adhere to binary membership (in or out), fuzzy sets introduce a revolutionary concept by assigning a degree of membership to each element, ranging from 0 to 1. This departure from the traditional binary logic allows for a more sophisticated and nuanced representation of concepts [1–15].

In classical sets, an element either fully belongs to a set (with a membership of 1) or does not belong at all (with a membership of 0) but Fuzzy sets consider that membership is not an all-or-nothing affair. By assigning partial membership degrees, fuzzy logic accommodates the inherent uncertainties and imprecisions prevalent in many real-world scenarios.

The graded membership concept facilitates a flexible and adaptive modelling approach, especially in situations where the boundaries between categories are unclear or when precise numerical values are challenging to determine. Fuzzy sets provide a powerful tool for capturing and expressing the inherent vagueness and ambiguity in human reasoning and linguistic descriptions, making fuzzy logic a valuable framework for handling complex systems with uncertain information.

3.2 Membership Functions

Membership functions serve as pivotal elements within the framework of fuzzy logic, playing a crucial role in capturing and quantifying the degree to which an element belongs to a fuzzy set. These functions are fundamental to the transition from crisp, precise data to the nuanced and fuzzy representation essential for handling uncertainties in real-world scenarios [12–16].

In essence, membership functions act as mathematical bridges, facilitating the mapping of input variables onto linguistic terms within the fuzzy logic system. These terms, such as “low,” “medium,” or “high,” are representative of fuzzy sets and help encode the qualitative nature of human reasoning. The design and shape of these membership functions are crafted to mirror the subjective and imprecise nature of human language, allowing for a more realistic representation of knowledge.

Membership functions are characterized by their ability to express the uncertainty inherent in linguistic terms. Instead of rigidly assigning values, they provide a graded spectrum of membership degrees, typically ranging from 0 to 1. This gradation enables a smoother transition between different linguistic categories, contributing to the flexibility and adaptability of fuzzy logic systems in handling vague and uncertain information.

By incorporating membership functions, fuzzy logic systems gain the capacity to process and interpret imprecise inputs, making them well-suited for applications where precise numerical data may be challenging to obtain or where human expert knowledge plays a crucial role. The use of membership functions enhances the capability of fuzzy logic to effectively model and reason about complex, real-world situations [18–21].

3.3 Rule Base

The rule base stands as a foundational element within fuzzy logic systems, constituting a repository of expert knowledge expressed through traditional semantic "if–then" rules. These rules serve as the medium that establish relationships between input variables and the corresponding output variable, encapsulating the inherent reasoning process.

In essence, the rule base embodies the expertise and insights of human operators or domain experts, translating their qualitative knowledge into a formalized structure that a computerized system can comprehend. Each rule in the rule base takes the form of an "if–then" statement, delineating the conditions under which certain actions or decisions should be taken.

The structure of these rules reflects the linguistic relationships between the input and output variables. For instance, a rule might state, "If the temperature is high, then decrease the cooling level." These rules enable the fuzzy logic system to make decisions based on imprecise and qualitative information, mirroring the way humans often make decisions in complex, uncertain environments.

The rule base, through its collection of rules, provides a roadmap for the system's decision-making process. During operation, the fuzzy logic system consults this rule base to determine the appropriate action or output corresponding to the given input conditions. The ability to encode expert knowledge in the form of rules makes fuzzy logic systems particularly adept at handling complex tasks across various domains, including control systems, artificial intelligence, and decision support systems. The flexibility inherent in the rule-based approach allows for the adaptation of the system's behaviour to different scenarios and contexts [1–8].

3.4 Inference Engine

Positioned at the heart of a fuzzy logic system, the inference engine serves as the computational powerhouse responsible for navigating the intricacies of the rule base. Its primary function is to interpret the "if–then" rules established within the system, leveraging the fuzzy sets defined by the membership functions.

When confronted with a specific set of input conditions, the inference engine systematically evaluates the applicable rules, considering the degree of membership of the input variables within the fuzzy sets. Fuzzy logic operators, including AND, OR, and NOT, play a crucial role in this evaluation, enabling the combination and aggregation of fuzzy sets as dictated by the rules.

The AND operator is often employed to model the simultaneous satisfaction of multiple conditions in a rule, whereas the OR operator accommodates scenarios where either or both of multiple conditions is met. The NOT operator, on the other hand, facilitates the representation of negation, allowing for the consideration of situations where a condition is not fulfilled.

Through the intricate interplay of fuzzy logic operators and rule evaluation, the inference engine derives a fuzzy output that encapsulates the system's response to the given input conditions. This fuzzy output reflects the uncertainty and imprecision inherent in the input data, mimicking human-like decision-making under conditions of ambiguity.

The inference engine, by simulating human-like reasoning, empowers fuzzy logic systems to excel in situations where conventional, crisp logic may fall short. Its ability to handle and process imprecise information makes fuzzy logic particularly well-suited for applications in which human expertise and intuition are crucial components of decision-making [8–15].

3.5 Defuzzification

The conclusive step or the final step in a fuzzy logic system is defuzzification, a process crucial for translating the fuzzy output derived from the inference engine into a concrete and actionable decision. This transformative stage is essential for bridging the gap between the fuzzy representation employed in the reasoning process and the practical output required for real-world applications.

Defuzzification methods serve as the means by the inherently fuzzy result is converted into a precise, understandable output. Two common defuzzification techniques are the centroid method (as mentioned in the example of the speed of the fan according to the temperature) and the maxima method.

Centroid Method: The centroid method determines the center of mass of the fuzzy output, effectively finding the weighted average of the fuzzy set. This approach considers the distribution of membership values across the output space, providing a representative and balanced crisp output. The centroid method is particularly effective when the fuzzy output exhibits a symmetrical distribution.

Maxima Method: In contrast, the maxima method identifies the point of maximum membership within the fuzzy output. This approach selects the most prominent value, reflecting the most influential region of the fuzzy set. While less concerned with the overall distribution, the maxima method is valuable when the fuzzy output is skewed or exhibits multiple peaks.

The choice between these defuzzification methods depends on the characteristics of the fuzzy output and the nature of the application. The selected method transforms the fuzzy result into a clear and actionable decision, providing a concrete value or set point that can be used in the real-world context. This output represents the system's final response to the given input conditions, allowing for straightforward implementation of decisions or control actions.

Defuzzification is a critical component in the application of fuzzy logic, ensuring that the inherent flexibility and adaptability of fuzzy reasoning culminate in practical, understandable outcomes. It facilitates the integration of fuzzy logic into various fields, including control systems, decision support systems, and artificial intelligence, where the ability to handle imprecise information is paramount [10–21].

4 Real-Time Applications of Fuzzy Logic

4.1 Control Systems

Fuzzy Logic Controllers (FLCs) have emerged as highly effective tools in the realm of real-time control systems, offering a distinctive advantage over traditional controllers. The adaptability and capacity of FLCs to handle imprecise input data make them particularly well-suited for diverse applications, ranging from temperature control and speed regulation to robotics like:

Imprecise Input Accommodation:

One of the key strengths of FLCs lies in their ability to handle imprecise and uncertain input data. Traditional controllers often struggle in situations where the precise numerical values required for input are challenging to obtain. FLCs, on the other hand, thrive in such scenarios by leveraging fuzzy sets and membership functions to model and process imprecision, allowing for more robust and flexible control.

Dynamic Adaptation:

FLCs excel in dynamic and changing conditions. Their capacity to adapt to variations in the environment or system parameters is a distinct advantage. This adaptability is particularly beneficial in scenarios where the relationship between inputs and outputs is complex and may evolve over time. FLCs can dynamically adjust their control strategies based on the current conditions, enhancing their performance in real-world, dynamic systems.

Applications in Temperature Control:

FLCs find widespread use in temperature control systems, where precise control is essential for maintaining optimal conditions. By employing linguistic variables such as "cold," "cool," "warm," and "hot," FLCs can regulate heating or cooling systems with a level of precision that traditional controllers may struggle to achieve.

Speed Regulation:

In applications involving speed regulation, such as in motor control or vehicle systems, FLCs offer an advantage in dealing with the inherent imprecision and nonlinearities of these systems. The linguistic terms and fuzzy rules enable FLCs to respond intelligently to varying speeds and loads, ensuring smooth and efficient operation.

Robotics:

Fuzzy logic has found extensive use in robotics, where the complex and often unpredictable nature of the environment requires adaptive control. FLCs enable robots to navigate and interact with their surroundings by interpreting imprecise sensor data and making dynamic decisions based on fuzzy rules.

Overall, the success of FLCs in control systems lies in their ability to handle uncertainty, adaptability to changing conditions, and their intuitive representation of linguistic variables. These qualities position FLCs as powerful tools for addressing the challenges posed by real-world control applications [1–21].

4.2 Traffic Management

Fuzzy logic has emerged as a pivotal technology in the field of traffic signal control systems, revolutionizing the way traffic is managed in urban environments. By incorporating fuzzy logic, traffic signal controllers can intelligently adapt to dynamic conditions, optimizing signal timings and ultimately reducing congestion while enhancing overall traffic flow in real-time.

Consideration of Multiple Factors:

Fuzzy logic in traffic signal control takes into account a multitude of factors that contribute to the complexity of urban traffic. Parameters such as traffic density, time of day, and historical data are considered as linguistic variables, allowing the system to interpret and respond to the imprecise and dynamic nature of these inputs. This holistic approach enables the traffic management system to make nuanced decisions that reflect the real-world intricacies of urban traffic.

Dynamic Signal Optimization:

Unlike traditional traffic signal control systems with fixed timing plans, fuzzy logic allows for dynamic and adaptive optimization of signal timings. The system continuously evaluates the current traffic conditions, adjusting signal timings in real-time based on the fuzzy rules embedded in the control algorithm. This adaptability enables the traffic management system to respond promptly to changing traffic patterns, reducing delays and improving overall efficiency.

Congestion Reduction:

Fuzzy logic contributes significantly to congestion reduction by providing a more sophisticated decision-making framework. In situations where traffic density is high or unexpected events occur, the fuzzy logic controller can allocate green time to the most congested directions, helping to alleviate bottlenecks and enhance the overall flow of traffic. This dynamic and context-aware approach contributes to a more responsive and efficient traffic management system.

Improved Traffic Flow:

The application of fuzzy logic in traffic signal control aims to enhance the overall flow of traffic. By considering various factors simultaneously and adjusting signal timings accordingly, the system can balance the needs of different intersections and road segments. This results in a more coordinated and synchronized traffic flow, minimizing stop-and-go patterns and improving the overall mobility of vehicles.

Adaptation to Peak Hours:

Fuzzy logic is particularly valuable in adapting signal timings to accommodate peak traffic hours. By analysing historical data and considering the time of day, the system can intelligently allocate green time to directions with higher demand during specific periods. This proactive approach aids in preventing congestion before it occurs and ensures a smoother traffic flow during peak hours.

In summary, the application of fuzzy logic in traffic signal control systems represents a paradigm shift in traffic management. The ability to consider multiple factors, adapt in real-time, and optimize signal timings dynamically contributes to reduced congestion, improved traffic flow, and a more efficient urban transportation system [1–21].

4.3 Medical Diagnosis

In the realm of healthcare, where uncertainty is inherent and symptoms often present in a nuanced manner, fuzzy logic plays a crucial role in medical diagnosis. Fuzzy systems, leveraging the principles of fuzzy logic, prove invaluable in processing vague and imprecise symptoms, offering probabilistic diagnoses and providing valuable support to medical practitioners in real-time decision-making.

Handling Uncertainty and Vagueness:

Healthcare scenarios frequently involve imprecise information, vague symptoms, and uncertainty. Fuzzy logic excels in handling these complexities by allowing for the representation of uncertainty through fuzzy sets and membership functions. This capability enables medical diagnosis systems to interpret ambiguous symptoms with a degree of flexibility that mirrors the inherent uncertainty in medical conditions.

Probabilistic Diagnoses:

Fuzzy logic systems in medical diagnosis do not provide binary outcomes but rather offer probabilistic diagnoses. Instead of categorizing a patient as either "healthy" or "diseased," fuzzy systems assign membership degrees to various potential diagnoses based on the observed symptoms. This probabilistic approach enhances the diagnostic process, acknowledging the likelihood of multiple conditions and providing a more nuanced understanding of the patient's health status.

Real-Time Decision Support:

Fuzzy logic contributes to real-time decision-making in medical diagnosis by swiftly processing complex and imprecise information. As symptoms are input into the system, fuzzy logic algorithms evaluate the degrees of membership to different diagnostic categories, aiding medical practitioners in making informed decisions promptly. This real-time support is particularly valuable in critical situations where swift and accurate diagnoses are crucial for effective treatment.

Adaptability to Patient-Specific Variations:

Each patient may exhibit unique variations in symptoms, and fuzzy logic accommodates this variability by allowing for patient-specific interpretation. The linguistic variables and fuzzy rules in medical diagnosis systems can be tailored to the specific characteristics of individual patients, enhancing the system's adaptability and ensuring a more personalized approach to diagnosis.

Integration of Expert Knowledge:

Fuzzy logic in medical diagnosis systems integrates expert knowledge from healthcare professionals. The rule base, constructed based on the expertise of medical practitioners, captures the nuanced relationships between symptoms and potential diagnoses. This collaborative approach combines the computational power of fuzzy logic with the experiential insights of healthcare experts, resulting in a comprehensive and reliable diagnostic tool.

In conclusion, the application of fuzzy logic in medical diagnosis addresses the inherent uncertainties in healthcare, providing a flexible and adaptive framework for interpreting vague symptoms. By offering probabilistic diagnoses and real-time decision support, fuzzy logic contributes to more nuanced and personalized patient care, ultimately enhancing the effectiveness of the diagnostic process in the medical field [1–21].

4.4 Financial Decision Support

Financial decision-making is inherently complex, often involving uncertain and imprecise factors. Fuzzy logic has found significant application in financial systems, particularly in the realms of risk assessment and decision support. Here's a more detailed exploration of how fuzzy logic is leveraged in this context:

Risk Assessment:

Fuzzy logic proves invaluable in assessing and managing financial risks due to its ability to handle imprecise and uncertain data. In traditional financial models, the assumption of precise and accurate information may not always hold, especially in volatile markets. Fuzzy logic allows for a more realistic representation of the imprecise nature of financial data, considering factors such as market fluctuations, economic indicators, and geopolitical events.

By employing fuzzy sets and membership functions, financial systems can model the degrees of risk associated with different assets or investment strategies. For instance, linguistic variables like "high risk," "moderate risk," and "low risk" can be defined, and fuzzy rules can capture the relationships between these variables and various financial indicators.

Realistic Predictions:

Financial markets are influenced by a myriad of interconnected variables, and their behaviour is often characterized by uncertainty. Fuzzy logic facilitates the modelling of these uncertainties, enabling financial decision support systems to generate more realistic predictions. Instead of providing binary outcomes, fuzzy logic offers a spectrum of possibilities, acknowledging the shades of truth in financial forecasts.

Through the utilization of historical data and real-time market information, fuzzy logic algorithms can evaluate the likelihood of different market scenarios. This nuanced approach allows investors and financial analysts to make informed decisions based on a more comprehensive understanding of the potential outcomes.

Timely Decision-Making:

In the fast-paced world of finance, timely decision-making is crucial. Fuzzy logic excels in providing quick and adaptive decision support by processing information with varying degrees of certainty. The inherent flexibility of fuzzy logic allows financial systems to swiftly adapt to changing market conditions, making it particularly well-suited for real-time decision support.

For example, if there is a sudden market shift or unexpected economic news, fuzzy logic-based decision support systems can dynamically adjust risk assessments and investment recommendations. This adaptability enhances the responsiveness of financial strategies, helping investors capitalize on opportunities or mitigate risks promptly.

Integration of Qualitative and Quantitative Factors:

Financial decision support often requires the integration of both quantitative and qualitative factors. Fuzzy logic excels in this regard by providing a framework to incorporate expert knowledge and linguistic variables alongside numerical data. This integration allows financial experts to express nuanced insights and considerations that may not be easily quantifiable.

Linguistic variables such as "market sentiment," "economic stability," or "political uncertainty" can be included in fuzzy models. The rule base of the fuzzy system can then capture the complex relationships between these qualitative factors and quantitative indicators, providing a more holistic foundation for decision-making.

In summary, the application of fuzzy logic in financial decision support contributes to a more robust and adaptive approach to managing risks and making investment decisions. By embracing the imprecise nature of financial data, fuzzy logic enhances the accuracy of predictions, facilitates timely decision-making, and accommodates the intricate interplay of qualitative and quantitative factors in the dynamic financial landscape [1–21].

4.5 Human–Machine Interaction

In the realm of human–machine interaction (HMI), the integration of fuzzy logic brings forth significant advancements, particularly in systems where the interpretation and response to user inputs demand a nuanced and context-aware approach. This section elaborates on the real-time applications of fuzzy logic in HMI, emphasizing its role in understanding the subtleties of natural language and contributing to the development of more intuitive and responsive human–computer interfaces.

Interpreting Natural Language:

Fuzzy logic excels in deciphering natural language, which is inherently nuanced and often imprecise. In HMI, where users communicate with machines through speech or text, the ability to understand the subtleties of language is crucial. Fuzzy logic facilitates the interpretation of linguistic variables and fuzzy sets, allowing systems to comprehend the shades of meaning in user inputs. Unlike traditional binary logic systems, which may struggle with the variability and ambiguity of language, fuzzy logic provides a more adaptive and context-aware framework for processing natural language commands.

Context-Aware Decision-Making:

Human–machine interaction often involves situations where the context of user inputs plays a significant role in determining appropriate responses. Fuzzy logic, through its rule-based systems and membership functions, enables systems to make context-aware decisions. For example, in a smart home environment, a fuzzy logic system can understand that a user saying "it's a bit warm" implies a different action compared to "it's too hot." This context-awareness contributes to more intelligent and personalized responses, enhancing the overall user experience.

Handling Uncertainty in User Inputs:

Users might express their preferences or commands with varying degrees of certainty or imprecision. Fuzzy logic is well-suited to handle such uncertainties by allowing for degrees of membership in linguistic terms. For instance, a user stating "increase the volume a little" introduces a level of uncertainty regarding the exact volume adjustment required. Fuzzy logic can model and respond to these imprecise inputs, providing a system that adapts to the user's preferences in a more natural and human-like manner.

Adaptive User Interfaces:

Fuzzy logic contributes to the development of adaptive user interfaces that evolve based on user interactions. Through continuous learning and adjustment, fuzzy logic systems can tailor the interface layout, colour schemes, or interaction modes based on user preferences and historical inputs. This adaptability enhances the user experience by creating interfaces that are more intuitive and aligned with individual user behaviours.

Gesture and Emotion Recognition:

Beyond natural language, fuzzy logic can be employed in the interpretation of gestures and emotions. Facial expressions, hand movements, and other non-verbal cues are inherently fuzzy in nature, and fuzzy logic systems can effectively capture and respond to these nuances. This is particularly relevant in applications such as virtual reality or human–computer interfaces where user emotions and gestures contribute to a richer interaction experience.

Enhanced User Feedback:

Fuzzy logic enables systems to provide more nuanced and human-like feedback to users. Instead of rigid responses, fuzzy systems can generate responses that reflect the uncertainty or imprecision in user inputs. This enhances the user-machine interaction by creating a more natural and empathetic communication channel, contributing to a sense of understanding and responsiveness from the machine.

On summarising, the integration of fuzzy logic in human–machine interaction introduces a new dimension of adaptability and intelligence. By understanding and responding to the nuances of natural language, contextual cues, and uncertain inputs, fuzzy logic enhances the overall user experience, making human–computer interfaces more intuitive, responsive, and aligned with human communication patterns [1–21].

5 Potential Challenges

5.1 Computational Complexity

Fine-tuning fuzzy systems to achieve optimal performance can be computationally intensive. The processing of fuzzy sets, rule-based systems, and inference engines in real-time applications requires significant computational resources. This complexity can pose challenges, especially in scenarios where rapid decision-making is essential.

5.2 Real-Time Processing of Large Datasets

Real-time applications often involve handling large datasets generated by sensors, IoT devices, or other sources. Processing such vast amounts of data in real-time, while maintaining the precision and adaptability of fuzzy logic, presents a significant challenge. Ensuring that fuzzy systems can operate efficiently in dynamic, data-rich environments is crucial for their widespread adoption.

5.3 Rule Base Complexity

The rule base, which encapsulates expert knowledge in the form of "if–then" rules, can become complex as systems expand to handle more variables and scenarios. Managing and updating intricate rule sets, especially in dynamic environments, can be challenging. Maintaining the interpretability and relevance of rules without overwhelming computational resources is a key challenge in the development of robust fuzzy logic systems.

5.4 Integration with Other Technologies

Fuzzy logic often needs to be integrated with other computational models or technologies, such as machine learning algorithms, to enhance its capabilities. The seamless integration of fuzzy logic with these approaches poses challenges related to interoperability, data compatibility, and maintaining the interpretability of fuzzy systems.

5.5 Lack of Standardization

While fuzzy logic has proven effective in various applications, there is a lack of standardized methodologies for designing and implementing fuzzy systems. This lack of standardization can hinder the widespread adoption of fuzzy logic, especially in critical domains where standardization is essential for safety and reliability.

6 Future Directions

6.1 Optimization of Fuzzy Algorithms

Ongoing research focuses on optimizing fuzzy algorithms to address computational challenges. This includes developing more efficient algorithms for fuzzy inference engines, exploring parallel processing techniques, and leveraging advancements in hardware, such as GPUs, to enhance the speed and scalability of fuzzy systems.

6.2 Hybrid Approaches

Hybrid approaches that combine fuzzy logic with other computational models, such as neural networks or evolutionary algorithms, are gaining attention. These approaches aim to leverage the strengths of different paradigms to create more robust and adaptive systems. The integration of fuzzy logic with machine learning techniques, for example, allows systems to learn from data and improve over time.

6.3 Explainable AI and Interpretable Fuzzy Systems

As artificial intelligence (AI) becomes more prevalent, there is a growing emphasis on the interpretability of AI systems. Future directions in fuzzy logic include research on developing explainable AI techniques, ensuring that fuzzy systems can provide transparent and understandable reasoning for their decisions. This is particularly important in applications where human operators need to trust and comprehend the system's decision-making process.

6.4 Standardization Efforts

Efforts towards standardizing methodologies for designing and implementing fuzzy systems are underway. Establishing industry standards can contribute to the wider acceptance of fuzzy logic, especially in safety–critical applications. Standardization efforts may involve defining best practices, common frameworks, and interoperability standards for fuzzy logic applications.

6.5 Adaptive Fuzzy Systems

The development of fuzzy systems that can adapt dynamically to changing environments and evolving datasets is a future direction. Research aims to create fuzzy systems that can autonomously adjust their rule bases, membership functions, and inference strategies based on real-time feedback and learning.

6.6 Edge Computing and Fuzzy Logic

With the rise of edge computing, there is a growing interest in deploying fuzzy logic directly at the edge, closer to where data is generated. This reduces latency and enhances the responsiveness of fuzzy systems in real-time applications. Future research explores the optimization of fuzzy logic for edge computing environments.

In conclusion, while challenges exist, ongoing research and development efforts in the field of fuzzy logic are focused on addressing these challenges and unlocking new possibilities. The future directions outlined above aim to enhance the efficiency, adaptability, and integration capabilities of fuzzy logic, ensuring its continued relevance and applicability in diverse real-time scenarios.

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Application of Bayesian Algorithm for Impacting Social Media Marketing on Smart Electric Motorcycle Purchase Intention

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Abstract Studying the influence of social media marketing on smart electric motorcycle purchase intention of customers in Ho Chi Minh City. The book chapter uses a quantitative method, with a survey sample of 228 samples and 6 independent variables. Based on Bayesian Algorithm results, the study has identified 5 factors that show the effect of social media marketing on smart electric motorcycle purchase intention of customers in Ho Chi Minh City, including (1) Electronic-word-of-mouth, (2) Social interaction, (3) Informative value, (4) Informative reliability, (5) Advertising entertainment. In addition, the study also examines the influence of smart electric motorcycle purchase intention in Ho Chi Minh City through 4 demographic variables: gender, age, occupation, and income. From there, the management implications of the customer's purchase intention can be created, which can make better marketing campaigns and the most appropriate social media marketing strategy to contribute to improving the smart electric motorcycle business. These businesses contribute to high revenue and the best profits. Previous investigations demonstrated that by using linear regression. The best option is used in this investigation by the Application of the Bayesian Algorithm (BIC) for impacting social media marketing on smart electric motorcycle purchase intention.

Keywords BIC algorithm · Smart electric motorcycle purchase intention · Electronic word of mouth · Social interaction · Informative value · Informative reliability · Advertising entertainment

1 Introduction

The research is based on the rapid development of innovation in the 4.0 or even 5.0 technology era. Since then, the marketing market has gradually become more exciting, helping to spread information on social media so that people can always

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access information promptly, quickly, and widely. In January 2024, there will be 5.04 billion social media users worldwide, or 62.3% of the world's population [1].

Thus, social networking becomes everyone's closest friend because they are potential young individuals who were born and raised to live and have access to the best technology.

Social media is a breakthrough for businesses, whether large or small, and it can help them promote their reputation. It has quickly become an extremely effective marketing and promotion tool [2]. In particular, marketing strategies have developed and are an effective method of attracting more customers to use products and services.

At present, in any place where information plays the role of maintaining and connecting close to each other, it is like a way of connecting longer and the existence and renewal of each country and people. Along with that, individuals' desire to experience new technology is also more interesting; smart smartphones and application software have become familiar to people, especially the Internet system. Traditional advertising, such as television, radio, radio, and leaflets, is no longer as useful as before. Instead, the power of digital technology has created a new concept for the field of marketing, which is Social Media Marketing.

In the social network space, an open space has been created, allowing people to connect people and people, between businesses and their customers. There is no denying the positive side of cyberspace. Businesses have directly applied it as a very useful and successful advertising and marketing method, so advertising and marketing play a central role in the current marketing environment.

In parallel with the great progress of social media marketing, the transport infrastructure is always changing, constantly transforming in order to develop and support people's travel, trade, and commerce more easily. In the capital of Hanoi and Ho Chi Minh City, where personal motorbikes are the primary means of transport, we always encounter scenes where the density of motorbikes is dense, which causes a decrease in air quality, living landscapes to the environment. are also significantly affected by this problem.

Ho Chi Minh City, which has an area of over 2 million square kilometers, is also home to a daily density of traffic participants. HCMC has 8.8 million registered vehicles [3]. A permanent problem in the city that is always mentioned is the status quo related to climate, living environment, atmosphere, pollution, etc. The greenhouse effect, diseases, many landfills, and carbon emissions from gasoline vehicles, accounting for 90% of all motor vehicles in the city, are also causes that cannot be ignored for the source of air pollution when car machines consume fossil fuels and emit harmful CO_2 into the environment.

Realizing the severe consequences of these problems, our state and specialized agencies have come up with alternative solutions, such as mobilizing people to consume green, live green, and encourage the use of vehicles. Especially green can be mentioned as bicycles and electric bicycles, and in recent years, electric motorbikes, a type of vehicle that does not use or consume gasoline, can maximize energy savings compared to conventional motorbikes. But also a most effective alternative to protect our living environment. Stemming from that, the market for smart electric motorcycles has been vibrant again and very popular.

In Vietnam, electric motorbikes are becoming a new, smart, and modern consumer trend, with the emergence of a series of domestic electric motor vehicle brands such as Vinfast, Datbike, Arevo, and Pega, among others. Foreign cars such as Yadea, Dibao, and MBigo are present in the Vietnamese market. According to the youth newspaper, in October 2022, there was a shortage of gasoline or gas. On November 1, 2022, out of 550 petrol stations, 4 were temporarily closed, and 108 were short of petrol or oil. This proves the essential nature as well as the urgency of the wave of electric motorbikes being used to replace gasoline-powered ones. It is because of this reason that electric motorcycles are making waves in online forums. We are more and more aware of the great power of the media without too much expense, and it also helps to attract a large number of people potential customers. Through this, the chapter shows the problem related to online communication that brings many efficiencies, opportunities, and innovation potential for businesses. The chapter explores the Bayesian algorithm for impacting social media marketing on smart electric motorcycle purchase intention.

This paper applies the Bayesian algorithm (BIC) to the workshop attractiveness of the career counseling programs presented: Part 1 gives an overview of the study, Sect. 2 reviews the literature on the variable employed in the study, and Sect. 3 explains the methodology. Section 4 presents the results and analysis along with some commentary and implications. This paper is finally concluded in Sect. 5.

2 Literature Review

2.1 Social Media Marketing

According to Meyers et al. [4], social media marketing is a means of communication, and building a business's brand is the most effective tool. Mandiberg [5] agrees and says that social media marketing is based on the Internet and websites that humans use for marketing today.

Social Media Marketing is the use of media networks to promote market-specific brands or organizations. Mandiberg [5] agrees and says that Social Media Marketing is based on the Internet. It is used in business cases to let customers know about brand or product features. The primary goal of social media marketing is to produce content that will draw in a large number of new viewers [6].

Creating content that will draw in a large number of new viewers is the primary goal of social media marketing., and it gives users countless conveniences for social interaction and centered networking [7]. This advancement contributes to most marketing activities. Social media has gained its position as a major source for people to refer to a product or service. This marketing also creates content that users can share via social media.

In short, Social Media Marketing shows that the benefits it brings or makes a brand famous for many customers is very easy. In the customer's mind, identify an image of a certain brand. Social media marketing gives the opportunity to develop the right plans and optimize the conversion engine that can create many new trends in the future.

Thanks to the use of social networks, all distances are eliminated and no longer limited by any unwanted agents. Therefore, marketing through social networks such as Facebook, Twitter, blogs, YouTube, Flickr, etc., always has more outstanding features than traditional marketing.

2.2 Purchase Intention (PI)

The user's first purchase process causes the resulting perception. It depends on the selling price, company brand, and customer income and is also affected by other factors such as attitude, understanding, belief, etc. [8].

Purchase intention is a combination of a buyer's buying preference and purchasing ability to perform a behavior, and consciously motivated intention is required [9]. We can also calculate the probability that the consumer intends to buy or not to buy.

Purchase intent is heavily used these days and delivers very good value with measured factors [10]. Therefore, it is considered a method to predict and guide buying behavior.

The key to applying the effect of social media marketing to purchase intention is to see if the level of communication effectiveness that the business applies is fantastic. Is the cost worth what the business is doing to receive?

2.3 Electronic Word of Mouth (e-WOM)

According to Hidayanto [11], e-WOM has a significant influence on finding sources and trusting suppliers. Users generate an intent to share their own experiences through the uploading of videos, comments or references, and relevant reviews of electronic services [12]. Electronic word of mouth also has a powerful impact on purchase intention in the mobile phone industry [13]. Through the benefits that e-WOM brings along with the feelings of users, this will be a premise that has much impact on the goal of information search and trust in businesses providing combined products [14]. As a result, a hypothesis is set in:

H1: Social media marketing electronic word of mouth has a positive effect on smart electric motorcycle purchase intention.

2.4 Social Media Marketing Distractions (SD)

Ducoffe [15] demonstrates dissenting attitudes related to distractions. In the same opinion, in other similar research, advertising also creates discomfort, causes much inconvenience, confuses consumers with many negative reactions and undesirable effects, and reduces purchase intention. Through that, businesses will face many disadvantages when implementing social media marketing [16]. Therefore, the following hypothesis is anticipated:

H2: Social media marketing distractions have a negative impact on smart electric motorcycle purchase intention.

2.5 Social Interaction (SI)

Through the use of the Internet, people are connected regardless of time and distance to communicate with each other and create interaction with various types of advertising where we can be creative and have fun in entertainment. Yaakop et al. [17] state that social media positively influences decision-making, public opinion, and attitude formation toward consumers. The interaction is expressed through the use of images, texts, and videos to increase curiosity and learn, leading to purchase intention if the higher the impact, the higher the purchase intention [18]. Therefore, the following hypothesis is established:

H3: Social Media Marketing interaction has a positive effect on smart electric motorcycle purchase intention.

2.6 Informative Value (IV)

The information provided provides additional product choices for them to make shopping behavior in the most satisfied state [19]. The amount of information that users can accept through advertising is considered important because the information needs to be provided in an accurate, timely, and useful way, and they often ask for it. They must have messages relevant to their needs [15]. Therefore, the following hypothesis is proposed:

H4: Informative value of Social Media Marketing has a positive effect on smart electric motorcycle purchase intention.

2.7 Informative Reliability (IR)

Buyer confidence in a message is based on the source from which they receive the content, the advertising message, which is the hope through marketing-based information. Goldsmith et al. [20] demonstrate the trust impact of trust on attitude towards advertising, brand reputation, and purchase intention. Customers who identify a reliable source of information will, therefore, increase their buying ability to the best level [21]. Therefore, the following hypothesis is proposed:

H5: Informative reliability of Social Media Marketing has a positive effect on smart electric motorcycle purchase intention.

2.8 Advertising Entertainment (AE)

Ducoffe [15] pointed out that this is the key factor affecting consumer attitudes. When the elements in the advertisement create positive information, they will directly affect the perceived value for the viewers, increasing the level of emotion, mood, and attraction as well as creating attractiveness. Guide to consumers. The value of advertising entertainment helps to create deep, engaging links that enhance the purchase intention of customers [22]. Therefore, the following hypothesis is proposed:

H6: Advertising entertainment of Social Media Marketing has a positive effect on smart electric motorcycle purchase intention.

All hypotheses and factors are shown in Fig. 1.

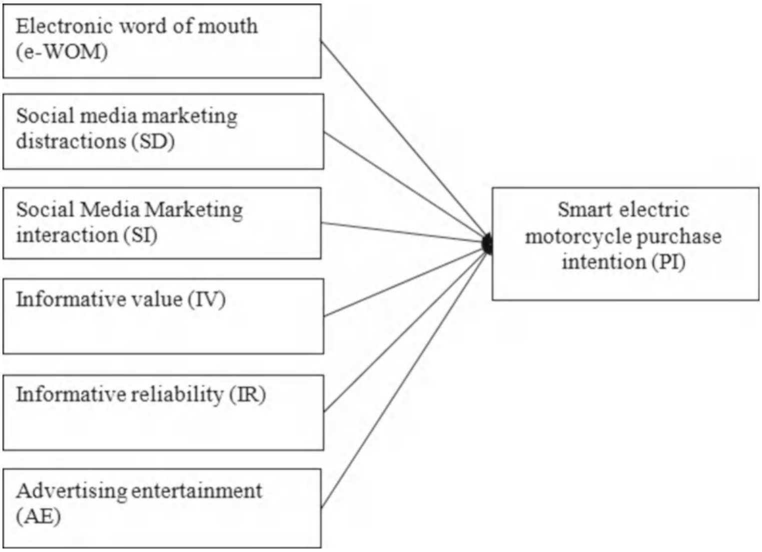

Fig. 1 Research model

3 Method

3.1 Sample Size

The sample size for the topic is determined based on two factors: the minimum sample level is 50, and if it is good, it can be 100 or more [23]. We investigated 243 observations to prevent invalid or non-compliant samples. Table 1 designates the statistics of sample characteristics.

For 243 responders, we assess the degree of consent for the relevant parameters using a 5-point Likert scale. Consequently, Table 2 in this paper used a 5-point Likert scale, with 1 denoting disagreement and 5 representing agreement, to assess the degree of permission for all observed variables.

Table 1 Statistics of sample

Characteristics		Sample size	
		Amount	Percent (%)
Sex and age	Male	82	36.8
	Female	141	63.2
	18–24	14	6.3
	25–30	157	70.4
	31–35	43	19.3
	Above 35	9	4.0
Monthly income	Below 212.13 USD	18	8.1
	212.13–424.27 USD	47	21.1
	466.69–848.54 USD	85	38.1
	Over 848.54 USD	73	32.7
Job	Student	18	8.1
	Staff	47	21.1
	Business	85	38.1
	Other	73	32.7
Reference source	Social network	106	47.5
	Family/friends/relatives	40	17.9
	Marketing	77	34.5

Table 2 Reliability

Factor	α	Item	Code	Mean	CITC
eWOM	0.869	The e-WOM information is important and useful to me	eWOM1	3.32	0.728
		I always trust e-WOM information recommendations provided by friends/relatives on social networks	eWOM2	3.22	0.720
		I often like to consult electronic reviews to choose smart electric motorbikes that appeal to me	eWOM3	3.26	0.693
		People's reviews and ratings on websites check whether the smart electric motorbike I choose is suitable or not	eWOM4	3.44	0.747
SD	0.433	Social media marketing interrupts viewers' entertainment	SD1	3.70	0.164
		Social media marketing content is often annoying	SD2	3.42	0.140
		Social media marketing brings negativity to viewers	SD3	3.87	0.234
		The appearance of social media marketing annoys viewers	SD4	3.80	0.338
		Many advertisements through social networks bring discomfort	SD5	3.75	0.268
SI	0.913	Social media makes viewers excited because it is highly interactive	SI1	3.49	0.735
		I know what I like to buy purely through interactions on the website	SI2	3.57	0.790
		Advertising news is always posted in a brief time	SI3	3.55	0.757
		Social marketing helps me know which products and services are suitable for my personal characteristics	SI4	3.76	0.802
		Social media marketing creates connections between people	SI5	3.59	0.814
IV	0.903	The information channel that social media marketing transmits is updated promptly and accurately	IV1	3.25	0.745
		The information provided is very practical and useful	IV2	3.22	0.757
		Social media marketing provides more information than other advertising channels	IV3	3.29	0.775
		The information provided on marketing is reliable	IV4	3.29	0.754
		Information is updated regularly through marketing	IV5	3.23	0.754
IR	0.883	Social media marketing is trustworthy	IR1	3.18	0.709

(continued)

Table 2 (continued)

Factor	α	Item	Code	Mean	CITC
		Marketing is the reference for shopping	IR2	3.40	0.666
		Social media marketing is more trustworthy than any other advertising	IR3	3.20	0.786
		Social networks should use social media marketing	IR4	3.17	0.800
		Choosing to accept or reject social media marketing advertising is important	IR5	3.21	0.643
AE	0.808	Advertising through social marketing is more interesting than traditional marketing	AE1	3.13	0.617
		Advertising through social media marketing has a very diverse and multi-dimensional content value	AE2	3.29	0.574
		The advertising content that social media marketing brings is very fun	AE3	3.24	0.635
		Social media marketing creates relaxation and comfort for viewers	AE4	3.27	0.677
PI	0.901	Through social marketing, increases the likelihood of purchasing and the desire to buy in the future	PI1	3.10	0.785
		Opinions shared on social networks can stimulate purchase intentions	PI2	3.28	0.767
		The richer the content ads posted on social networks lead to higher purchasing intentions	PI3	3.11	0.792
		Social marketing effectively promotes the intention to buy smart electric motorbikes	PI4	2.95	0.772

$\alpha = \frac{k}{k-1}\left[1 - \frac{\sum \sigma^2(x_i)}{\sigma_x^2}\right]$

3.2 *Reliability Test*

According to Hair [23], it is necessary to test the reliability and value of the scale to avoid inappropriate cases because of other factors arising from the environment. The value of Cronbach's Alpha coefficient is evaluated as follows:

- Cronbach's Alpha < 0.6: Not suitable.
- Cronbach's Alpha 0.6–0.7: Acceptable with new research.
- Cronbach's Alpha 0.7–0.8: Acceptable.
- Cronbach's Alpha 0.8–0.95: Good.
- Cronbach's Alpha ≥ 0.95: Acceptable but not good, may occur by coincidence.

The coefficient showing a relationship between one observed variable in the factor and the other variables is called corrected item-total correlation, or CITC. This coefficient shows how much a given observable variable contributes to the factor's conceptual value. In most cases, a scale with a Cronbach's Alpha coefficient between 0.7 and 0.8 is utilized; an excellent scale is thought to have a coefficient between 0.8 and 1.0. Nevertheless, Cronbach's Alpha values greater than 0.6 can be applied. During the study process, poor observed variables or unsatisfactory scales can be eliminated with the aid of Cronbach's Alpha tool, as these variables have the potential to produce fictitious factors.

3.3 Bayesian Information Criteria

Theoretically, prior information provides the foundation for Bayesian statistics, and its conclusions are combined with observed data [24]. The Bayesian method states that probability is information about uncertainty and that probability quantifies the degree of uncertainty in the information [25]. The Bayesian method is gaining popularity, particularly in the social sciences. Bayesian statistics gained popularity as big data, data science, and computer processing advanced quickly [26]. A useful and useful metric for choosing a comprehensive and simple model is the Bayesian Information Criterion (BIC). Based on the BIC information standard, a lower BIC model is selected. The optimal model will terminate when the least BIC value is attained [27].

4 Results

4.1 Reliability Test

One technique to assess the quality and dependability of the observed variables for the significant factor is Cronbach's Alpha test. This test ascertains whether the conditions for concordance and compatibility among dependent variables in the same major factor are closely related [28]. Cronbach's Alpha coefficient boosts the factor's reliability. A very good scale has a Cronbach's Alpha value coefficient of 0.8–1, a good usage scale has a value of 0.7–0.8, and a qualified scale has a value of 0.6 and above. When a measure has a corrected item-total correlation (CITC) of more than 0.3, it is said to meet the criteria [29].

Table 2 displays that Cronbach's Alpha coefficient of Purchase Intention (PI), Electronic word of mouth (e-WOM), Social interaction (SI), Informative value (IV), Informative reliability (IR), Advertising entertainment (AE) are all greater than 0.6 since they are accepted. Social media marketing distractions (SD) equal to 0.433 are lower than 0.6, so it is received. Table 2 indicates that there is a corrected item-total correlation greater than 0.3 since they are accepted. Purchase Intention (PI4) item equal to 0.295 is lower than 0.3 since it is received. This demonstrates how closely related the items are to each other inside the factor and how they help to evaluate the concept and characteristics of each component accurately.

4.2 BIC Algorithm

A multitude of techniques have been devised and examined to locate association rules inside transaction databases. Additional mining algorithms, such as incremental updating, multilevel and generalized rule mining, quantitative rule mining, multidimensional rule mining, constraint-based rule mining, mining with multiple minimum supports, mining associations among correlated or infrequent items, and mining of temporal associations, were presented. These algorithms provided more mining capabilities [30]. Big data analytics and deep learning are two subfields of data science that are receiving much attention. Large volumes of deep learning algorithms have been accumulated by individuals and businesses, making big data increasingly important [31]. Bayesian Information Criteria, or BIC, was utilized by the R software to identify the optimal model. In the theoretical setting, models have been chosen using BIC. BIC can be used as a regression model to estimate one or more dependent variables from one or more independent variables [36]. When choosing a comprehensive and straightforward model, the BIC is an important and useful metric. A model with a lower BIC is chosen in accordance with the BIC information standard [27, 32, 33]. Every phase of the quest for the perfect model is shown in the R report [34, 35]. BIC's selection of the top two models is shown in Table 3.

The models in Table 3 have one dependent variable and five independent variables. The Electronic word of mouth (e-WOM), Social interaction (SI), Informative value (IV), Informative reliability (IR), Advertising entertainment (AE) have a probability of 100%. Advertising entertainment (AE) has a probability of 68.4%.

Table 3 BIC model choice

PI	Probability (%)	SD	Model 1	Model 2
Intercept	100.0	0.217	−0.883	−0.774
eWOM	100.0	0.055	0.273	0.291
SI	100.0	0.049	0.312	0.316
IV	100.0	0.056	0.258	0.303
IR	100.0	0.059	0.207	0.246
AE	68.4	0.080	0.142	

Table 4 Model test

Model	nVar	R^2	BIC	Post prob
Model 1	5	0.657	−211.723	0.684
Model 2	4	0.646	−210.174	0.316
BIC = -2 * LL + log (N) * k				

4.3 Model Evaluation

Based on the data from Table 4, BIC indicates that Model 1 is the best option because BIC (−211.723) is the minimum. Electronic word of mouth (e-WOM), Social interaction (SI), Informative value (IV), Informative reliability (IR), Advertising entertainment (AE) impact Purchase Intention (PI) is 65.7% ($R^2 = 0.657$) in Table 4. According to BIC, model 1 is the best option, and the probability of the three variables is 68.4% (post-prob = 0.684). The aforementioned analysis demonstrates the statistical significance of the regression equation below.

$$PI = -0.883 + 0.273eWOM + 0.312SI + 0.258IV + 0.207IR + 0.142AE$$

4.4 Discussion

Social interaction is accepted with a Beta coefficient = 0.312, proving that the relationship between the Intention to buy smart electric motorbikes and this factor is in the same direction. So when the social interaction factor increases by 1 unit, the intention to buy a smart electric motorbike increases by 0.312 units, which is the best-influencing factor.

Electronic word of mouth (e-WOM) is accepted with a Beta coefficient of 0.273, proving that the relationship between the Intention to buy smart electric motorbikes and this factor is in the same direction. So when the electronic word of mouth factor increases by 1 unit, the intention to buy a smart electric motorbike increases by 0.273 units, making it the second most influential factor.

The informative value with a Beta coefficient of 0.258 proves that the relationship between the Intention to buy smart electric motorbikes and this factor is in the same direction. So when the Information Value factor increases by 1 unit, the Intention to buy a smart electric motorbike increases by 0.258 units, and this is the third influencing factor.

Informative reliability is accepted with a Beta coefficient of 0.207, proving that the relationship between the Intention to buy smart electric motorbikes and this factor is in the same direction. So when the reliability of information increases by 1 unit, the intention to buy a smart electric motorbike increases by 0.207 units, making it the fourth most influential factor.

Advertising entertainment is accepted with a Beta coefficient = 0.142, proving that the relationship between the Intention to buy smart electric motorbikes and this factor is in the same direction. So when the entertainment-advertising value factor increases by 1 unit, the intention to buy smart electric motorbikes increases by 0.142 units, which is the fifth most influential factor.

5 Conclusions

The results identified these factors that influence social media marketing on purchase intention, including Electronic word of mouth (e-WOM), Social interaction (SI), Informative value (IV), Informative reliability (IR), and Advertising entertainment (AE) with 23 observed variables. The process to perform data analysis and interpret and comment on the results goes through the following stages:

After analyzing the data, it shows that five factors have a strong to low-affected level: social interaction (SI), Electronic word of mouth (e-WOM), Informative value (IV), Informative reliability (IR), and Advertising entertainment (AE). The analyzed results are the basis for the author's recommendations and solutions on social media marketing affecting the intention to buy smart electric motorbikes.

These analysis results show that smart electric motorbike businesses have not really taken advantage of social marketing to promote smart electric motorbikes. This is most clearly demonstrated through the R2 result (adjusted R2 = 0.657), equivalent to 65.7% determined from 5 factors. Therefore, in addition to these 5 factors, including 23 observed variables, other factors have not been mentioned by the author in this research model.

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Optimizing Traffic Signal Control Using Fuzzy Logic: A Solution for Urban Congestion Management

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Abstract Urban traffic congestion poses a significant challenge to modern cities, leading to increased travel times, fuel consumption, and environmental pollution. To address this issue and improve urban mobility, this research paper explores the utilization of fuzzy logic-based traffic signal control mechanisms. In order to optimize traffic light regulation, this study uses the Analytic Hierarchy Process (AHP) to systematically find the weights of different criteria. The research creates a solid basis for determining the most important criteria by using fuzzy matrices and fuzzy pairwise contrasts. The optimized traffic light signals are built around the resultant weighted criteria, which help prioritize variables that are crucial for improving urban traffic flow and efficiency. Because of its flexibility and capacity to learn from experience, fuzzy logic shows promise as a tool for dynamically adjusting traffic light timings. This study explores the fundamentals of traffic light control using fuzzy logic, including linguistic variables, fuzzy rule bases, rule aggregation, and defuzzification. It demonstrates the flexibility of fuzzy logic in dealing with variables like traffic volume, wait times, and even the weather.

Keywords Fuzzy logic · Urbanization · Traffic management · Traffic light control criteria

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1 Introduction

Fuzzy logic provides a sophisticated and adaptable method for dealing with imprecision and uncertainty in decision-making [1]. True or false is not always the best way to evaluate a statement; this is the essential premise of fuzzy logic. It takes a more practical approach by accepting partial truth values between 0 and 1, embracing the idea of "fuzziness" and so better reflecting the complicated and unpredictable character of real-world problems. Two fundamental components of fuzzy logic are fuzzy sets and linguistic variables. Lotfi Zadeh originally put up the idea of a fuzzy set in 1965 [2]. It goes beyond the rigid membership/non-membership dichotomy that is used in traditional set theory. Instead, items can be part of a fuzzy set to variable degrees. Sets can be defined with more subtle ambiguity when elements have degrees of membership defined by fuzzy membership functions. One component of fuzzy logic, linguistic variables, connects the two extremes of mathematical expression precision and human language's inherent imprecision [3]. They provide a way to make quantitative decisions that also take qualitative factors into account. One example is the use of linguistic variables to convey concepts like "high," "medium," or "low" in a form that people understand, making it easier to explain and analyze complicated systems. In many different fields, fuzzy logic, fuzzy sets, and language variables have proven to be practical. By applying fuzzy logic to control systems, intelligent and adaptive controllers that can deal with uncertain input data and changing situations can be built. Due to its ability to represent human reasoning, fuzzy logic is well-suited for use in decision support systems in industries including healthcare, banking, and transportation [4]. The inherent ambiguity, vagueness, or subjectivity in many real-world problems can be effectively handled by fuzzy systems, which provide a potent tool for solving these problems with greater sophistication and precision. Future technological developments will likely see fuzzy logic and its components used more frequently in routine decision-making, where they will be crucial in enhancing the adaptability and effectiveness of various systems.

A new phase of extraordinary urban development and change has begun as a result of the rapid acceleration of urbanization [5]. Vehicle traffic has risen dramatically due to the exponential growth of cities and the accompanying population and economic activity growth. Although it is a sign of development, the enormous problem of efficiently managing traffic congestion is brought about by this fast urbanization [6]. When it comes to the ever-changing traffic patterns of growing cities, the old-fashioned fixed durations for green lights at junctions just aren't worth it. This has led to an increased awareness of the need to use novel strategies, most notably a system to automatically choose the green light period [7]. This study aims to enhance green light durations dynamically using fuzzy logic inference algorithms to tackle the complex connection between development and traffic congestion. In order to deal with the growing complexity of urban traffic and promote long-term mobility solutions, these kinds of adaptive measures are essential. Although they are useful as a control device, they are too rigid to accommodate the unpredictable flow of urban traffic [8].

The concept of traffic management encompasses a holistic approach to addressing urban congestion and improving transportation efficiency. Owing to the aforementioned issues, numerous studies are being conducted on intelligent transportation systems. This encompasses a wide range of research topics, including traffic control with automated traffic signals [9], fuzzy logic [10, 11], swarm intelligence [11], genetic algorithms [12], and multiagent-oriented networks [10]. A Graphical User Interface (GUI) was created by Taha and Ibrahim [13] to model and assess the efficacy of a fuzzy-based approach in terms of various attributes for the traffic systems navigating roads, such as average waiting time and traffic queue length. In order to integrate connectivity with fuzzy logic, the authors constructed a junction between a road network and a Graphical User Interface (GUI). The user can design a variety of fuzzy logic factors, including input variables, fuzzy rules, inference frameworks, involvement operations, and outcomes. One of the paper's drawbacks was that the authors did not provide a specific method for managing traffic, a mechanism for predicting traffic, or a route recommendations system for drivers. Fuzzy logic was used by Kanan [14] to present the intelligent traffic control technique. The input consisted of the number of cars and their average speed, which defined the small, medium, and large membership functions. The author created several fuzzy rules to assist in determining the output variable values, waiting period, and green light measure. A model for predicting traffic for a specific day and time was presented by Sharma et al. [15]. The day and hour at which traffic is taken into consideration served as the modeling's input. Several fuzzy sets were established in order to accurately forecast the traffic. The triangle membership function has also been applied by authors to input and output variables. Collotta and colleagues [12] utilized a sensor network that can be used without wires to monitor traffic in the present moment. They then combined this data with fuzzy logic to determine the duration of the green light and dynamically regulate traffic at crossings. The total measurement of the line is used to define the importance of every stage of the traffic signal's cycle according to every instruction, which is then utilized for calculating the period of the green light calculation of the period of wait or vehicle count at the segment of roadway in each route when the red light is examined, and then it will determine how long the green light will last. This model's drawback is that it doesn't give the user access to real-time path information. There is uncertainty on whether the driver should take the alternate route or the same one. Fuzzy logic-based traffic management systems represent a paradigm shift in how cities approach this multifaceted challenge. These systems are designed not only to optimize traffic signal timings but also to provide real-time traffic information to commuters, enabling them to make informed choices about their routes and modes of transport [16–18]. Deshpande and Bajaj [19] have blended neural networks and fuzzy logic to anticipate short-term congestion. The five-layer neuro-fuzzy model was presented. The input data of the first stage fuzzies the input by determining the membership function value, which then delivers the crisp value to the second layer. Fuzzy rules were found in the third layer, which is where neurons are learned. After defuzzification, the result was generated on layer five, with the output membership function located on stage four. The proposed approach integrated the benefits of fuzzy logic handling uncertainty and neuronal training for output generation. Deshpande and Bajaj [21] employ the machine technique for learning Support Vector Machines (SVMs) in order to provide an accurate road traffic prediction [20]. Additionally, they use mean square error, root mean square error, and normalized mean square error to measure the efficiency of the system. Cong et al. [22] proposed the traffic movement fore-casting scenario using the Fruit Fly Optimization Process and the Least Squares Support Vector Machine [23] with the goal reach at a more efficient result. Additionally, they contrasted the method's effectiveness with the earlier method. Fuzzy logic's arrival marked a significant change in the status quo of traffic light management. Systems based on fuzzy logic can make instantaneous adjustments to signal timings in response to incoming data. Fuzzy logic is effective for this because it can deal with vague or incomplete data. Fuzzy logic-based traffic signal control has been shown to improve traffic flow and reduce congestion in several studies [24, 25]. Using fuzzy logic, its applications go far beyond tweaking the timing of signals. It is becoming more commonplace in real-time traffic management systems that account for things like congestion, road conditions, and even the weather. In this, we saw how fuzzy logic's malleability in dealing with intricate decision-making was demonstrated by introducing a real-time traffic control system based on it.

Since several criteria oversee managing traffic lights, but since a growing number of criteria will make it difficult to compute inference rules, a process to determine the relative weightage of each criterion is required to select the more appropriate criteria for calculation. To provide insight into the relative relevance of the choice criteria under consideration, Multi-Criteria Decision-Making (MCDM) approaches make use of criterion weights [26]. To assess the relative importance of these characteristics, various models have been developed. Methods such as the Best Worst (BWM), Full Consistency (FUCOM), Level-Based Weight Assessment (LBWA), and Analytical Hierarchy Process (AHP) are well-known. Several types of investigations have made use of the Stepwise Weight Assessment Ratio Analysis (SWARA) approach because of its straightforward and minimally invasive procedure [27]. But its biggest flaw is that it can't verify results using consistency levels [28]. The ability to identify the measurement of consistency has led to FUCOM, BWM, LBWA, and AHP seeing increased use recently. Among the aforementioned methods, the FUCOM algorithm features the fewest pairwise contrasts [29]. Nevertheless, this method's additional calculating step is rather intricate. Two and a half times as many pairwise comparisons are required by the BWM approach as by the FUCOM method [30]. The BWM technique employs a nonlinear min–max model to ascertain the ideal criterion weights. Nevertheless, BWM becomes exceedingly complex when dealing with a large number of pairwise comparisons [31]. The LBWA model [31] is comparable to the FUCOM approach in that it enables the computation of weights using a minimum number of pairwise comparisons. Adding more criteria does not make the algorithm more complicated, which is another advantage of this strategy. In addition to these advantages, the LBWA model should emphasize how the weight coefficients can be further adjusted using the elasticity coefficient to suit the preferences of the decision maker [32]. The AHP, which has been extensively referenced in scientific article [33], is an arrangement for multiple criteria decision-making. Its purpose is to simplify the process of organizing problems with multiple criteria and to offer an unbiased approach to selecting the best solution from multiple choices. According to [34], the AHP method's hierarchical structure allows for more efficient and transparent targeting of each criterion. However, when dealing with subjective human assessments, the ambiguity and vagueness make the AHP approach ineffective. Chang [35] improved this strategy to cope with variability and ambiguity in order to address this issue. By fusing AHP with fuzzy set theory, this modification, also known as Fuzzy AHP, provides more plausible and precise illustrations of the decision-making procedure. The relative importance of each set of criteria can be expressed using fuzzy integers and linguistic variables in fuzzy AHP. The fuzzy AHP is selected for this paper due to its various benefits, one of which is its relatively straightforward handling of multiple criteria. Information that is qualitative as well as quantitative may be processed rapidly by this decision-making tool, and it is also straightforward to grasp. The ease of use and the fact that users can input the judgment data easily without needing complicated mathematical expertise are two major benefits of the AHP technique. Another advantage is that it accepts errors in perceptions and judgments. One of AHP's strongest points is the way it can structure complicated problems into manageable tiers, much like a hierarchy.

In this research, a total of seven sections are present. The Sect. 1 contains the introduction part which includes the basic information about the terms included in the research and the work done till now by researchers. The Sect. 2 includes the definitions related to fuzzy set theory. The Sect. 3 includes the methodology part which specifies the steps that we must take for the criteria preference and implementation of fuzzy logic in traffic light signals. The Sect. 4 contains a hypothetical numerical experiment that enhances the knowledge of the implementation of the methodology. The Sect. 5 gives the result of this research. The Sect. 6 gives the discussion and future scope for new researchers in this area. The Sect. 7 contains the conclusion part which tells about the benefits of the research.

2 Key Concepts in Fuzzy Set Theory

2.1 Fuzzy Set

Classical set theory states that components can only have one of two relationships with sets: either they are members of the set, or they are not. There is a degree scale from 0 to 1 that describes how involved one is with a fuzzy set. To the extent that a piece of data is a part of a fuzzy set, it can be considered that it contributes some of its distinguishing features. $\acute{m}_{\tilde{F}}$ is the association function associated with every component within the fuzzy set $\tilde{F}$, given that X is an ambiguous space is given in the equation (Eq. 1). Next, we have the following notation of a fuzzy set:

$$\tilde{F} = \left\{ \left(x, \underset{\tilde{F}}{\acute{m}}(x) \right) | x \in X \right\} \tag{1}$$

2.2 Membership Function

It assigns an association value to each component in the universal set, which represents the item's likelihood of being in the fuzzy set. The expression that represents it mathematically over $\tilde{F}$ is provided by the equation (Eq. 2):

$$\acute{m}_{\tilde{F}} : X \to [0, 1] \tag{2}$$

2.3 α-Cut

For every fuzzy set $\tilde{F}$, the α-cut, includes all the elements of the universal set X for which the degrees of membership in A are larger than, equal to the given value of α given in the equation (Eq. 3).

$$\tilde{F}^{\alpha} = (x | \acute{m}_{\tilde{F}}(x)) \geq \alpha) \tag{3}$$

2.4 Support

Support refers to those elements in a fuzzy set that have a participation value that is not zero. The fuzzy set represents anything that is only partly contained inside it. The following is the mathematical expression for $\tilde{F}$ is given in the equation (Eq. 4):

$$S = (x | \acute{m}_{\tilde{F}}(x)) > 0) \tag{4}$$

2.5 Convex Fuzzy Set

If an association function ḿ corresponds with a fuzzy set, $\tilde{F}$ and that set is convex as given in the equation (Eq. 5).

$$\acute{\mathrm{m}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\acute{\mathrm{m}}(x_1), \acute{\mathrm{m}}(x_2)\right) \tag{5}$$

for all x_1, x_2 ε X, and all λ ε [0, 1].

2.6 Normal Fuzzy Set

A fuzzy set A is called normal when the largest membership grade obtained by any element in that set is equal to 1.

2.7 Fuzzy Number

Any fuzzy set $\tilde{F}$ satisfying the following four conditions is a fuzzy number on X:

- $\tilde{F}$ needs to be normal.
- $\tilde{F}^{\alpha}$ need to be closed interval.
- The support of $\tilde{F}$ need to have boundaries.
- Fuzzy set to be convex is required.

2.8 Triangular Fuzzy Number (TFN)

A TFN, denoted as $\tilde{T}$is explained in the equation (Eq. 6) [36]:

$$\tilde{T} = (a, b, c) \tag{6}$$

Furthermore, the membership function can be seen in the equation (Eq. 7):

$$\mathrm{m}'_{\tilde{T}}(t) = \begin{cases} 0; & if \quad t \leq a \\ \frac{t-a}{b-a} & if \ \ a \leq t \leq b \\ \frac{c-t}{c-b} & if \ \ b \leq t \leq c \\ 0; & if \quad c \geq l \end{cases} \tag{7}$$

In this scenario, the degree of lack of participation is equal to the degree of participation minus 1.

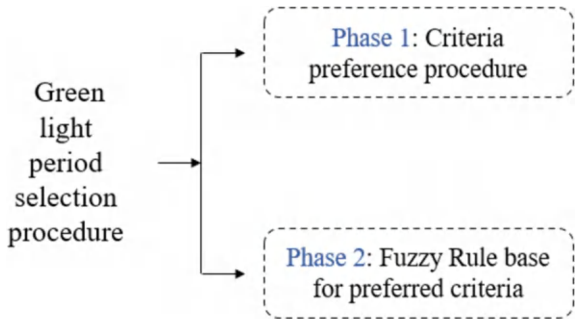
3 Research Methodology

Due to the intricacy of fuzzy logic operations, careful selection is required, especially when dealing with many criteria. Considering the complexity of dealing with several fuzzy sets and rules, it is not feasible to combine all criteria without a systematic method. In order to overcome this obstacle, we offer a weight-based selection process that prioritizes consumer preferences. The Analytic Hierarchy Process (AHP) allows for the assignment of weights to various criteria, which streamlines the selection process. Prioritizing criteria according to their perceived relevance is made possible by this weight assignment, which is in line with the preferences and priorities of end-users. By taking a strategic approach, computational complexities can be reduced, and the criteria used in decision-making can accurately reflect the concerns and expectations of all parties involved. So, Customer preference analysis and green light period management are the two primary components of the proposed method as shown in Fig. 1.

3.1 Examining the Preferences of the Customers

Criteria selection becomes more complicated when customer preferences are factored into this decision-making process. Moreover, figuring out what people want is not a simple task. Our automatic traffic signal period selection system can take input from drivers through a designated mobile app. Having this information at their disposal can help Decision-Makers (DM) understand consumers' views on service and, consequently, choose the right criteria to satisfy customers' expectations. Figure 2 shows the research flow diagram for Phase 1.

Fig. 1 Different phases of research work



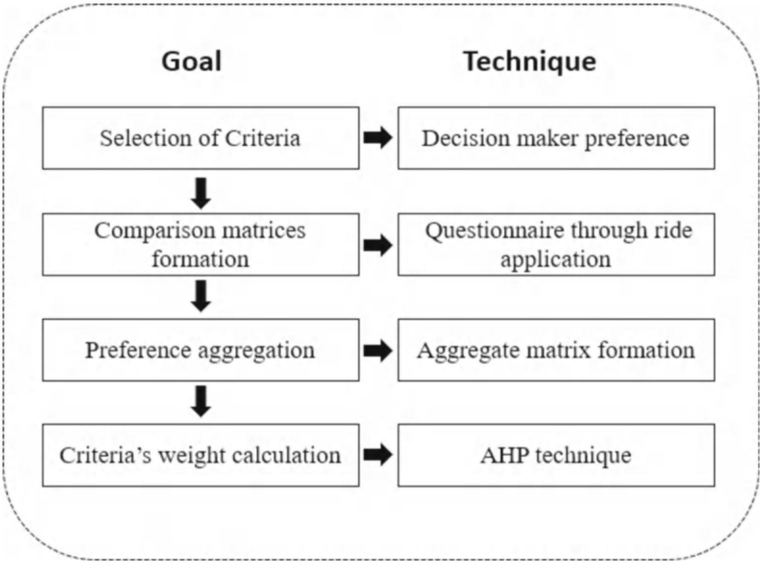


Fig. 2 Research flow diagram for Phase 1

Step 1: Selection of Criteria

An essential part of controlling traffic signals to make sure there is minimal congestion and maximum efficiency is choosing the right length of time for the green light to be on. Different traffic situations, urban planning objectives, and safety concerns may call for different standards for deciding how long a green light period should last. An explanation of each desired criterion follows:

i. *Vehicle Density*: The density of vehicles measures how many automobiles there are per unit area. The duration of green lights should be adjusted based on the number of vehicles on the road in order to avoid congestion and keep traffic moving smoothly.
ii. *Intersection Type and Geometry*: Vehicles' travel times are affected by the geometry and intricacy of the intersection. Think about things like turn lanes and different approaches.
iii. *Pedestrian and Cyclist Activity*: A more inclusive and safe traffic signal design would include enough green time for pedestrian crossings and room for bicycles.
iv. *Waiting time*: Improving traffic flow requires reducing wait times at crossings. Vehicles are expected to spend less time waiting at red lights if green light lengths are adjusted.
v. *Rain intensity*: Inconvenient weather conditions, including rain, can affect the flow of transportation. To compensate for decreased visibility and changed driving conditions caused by heavy rain, the length of time that green lights remain on might be changed.

vi. *Safety concern*: Priority one is making sure that vehicles are operated safely. The durations of green lights should be sufficient to provide safe acceleration and deceleration, therefore decreasing the likelihood of accidents.

Because of the interconnected nature of these factors, a holistic strategy is required when deciding on green light lengths to account for the specifics of each intersection. To further optimize the aforementioned criteria, advanced traffic control systems like adaptive signal control can dynamically alter green times depending on real-time traffic circumstances.

Step 2: Prioritization of Green Light Period Selection Criteria

At this point, We have determined each criterion's strengths and weaknesses based on the perspective of an individual traveler.

By asking users to rank the importance of two travel variables simultaneously, the transportation app may learn which one's passengers value most and use that information to prioritize their experiences.

Coding these choices as fuzzy response matrices would follow collection. After being transformed into fuzzy integers, fuzzy language parameters are used to represent every set of criteria's rank in importance.

The following is the formula for the fuzzy evaluation matrix (Eq. 8), which uses triangular fuzzy numbers and comparisons between pairs as its foundation:

$$\tilde{M} = \begin{bmatrix} \widetilde{m_{11}} & \widetilde{m_{12}} & \dots & \widetilde{m_{1k}} \\ \widetilde{m_{21}} & \widetilde{m_{22}} & \dots & \widetilde{m_{2k}} \\ \dots & \dots & \dots & \dots \\ \widetilde{m_{l1}} & \widetilde{m_{l2}} & \dots & \widetilde{m_{lk}} \end{bmatrix} \tag{8}$$

in which $\widetilde{m_{lk}}$ denotes the relative weight of criterion l corresponding to criterion m, with l = k = 1, 2 …, n.

In this case, the number of matrices obtained corresponds with the number of travelers who agreed to use the upon-request method to complete the questionnaire.

Step 3: Create a Fuzzy Judgment Matrix that is Collected

A group judgment and an approximation of the collective choices can be obtained by fusing these individual traveler opinions once the prioritization of the green light period evaluation criteria has been completed. To do this, we employ the definitions offered by [37] to build an aggregated fuzzy decision matrix. Let $\tilde{m}_{lk} = (a_{lk}, b_{lk}, c_{lk})$ and $\widetilde{M}'$ is the aggregated matrix given in the equation (Eqs. 9–11):

$$a'_{lk} = \min_{m \in \{1,2,..p\}} a_{lkm} \tag{9}$$

$$b'_{lk} = \left(\prod_{m=1}^{p} b_{lkm} \right)^{1/p} \tag{10}$$

$$c'_{lk} = \max_{m \in \{1,2,..p\}} c_{lkm} \tag{11}$$

Step 4: Determining the Scale of Significance of the Criteria

This phase allows us to calculate the weight vector $W = (w1, w2, ..., wn)$ of the n criteria that were chosen, where *wj* represents the weight of the criterion j. We apply the steps given in the following section of [38] to determine the precedence variable of the fuzzy matrix $\widetilde{M'}$ using the extent analysis technique given in the equation (Eq. 12).

$$\widetilde{M'} = \begin{bmatrix} \widetilde{m'_{11}} & \widetilde{m'_{12}} & \ldots & \widetilde{m'_{1k}} \\ \widetilde{m'_{21}} & \widetilde{m'_{22}} & \ldots & \widetilde{m'_{2k}} \\ \ldots & \ldots & \ldots & \ldots \\ \widetilde{m'_{l1}} & \widetilde{m'_{l2}} & \ldots & \widetilde{m'_{lk}} \end{bmatrix} \tag{12}$$

Here; $\widetilde{m}'_{lk} = (a'_{lk}, b'_{lk}, c'_{lk})$.

To begin, utilize fuzzy arithmetic procedures to determine the total number of entries in each row of the fuzzy array $\tilde{M'}$ given in the equation (Eq. 13).

$$RS_l = \sum_{k=1}^{n} \widetilde{m}'_{lk} \tag{13}$$

Next, standardize the sums of the rows by doing the following equation (Eq. 14):

$$\widetilde{N}_l = RS_l / \sum_{k=1}^{n} RS_k \tag{14}$$

Thirdly, determine the level of certainty that $\widetilde{N}_l \geq \widetilde{N}_k$, which is described by the equation (Eq. 15)

$$D\big(\widetilde{N}_l \geq \widetilde{N}_k\big) = \begin{cases} 1 & if \quad b_l > b_k \\ \frac{c_l - a_k}{(c_l - b_l) + (b_k - a_k)} & if \quad a_k < c_l \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

Lastly, the level of possibility for the remaining $(n-1)$ fuzzy values is determined using the following equation (Eq. 16).

$$D(\widetilde{N}_l \geq \widetilde{N}_k | k = 1, 2 \ldots n; l \neq k) = \min_{k \in \{1,2,..n; l \neq k\}} D\big(\widetilde{N}_l \geq \widetilde{N}_k\big) \tag{16}$$

The weight variables of the imprecise array $\widetilde{M'}$ are defined by the equation (Eq. 17), where $W = (w1, w2, w3, ..wl)$

$$wl = \frac{D(\widetilde{N}_l \geq \widetilde{N}_k | k = 1, 2 \ldots n; l \neq k)}{\sum_{i=1}^{n} D(\widetilde{N}_l \geq \widetilde{N}_k | k = 1, 2 \ldots n; l \neq k)} \tag{17}$$

3.2 Fuzzy Rule Base in Traffic Light Control

An essential part of a fuzzy logic-based traffic light management system includes a flexible rule basis. It consists of a set of IF–THEN rules that govern how traffic lights should be controlled based on real-time traffic conditions. These rules are designed to capture the complex and often imprecise relationships between different variables that affect traffic flow. Below is an explanation of the fuzzy rule base, along with a flow diagram illustrating its functioning in a traffic light control system as shown in Fig. 3.

- **Linguistic Variables**

Before creating fuzzy rules, you need to define linguistic variables that represent aspects of the traffic situation, such as "vehicle density," "traffic flow," "waiting

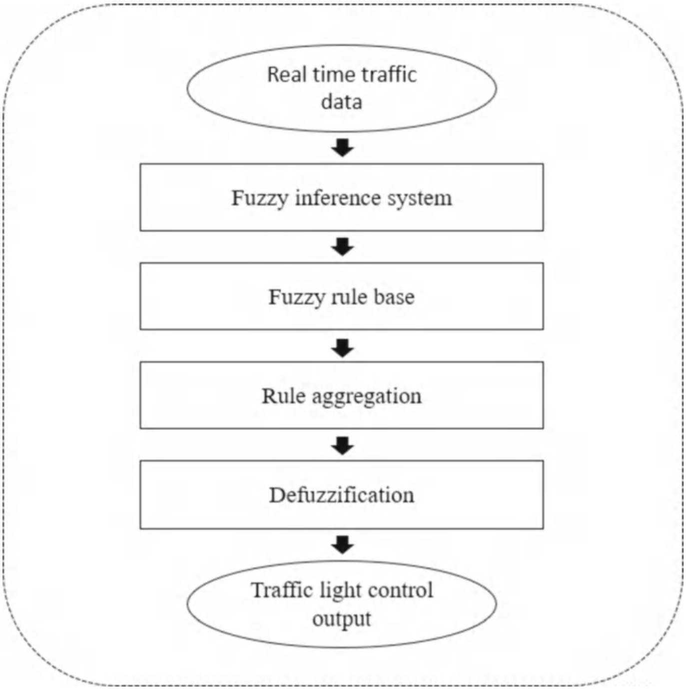


Fig. 3 Research flow diagram for phase 2

time," and "road occupancy." These variables are typically divided into different membership functions, representing categories like "low," "medium," and "high".

- **Fuzzy Rules**

Fuzzy rules are created based on these linguistic variables. Every rule has two parts: THEN (consequent) and IF (antecedent). The circumstances or input parameters are specified in the IF section, and the command activity or output parameter is specified in the THEN section. For example, a fuzzy rule might be: IF vehicle density is high AND waiting time is long THEN increase the green time for that direction.

- **Rule Aggregation**

When multiple rules are applied concurrently, their outputs need to be aggregated. This is typically done by taking into account every rule's "eliminating power", which depends on the extent to which the input variables hold the conditions in the IF part of the rule.

- **Defuzzification**

The aggregated result needs to be converted back into a crisp, non-fuzzy value that can be used to control the traffic lights. This process is called defuzzification. Common methods include centroid defuzzification, weighted average, or the max membership principle.

- Real-Time Traffic Data: This is the input to the fuzzy logic-based traffic light control system, consisting of information like vehicle density, traffic flow, waiting time, and road occupancy.
- Fuzzy Inference System: This component interprets received traffic information in actual time utilizing linguistic variables and fuzzy rules to determine the optimal traffic light control actions.
- Fuzzy Rule Base: The heart of the approach, where linguistic variables and fuzzy rules define the relationships between input conditions and control actions.
- Rule Aggregation: combines the output from several rules while accounting for the extent to which each regulation is met about the intake circumstances.
- Defuzzification: Converts the aggregated fuzzy output into crisp, actionable control signals for the traffic lights.
- Traffic Light Control Output: The final result determines how the traffic lights should be adjusted based on the fuzzy inference.

4 Numerical Experiment

Problem 1: Examining the Preferences of the Customers Here we have a decision-making situation where six criteria, C1, C2, C3, C4, C5, and C6, are being considered. Two reaction matrices, M_1 and M_2, are produced when two decision-makers separately offer their respective comparisons of the criteria. The matrix elements m_{ij}^1 and m_{ij}^2 show the preference scores for the relative comparison of criteria Ci and Cj, as given by the first and second decision-makers respectively. The given matrices should be used to get the aggregated weight preferences for each criterion. Here the estimation values are given from [0, 10] intervals as shown in Tables 1 and 2.

Specifically, we want to find the weight vector W in a mathematical sense so that:

An array of weights $W = (w1, w2, w3, w4, w5, w6)$ exists. When the preferences in both matrices are combined to give the aggregated weight for criterion Ci, then wi is the result. The final weight vector ought to show an all-encompassing perspective that takes into account the feedback from both decision-makers. To determine appropriate and consistent weights for each criterion, the calculation makes use of a technique like AHP, which makes use of the arrays for comparisons between pairs.

Proposed approach: After aggregation of the above two matrices as shown in Table 3.

By using the formula given in the equation (Eqs. 13–17):

$$RS_1 = (23, 32.8, 45)$$

$$RS_2 = (11, 20.7, 33)$$

$$RS_3 = (11, 24.3, 43)$$

Table 1 Response of the first rider toward different criteria

M_1	C1	C2	C3	C4	C5	C6
C1	(1, 1, 1)	(8, 9, 10)	(8, 9, 10)	(6, 7, 8)	(2, 3, 4)	(2, 3, 4)
C2	$(8, 9, 10)^{-1}$	(1, 1, 1)	(6, 7, 8)	(4, 5, 6)	(2, 3, 4)	(2, 3, 4)
C3	$(8, 9, 10)^{-1}$	$(6, 7, 8)^{-1}$	(1, 1, 1)	(4, 5, 6)	(2, 3, 4)	(2, 3, 4)
C4	$(6, 7, 8)^{-1}$	$(4, 5, 6)^{-1}$	$(4, 5, 6)^{-1}$	(1, 1, 1)	(4, 5, 6)	(4, 5, 6)
C5	$(2, 3, 4)^{-1}$	$(2, 3, 4)^{-1}$	$(2, 3, 4)^{-1}$	$(4, 5, 6)^{-1}$	(1, 1, 1)	(4, 5, 6)
C6	$(2, 3, 4)^{-1}$	$(2, 3, 4)^{-1}$	$(2, 3, 4)^{-1}$	$(4, 5, 6)^{-1}$	$(4, 5, 6)^{-1}$	(1, 1, 1)

Table 2 Response of the second rider toward different criteria

M_2	C1	C2	C3	C4	C5	C6
C1	(1, 1, 1)	(8, 9, 10)	(4, 5, 6)	(8, 9, 10)	(8, 9, 10)	(2, 3, 4)
C2	$(8, 9, 10)^{-1}$	(1, 1, 1)	(2, 3, 4)	(4, 5, 6)	(4, 5, 6)	(8, 9, 10)
C3	$(4, 5, 6)^{-1}$	$(2, 3, 4)^{-1}$	(1, 1, 1)	(8, 9, 10)	(6, 7, 8)	(8, 9, 10)
C4	$(8, 9, 10)^{-1}$	$(4, 5, 6)^{-1}$	$(8, 9, 10)^{-1}$	(1, 1, 1)	(2, 3, 4)	(6, 7, 8)
C5	$(8, 9, 10)^{-1}$	$(4, 5, 6)^{-1}$	$(6, 7, 8)^{-1}$	$(2, 3, 4)^{-1}$	(1, 1, 1)	(6, 7, 8)
C6	$(2, 3, 4)^{-1}$	$(8, 9, 10)^{-1}$	$(8, 9, 10)^{-1}$	$(6, 7, 8)^{-1}$	$(6, 7, 8)^{-1}$	(1, 1, 1)

Table 3 Aggregation of matrices given in Tables 1 and 2

$\widetilde{M'}$	C1	C2	C3	C4	C5	C6
C1	(1, 1, 1)	(8, 9, 10)	(4, 6.7, 10)	(6, 7.9, 10)	(2, 5.2, 10)	(2, 3, 4)
C2	(0, 1, 2)	(1, 1, 1)	(2, 4.6, 8)	(4, 5, 6)	(2, 3.9, 6)	(2, 5.2, 10)
C3	(0, 2.2, 6)	(2, 4.6, 8)	(1, 1, 1)	(4, 6.7, 10)	(2, 4.6, 8)	(2, 5.2, 10)
C4	(0, 1.7, 4)	(4, 5, 6)	(0, 2.2, 6)	(1, 1, 1)	(2, 3.9, 6)	(4, 5.9, 8)
C5	(0, 2.6, 8)	(4, 5.9, 8)	(2, 4.6, 8)	(4, 5.9, 8)	(1, 1, 1)	(4, 5.9, 8)
C6	(6, 7, 8)	(0, 2.6, 8)	(0, 2.6, 8)	(2, 3.9, 6)	(2, 3.9, 6)	(1, 1, 1)

$$RS_4 = (11, 19.7, 31)$$

$$RS_5 = (15, 25.9, 41)$$

$$RS_6 = (11, 21, 37)$$

$$\sum_{k=1}^{6} RS_k = (82, 144.4, 230)$$

$$\widetilde{N}_1 = (0.1, 0.23, 0.55)$$

$$\widetilde{N}_2 = (0.05, 0.147, 0.4)$$

$$\widetilde{N}_3 = (0.05, 0.17, 0.52)$$

$$\widetilde{N}_4 = (0.05, 0.146, 0.38)$$

$$\widetilde{N}_5 = (0.07, 0.18, 0.5)$$

$$\widetilde{N}_6 = (0.05, 0.145, 0.45)$$

$$D\left(\widetilde{N}_1 \geq \widetilde{N}_2, \widetilde{N}_3, \widetilde{N}_4, \widetilde{N}_5, \widetilde{N}_6\right) = \min(1, 1, 1, 1, 1) = 1$$

$$D\left(\widetilde{N}_2 \geq \widetilde{N}_1, \widetilde{N}_3, \widetilde{N}_4, \widetilde{N}_5, \widetilde{N}_6\right) = \min(0.78, 0.94, 1, 0.91, 1) = 0.78$$

$$D\left(\widetilde{N}_4 \geq \widetilde{N}_2, \widetilde{N}_3, \widetilde{N}_1, \widetilde{N}_5, \widetilde{N}_6\right) = \min(0.77, 0.99, 0.93, 0.9, 1) = 0.77$$

$$D(\widetilde{N}_5 \geq \widetilde{N}_2, \widetilde{N}_3, \widetilde{N}_4, \widetilde{N}_1, \widetilde{N}_6) = \min(0.89, 1, 1, 1, 1) = 0.89$$

$$D(\widetilde{N}_6 \geq \widetilde{N}_2, \widetilde{N}_3, \widetilde{N}_4, \widetilde{N}_5, \widetilde{N}_1) = \min(0.8, 0.995, 0.94, 0.997, 0.92) = 0.8$$

$$W = (0.196, 0.152, 0.171, 0.15, 0.173, 0.156)$$

Problem 2: Green Light Period Calculation Here is a hypothetical example to include weather factors, vehicle density, and waiting time in our fuzzy logic-based traffic light control. Weather conditions can significantly affect traffic flow and integrating them into the control system can further enhance its adaptability.

Proposed Approach:

Step 1: Data Collection and Preparation

We keep getting the latest information via detectors at the intersection, including vehicle density (V) and waiting time (W) as shown in Figs. 4 and 5 respectively. The membership function for rain intensity and green light period is shown in Figs. 6 and 7 respectively. Additionally, we now collect weather-related data.

- Rain Intensity (R) in millimeters per hour.
- For this example, let's assume we have the following data.
- Vehicle Density (V) = 40 vehicles/min.
- Waiting Time (W) = 60 s.
- Rain Intensity (R) = 5 mm/h.

Step 2: Development of Fuzzy Logic-Based Models

Linguistic Variables Identification:

We fuzzy numbers for linguistic variables for vehicle density (V), waiting time (W), and rain intensity (R) as shown in Table 4. Table 5 displays a variety of the symbols employed in the programming.

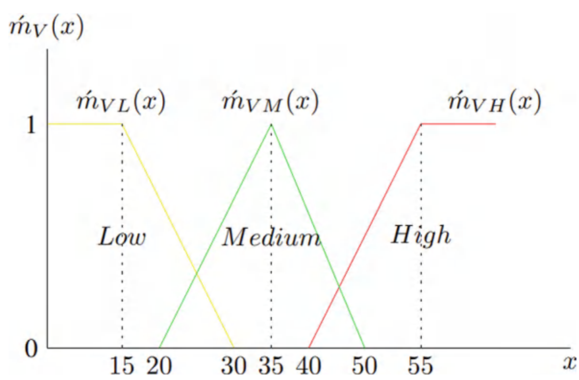


Fig. 4 Participation function for vehicle density

Fig. 5 Participation function for waiting time

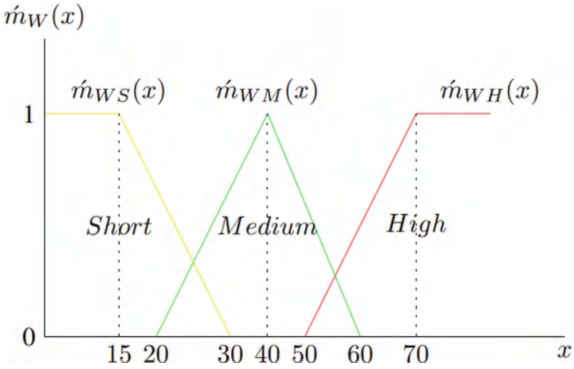


Fig. 6 Participation function for rain intensity

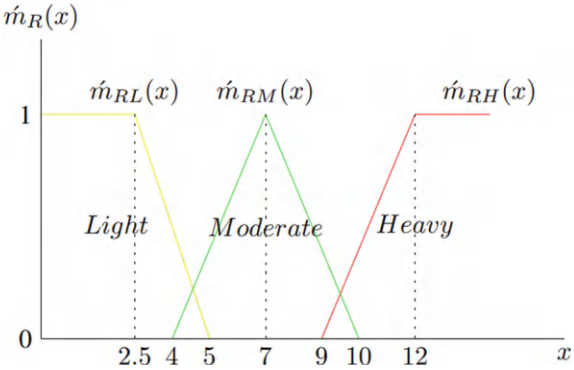
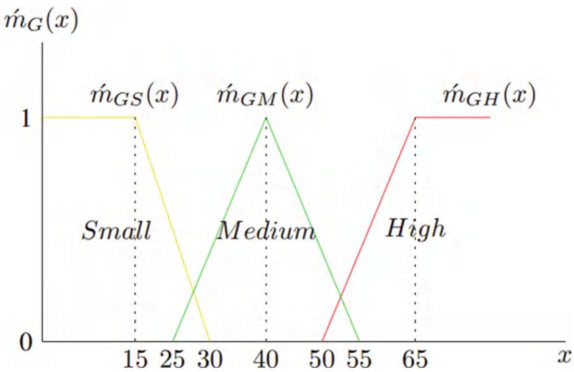


Fig. 7 Participation function for the green light period



Step 3: Fuzzy Inference Rules:

Here 27 fuzzy rules will be held (these will be decided according to DM) which are given below in Table 5.

Table 4 Data used for analysis according to Decision-Maker (DM)

Criteria	Fuzzy number	Linguistic variable
Vehicle density (vehicles/min)	(0, 15, 30)	Low (L)
	(20, 35, 50)	Medium (M)
	(40, 55)	High (H)
Waiting time (s)	(0, 15, 30)	Short (S)
	(20, 40, 60)	Medium (M)
	(50, 70)	Long (L)
Rain intensity (mm/hr)	(0, 2.5, 5)	Light (L)
	(4, 7, 10)	Moderate (M)
	(9, 12)	Heavy (H)
Green light period (s)	(0, 15, 30)	Small
	(25, 40, 55)	Medium
	(50, 65)	High

Table 5 Fuzzy rules decided by Decision-Maker (DM)

Green light period	Cases (Vehicle density, waiting time, rain intensity)
High	LLL, LLM, MLL, MLM, HSL, HSM, HML, HMM, HLL, HLM, HLH
Small	LSM, LSH, LMH, MSH, MMH,
Medium	LSL, LML, LMM, LLH, MSL, MSM, MML, MMM, MLH, HSH, HSM, HMH

Criteria	Level	Membership function
Vehicle density (vehicle/min)	Low (L)	$\acute{m}_{VL}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 30 \\ \frac{(30-\alpha)}{15} & \text{if } 15 \leq \alpha \leq 30 \\ 1 & \text{if } \alpha \leq 15 \end{cases}$
	Medium (M)	$\acute{m}_{VM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 20 \\ \frac{(\alpha-20)}{15} & \text{if } 20 \leq \alpha \leq 35 \\ \frac{(50-\alpha)}{15} & \text{if } 35 \leq \alpha \leq 50 \\ 0 & \text{if } \alpha \geq 50 \end{cases}$
	High (H)	$\acute{m}_{VH}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 40 \\ \frac{(\alpha-40)}{15} & \text{if } 40 \leq \alpha \leq 55 \\ 1 & \text{if } \alpha \geq 55 \end{cases}$
Waiting time (s)	Short (S)	$\acute{m}_{WS}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 30 \\ \frac{(30-\alpha)}{15} & \text{if } 15 \leq \alpha \leq 30 \\ 1 & \text{if } \alpha \leq 15 \end{cases}$

(continued)

(continued)

Criteria	Level	Membership function
	Medium (M)	$\acute{m}_{WM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 20 \\ \frac{(\alpha-20)}{20} & \text{if } 20 \leq \alpha \leq 40 \\ \frac{(60-\alpha)}{20} & \text{if } 40 \leq \alpha \leq 60 \\ 0 & \text{if } \alpha \geq 60 \end{cases}$
	Long (L)	$\acute{m}_{WL}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 50 \\ \frac{(\alpha-50)}{20} & \text{if } 50 \leq \alpha \leq 70 \\ 1 & \text{if } \alpha \geq 70 \end{cases}$
Rain intensity (mm/h)	Light (L)	$\acute{m}_{RL}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 5 \\ \frac{(5-\alpha)}{2.5} & \text{if } 2.5 \leq \alpha \leq 5 \\ 1 & \text{if } \alpha \leq 2.5 \end{cases}$
	Moderate (M)	$\acute{m}_{RM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 4 \\ \frac{(\alpha-4)}{3} & \text{if } 4 \leq \alpha \leq 7 \\ \frac{(10-\alpha)}{3} & \text{if } 7 \leq \alpha \leq 10 \\ 0 & \text{if } \alpha \geq 10 \end{cases}$
	Heavy (H)	$\acute{m}_{RH}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 9 \\ \frac{(\alpha-9)}{3} & \text{if } 9 \leq \alpha \leq 12 \\ 1 & \text{if } \alpha \geq 12 \end{cases}$
Green light period (G)	Small	$\acute{m}_{GS}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 30 \\ \frac{(30-\alpha)}{15} & \text{if } 15 \leq \alpha \leq 30 \\ 1 & \text{if } \alpha \leq 15 \end{cases}$
	Medium	$\acute{m}_{GM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 25 \\ \frac{(\alpha-25)}{15} & \text{if } 25 \leq \alpha \leq 40 \\ \frac{(55-\alpha)}{15} & \text{if } 40 \leq \alpha \leq 55 \\ 0 & \text{if } \alpha \geq 55 \end{cases}$
	High	$\acute{m}_{GH}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 50 \\ \frac{(\alpha-50)}{15} & \text{if } 50 \leq \alpha \leq 65 \\ 1 & \text{if } \alpha \geq 65 \end{cases}$

Step 4: Defuzzification.

The aggregated result needs to be converted back into a crisp, non-fuzzy value that can be used to control the traffic lights. This process is called defuzzification. Common methods include centroid defuzzification, weighted average, or the max membership principle. In our study, we will focus on the mean-max rule.

5 Result

Problem 1 Let us take an example in which we have considered all six criteria, but we have to choose only three criteria for the simplification of calculation and then find these three criteria.

Result:

The preferences of the criterion based on their weight preferences are given by:

$$C1 > C5 > C3 > C6 > C2 > C4$$

Based on this order, we can choose any number of criteria according to our suitability. The preference is given to the criteria having more weight parameters. So, the preferred three criteria are:

$$C1,\ C5,\ C3.$$

Problem 2 Step 1:

Let us take an example of a dataset for which at a time vehicle density is 22, waiting time is 20 s and rain intensity is 5 mm/hr then we have to find the time period of the green light signal.

Result:

Step 2: Membership functions defined for different criteria are shown in Table 6.

Step 3:

Total resulted in combinations: LSL, LSM, LML, LMM, MSL, MSM, MML, MMM.

Step 4:

By applying the min-max rule of defuzzification for all the eight combinations:

- LSL: min (8/15, 2/3, 0) = 0
- LSM: min (8/15, 2/3, 1/3) = 1/3
- LML: min (8/15, 0, 0) = 0

Table 6 Membership functions are defined for different criterion

Membership functions satisfied by vehicle density (22)	$\acute{\mathrm{m}}_{VL}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 30 \\ \frac{(30-\alpha)}{15} & \text{if } 15 \leq \alpha \leq 30 \\ 1 & \text{if } \alpha \leq 15 \end{cases} = 8/15$
	$\acute{\mathrm{m}}_{VM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 20 \\ \frac{(\alpha-20)}{15} & \text{if } 20 \leq \alpha \leq 35 \\ \frac{(50-\alpha)}{15} & \text{if } 35 \leq \alpha \leq 50 \\ 0 & \text{if } \alpha \geq 50 \end{cases} =$ $2/15$
Membership functions satisfied by waiting time (20)	$\acute{\mathrm{m}}_{WS}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 30 \\ \frac{(30-\alpha)}{15} & \text{if } 15 \leq \alpha \leq 30 \\ 1 & \text{if } \alpha \leq 15 \end{cases} = 2/3$
	$\acute{\mathrm{m}}_{WM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 20 \\ \frac{(\alpha-20)}{20} & \text{if } 20 \leq \alpha \leq 40 \\ \frac{(60-\alpha)}{20} & \text{if } 40 \leq \alpha \leq 60 \\ 0 & \text{if } \alpha \geq 60 \end{cases} = 0$
Membership functions satisfied by rain intensity (5)	$\acute{\mathrm{m}}_{RL}(\alpha) = \begin{cases} 0 & \text{if } \alpha \geq 5 \\ \frac{(5-\alpha)}{2.5} & \text{if } 2.5 \leq \alpha \leq 5 \\ 1 & \text{if } \alpha \leq 2.5 \end{cases} = 0$
	$\acute{\mathrm{m}}_{RM}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 4 \\ \frac{(\alpha-4)}{3} & \text{if } 4 \leq \alpha \leq 7 \\ \frac{(10-\alpha)}{3} & \text{if } 7 \leq \alpha \leq 10 \\ 0 & \text{if } \alpha \geq 10 \end{cases} = 1/3$

- LMM: min (8/15, 0,1/3) = 0
- MSL: min (2/15, 2/3, 0) = 0
- MSM: min (2/15, 2/3, 1/3) = 2/15
- MML: min (2/15, 0, 0) = 0
- MMM: min (2/15, 0, 1/3) = 0.

Now, max (0, 1/3, 2/15) = 1/3 which corresponds to the LSM condition and according to fuzzy rules it gives a small time period of green light.

Now, after applying membership functions of a small portion of the green light period along the membership degree 1/3 the crisp values are: $\frac{(30-\alpha)}{15} = \frac{1}{3} => \alpha = 25$

So, the time for the green light signal will be 25 s.

6 Discussion and Future Work

An important step forward in traffic signal control methodology is the use of fuzzy inference rules in conjunction with AHP to choose criteria that influence the entire length of the green light duration. By methodically evaluating the significance of different factors, like environmental conditions, passenger activity, and traffic density, the AHP technique is useful for organizing the decision-making process. To ensure that the riders' interests and needs are taken into consideration during the process of choice, the amounts of weight assigned using AHP provide a clear and reasonable basis for the following fuzzy inference system.

An additional layer of flexibility and versatility is added to the traffic signal control system by including fuzzy inference algorithms in the estimation of green light durations. In order to describe the ambiguous and imprecise data seen in actual transportation situations, fuzzy logic permits the incorporation of linguistic variables and fuzzy sets. Because of the complexity and volatility of traffic dynamics in metropolitan areas, this is of the utmost importance. To improve the system's adaptability to different traffic situations, fuzzy inference rules dynamically change the lengths of green lights according to the language descriptions and fuzzy sets linked with the chosen criteria.

In addition, the combination of AHP with fuzzy logic works effectively for managing urban traffic, which is inherently complicated and fraught with ambiguity. While AHP's organized hierarchy makes sure all criteria are considered, fuzzy logic accounts for the uncertainty in human decision-making and real-time traffic data. By working together, we can optimize traffic flow, reduce congestion, and improve urban mobility as a whole, in addition to overcoming the constraints of fixed-time signal control.

6.1 Future Work

The approach suggested is promising, but there is a need for improvement and further study. To begin, it is critical to continuously validate and improve the fuzzy inference system with real-world data and various urban contexts. By using methods such as machine learning, the system could be able to learn from past traffic trends and modify its rules appropriately, further increasing its adaptability.

Improving the traffic light control system's overall effectiveness may also involve investigating the possibility of integrating new technology like linked and autonomous cars. Better and more accurate traffic control techniques could be the result of research into the effects of these advancements on traffic dynamics and the use of their data in making decisions.

To further understand the suggested system's actual efficacy and any obstacles, it would be beneficial to conduct a test trial in real-world urban environments. The deployment of experimental solutions might be facilitated by collaborations with transportation organizations and city planning authorities. This would allow for incremental improvements based on consumer evaluations and actual performance.

7 Conclusion

Ultimately, a strong and promising method for controlling traffic signs is the combination of fuzzy inference rules for determining green light durations and the Analytic Hierarchy Process (AHP) for criterion determination. The AHP-provided hierarchical decision hierarchy allows for an open and methodical evaluation of criteria, laying the groundwork for future adaption using fuzzy logic. To optimize traffic flow and blockages in metropolitan areas, the fuzzy inference system improves the adaptability and understanding of traffic signal control by handling imprecise input and adapting to fluctuating traffic patterns.

The results of this study demonstrate the necessity of an all-encompassing approach that considers the relative importance of criteria as well as the dynamic aspect of traffic conditions. By combining AHP with fuzzy logic, a versatile and adaptive system is created that can handle the intricacies of urban traffic management and react to problems as they arise.

In this research, we investigated the feasibility of using fuzzy logic to improve traffic signal management. As a method for dealing with the challenges of traffic management and optimization, fuzzy logic shows promise due to its capacity to deal with imprecise and uncertain data. We developed a fuzzy inference system to examine how variables like vehicle density, waiting time, and rainfall intensity affect the decision to lengthen or shorten the time that green lights are on at traffic signals.

Our research illuminated the complex relationship between these variables and their effect on traffic patterns. Maintain Green Time, Decrease Green Time, or Increase Green Time were the three possible courses of action that might be deduced from the fuzzy inference rules that were based on linguistic terms and membership functions. Due to the interconnected nature of real-world situations, it's important to remember that the results weren't always ironclad.

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Optimizing Energy Efficiency in Smart Grids Using Deep Fuzzy Nets: A Comprehensive Approach to Power Regulation and Control

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Abstract In order to achieve sustainable and effective energy management, smart grids are now essential. Improving smart grid performance in terms of energy efficiency through the regulation and control of power generation, transmission, and distribution is a major concern. The Deep Fuzzy Nets (DFN) method combines deep learning and fuzzy logic to find the most efficient way to use smart grid energy. The suggested method deduces the intricate interrelationships between smart grid system characteristics by means of a deep learning architecture. The DFN method is appropriate for practical energy management applications since the fuzzy logic part deals with data uncertainties and imprecisions. In a smart grid environment that is always changing, the suggested method can improve energy efficiency while providing accurate predictions. The suggested method using deep fuzzy nets achieved a critical success index of 96.54%, a prevalence threshold of 92.37%, a sensitivity of 91%, and a specificity of 94.55%. Several energy systems have put this method to the test, and the results show that it can increase efficiency at the system level while still letting consumers manage their own energy consumption. Energy researchers

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are still primarily focused on optimizing intelligent grids' energy efficiency; DFN could be a powerful answer to this problem.

Keywords Energy efficiency · Smart grid · Optimization · Electricity transmission · Deep fuzzy nets · Advanced metering infrastructure (AMI)

1 Introduction

Modern innovations in the distribution of electrical power have culminated in smart grids. Smart grids provide for better control and performance of electricity by integrating innovations in IT with conventional power transmission systems [1]. Finding alternative energy solutions is becoming more and more crucial as populations rise and urban areas become additional densely inhabited. Improving these smart grids' energy efficiency is a great place to start. Utilizing smart meters and appliances is one approach to maximize the energy efficiency of a smart grid. Using high-precision sensors called intelligent meters, homeowners and business owners may keep a careful eye on their electrical consumption in real time [2]. The smart grid enables more efficient operation of smart equipment. Air conditioners and refrigerator-freezers are two examples of such networked home appliances. To further maximize smart grid energy efficiency, load management techniques can be implemented. Programs like these are crucial for regulating peak load times because they let utilities change energy prices to get people to switch to using less power during off-peak hours [3]. In addition to lowering carbon emissions, these initiatives make it possible to change energy consumption in order to avoid costly standby generation. Utilizing sophisticated demand response capabilities, smart grids can further maximize energy efficiency. One way to accomplish this is by responding to real-time energy prices by turning on or off individual loads; this helps to lower pricing during peak hours [4]. By implementing demand-side activities like peak load shedding and energy storage, these strategies can further contribute to lowering energy costs. In spite of deregulation and peak-hour pricing, renewable energy sources continue to be useful since they provide more energy supplies and, in certain instances, lower energy prices [5]. Incorporating this cutting-edge technology into smart grids allows utilities to maximize energy efficiency while simultaneously decreasing consumer bills. But public and private organizations must keep cooperating to maximize energy efficiency. Utilities can create affordable, long-term energy solutions for all by promoting energy literacy, raising public awareness, and incentivizing renewable energy investment [6]. In order to maximize smart grid energy efficiency, there are a number of technological considerations, some of which will be discussed further below. Energy losses during power transmission from generator to consumer are the main concern. Flexible power cable designs that can be adjusted to decrease transmission temperature and use contemporary insulation materials can minimize transmission losses. Grid load losses can be further decreased with the use of upgraded technology like HVDC and sophisticated power line communications [7]. The requirement for smart energy systems that can adapt to customer demand and run efficiently and flexibly is another technological hurdle to smart grids reaching their full potential for energy efficiency. As an example, load frequency management has the potential to immediately control the grid's supply and demand balance. It is possible to utilize distributed energy resources (DER) like wind turbines and solar PV to manage power supply and demand fluctuations at the same time. To make smart grids as energy efficient as possible, advanced digital technology development is also essential [8]. To save money on costly human interventions, intelligent grids can respond automatically to unanticipated demand changes with the help of predictive models developed using big data analytics. With blockchain technology, data can be stored securely and protected from corruption or manipulation, leading to increased trust in the new grid system and more transparency. Optimizing smart grids' energy efficiency requires fixing a number of technological problems. Research and development into smart energy systems, more flexible power cable designs, and advanced digital technologies is required for smart grids to work at peak efficiency [9]. For smart grid energy efficiency improvement, the DFN method is a potent tool for both public and commercial companies. Organizations can maximize their energy efficiency by rationally and systematically tackling complex and ever-changing situations [10]. This method integrates Deep Learning (DL) with Fuzzy Logic (FL), a well-established Artificial intelligence (AI) concept, to take into account numerous parameters at once. The capacity to include data from numerous sources, such as consumer usage patterns, market prices, and weather is the main contribution of the DFN method. Improving decision support and identifying complicated correlations are both made easier by combination these data sources with knowledge acquired from them. With integration of more data sources into smart grids, the complication of energy utilization grows, making it more difficult for companies to make accurate long-term energy usage forecasts [11]. Conventional optimization methods and traditional forecasting techniques are challenged by the demands of large datasets (such as real-time energy usage). When processed with advanced machine learning algorithms, large-scale smart grid data generated by Internet of Things devices could potentially provide valuable insights. Computer analysis and learning from this data allows for improved power distribution management, more precise energy demand estimates, and improved grid failure detection and abnormality detection [12]. By incorporating Internet of Things (IoT) devices like smart meters, sensors, and actuators into smart grids, consumption, real-time data on energy construction, and grid situations possibly acquired. In order for machine learning procedures to generate accurate forecasts and assessments, this data is crucial. Internet of Things also enables automated and remote control of grid systems, which boosts their efficiency and reliability [13]. Energy efficiency, cost savings, and grid performance may all see a major boost with the integration of state-of-the-art machine learning and IoT technology into smart grids. By implementing these technologies, utility companies have a better chance of enhancing energy sustainability, decreasing downtime and outages, and optimizing and controlling energy distribution. As the need for renewable energy sources increases, these technologies can help integrate them into the grid, making it smarter and more resilient [14]. The DFN method employing Self-Organizing Maps

(SOM) optimizes the energy efficiency by iteratively refining the model, enhancing user-friendliness, and reducing computing time. By leveraging clustered nodes, SOM illustrates the interdependencies among variables, facilitating stakeholders' understanding of elements impacting energy system performance. This approach enables better design and development of technologies aimed at increasing energy efficiency. In the context of smart grid energy optimization, the DFN method, combined with SOM and Fuzzy Logic, proves invaluable by allowing firms to analyze and optimize the system while considering various data sources and uncovering complex relationships. However, smart grids must recognize the method's drawbacks, such as high computational demands, sluggish convergence rates, and potential failures due to system nonlinearity, to effectively harness the benefits of Deep Fuzzy Nets.

2 Related Works

A building's energy consumption can be managed by an AI program [15]. Over time, Artificial intelligence algorithm is programmed to optimize the building's energy performance by predicting the ideal utilization of energy. To do this, information regarding the building's energy use as well as weather and environmental conditions are required. Based on this data, the AI algorithm can take action, like altering the ventilation or temperature to save energy. Buildings can improve their efficiency and reduce their energy use through this method. In the discussion of AI methodologies, authors focused on one approach that can optimize intelligent grid energy resources [16]. In order to reduce electricity costs and optimize grid power efficiency, this solution makes use of FL to enable self-learning algorithms that can forecast how vehicles would charge. The fuzzy technique can be used to design strategies for identifying the optimal charging times and for reducing or stopping charging to avoid peak prices. In their discussion of advanced ML techniques, researchers found a way to prevent and detect energy theft in intelligent grid networks using AMI [17]. Deep learning models can use advanced metering infrastructure data sets gathered from linked smart meters to spot irregularities and suspicious behaviour that may indicate power theft. The models obtain pre-existing data (voltage magnitude, meter usage history, etc.) and modify it using feature engineering to produce purpose-built features. The models can use supervised learning, unsupervised learning, or mix of the two to identify bad actors and suspicious conduct. These models can also identify operational environment anomalies like physical meter manipulation or high energy usage. Home energy management systems integrate ideas, setups, and technology to help homeowners track, evaluate, and improve their energy consumption [18]. Improved home energy consumption efficiency, lower energy bills, and supplementary services like electrical device control and safety monitoring are all possible with a home energy management system (HEMS). In order to monitor energy consumption and produce reports, homeowners can install home energy management systems (HEMS), which usually include hardware, software, and communication protocols.

A method that integrates data-driven DL models with more conventional deterministic and stochastic forecasting models [19]. This methodological combination allows for the use of diverse data as well as a broad range of features to capture the intricate dynamics of energy usage. In their discussion, in order to safeguard intelligent grids and prevent theft of power resources, a new area of research called "broad and deep CNN for electricity theft detection" is gaining traction. In order to analyse data and identify patterns in power consumption, these networks use convolutional layers. Using this data, we can build models that can prevent power theft by detecting it in its earliest stages. Researchers can utilize the network's wide layers to pick up on general patterns of power consumption, and the deep layers to pick up on more complex patterns. A mechanism for identifying anomalous changes in the operating characteristics of an intelligent grid over time [20]. It focuses on the detection of deviations from typical drift patterns, such as seasonality or slow changes over time. Streaming data anomaly detection, dynamics-aware analytics, and kinetic energy-aware feature discovery are the primary methods employed. In their discussion of machine learning techniques, authors outlined a method for predicting future load demand that integrates deep neural networks with an ensemble of fuzzy systems. Compared to more conventional methods, this one should be able to produce more precise predictions. In order to produce more precise forecasts, it employs fuzzy logic to spot trends in past data. In order to make it even more accurate, it incorporates deep neural networks that learn from different kinds of data. When compared to more conventional approaches, this model performs better in load forecasting tasks. To capture the local and seasonal aspects of electricity use, researchers have proposed an architecture of feed forward convolutional neural networks that uses TC with global temporal attention mechanism (GTA) [21]. An adversarial regularization strategy is used to optimize TC-GTA model, which incorporates shared temporal convolutional encoder-decoder construction. Precise short-term energy prediction in big and complicated systems is now possible with this model, which captures various points of temporal info from a variety of consumption statistics, thereby surpassing the limitations of existing models. The model's speed and accuracy are enhanced by its ability to predict power usage within a few seconds. Smart Grids use this kind of learning to make smarter decisions, run more efficiently, and boost the accuracy and performance of predictive analysis [22]. The two objectives of reinforcement learning, which is an extra form of machine learning, are enhancing decision-making by prioritizing long-term rewards and maximizing future rewards over time. Reinforcement learning can help smart grids optimize electric grid processes, improved manage energy storage, and better balance electricity output with demand. A technique for predicting the demand for electrical load founded on past data, present weather, and further inputs. While heuristic methods optimize load estimates in accordance with predefined goals or objectives, deep learning techniques improve their accuracy. Next, the electricity grid's supply and demand sides are fine-tuned using the predictions. Optimal management of electrical demand in an intelligent grid has been addressed in a control [23]. Better control of electricity demand, supply, and pricing is made possible by this method, making it crucial for smart grids. One approach to RNNs has been detailed by authors; it involves "stacked" layers of networks that work together to create additional robust system. Operators can better manage power grids by allocating resources and creating schedules based on the network's very accurate short-term projections of energy load. Regional manufacture and delivery of warm and cool water to multiple buildings in town region is referred to as DHC [24]. It offers a district-level energy delivery system that is both efficient and flexible enough to meet the demands of a wide variety of customers. A potential method for managing PV systems in an intelligent grid setting with non-uniform shading. An excellent complement to current MPPT algorithms, the DRL method may be fine-tuned to increase power distribution precision and account for partial shadowing of PV arrays. In the discussion of the smart grid's dynamic pricing management challenge, authors delve into the solution that involves reinforcement learning and a supply–demand Stackelberg game. To maximize efficiency in relation to demand and supply factors, the smart grid uses this methodology to control pricing and consumption of power. The development of optimal solutions that can withstand environmental uncertainty is made easier by this. Micro grid systems can safely manage their power resources by power markets or other methods. The capabilities of DRNN learning for autonomous learning of system dynamics and approximate dynamic programming for effectively calculating optimal control actions for every period are combined in this type of DEM. By utilizing complex metering systems and grid fraud detection, researchers have tackled a troubling issue. Their proposed method is an entirely new take on CNN and LSTM. This approach searches intelligent grid systems for fraudulent events using machine learning techniques. An accurate method of detecting power theft is provided by the proposed approach. An innovative study was conducted by authors that optimizes smart city traffic flow using fuzzy logic and deep learning models [2]. It aspires to improve smart city transportation as a whole by providing reliable traffic management predictions. A combination of deep learning and ensemble learning, as outlined by researchers, can be used to detect damaged power lines in smart grids more effectively and efficiently. The deep learning part makes advantage of DNN's capabilities to retrieve features and find patterns in the data, while the ensemble learning part employs several models to make better predictions. This combined strategy provides an efficient way for smart grid technologies to detect and locate downed power lines. Authors discussed how smart grids use deep learning techniques for demand forecasting. Due of their ability to manage large and complex datasets, these strategies have shown encouraging outcomes. With the help of RNN and LSTM models, we can adapt to changing demand patterns and produce accurate projections. This is the outcome of being able to control both the provision and the request for electricity. Researchers outlined how integrating DL methods with Coati optimization algorithm might improve the precision of PV/wind power forecasts for smart grid applications. Because the Coati technique optimizes the parameters of the deep learning model, the resulting predictions are more accurate. This makes the concept a good fit for smart grid energy control. Authors talked about how smart meters can be used with advanced analytics and algorithms to classify and locate power distribution network problems. It allows for precise predictions and quick decisions in response to changes in energy supply and demand. This approach has the potential to improve the reliability and efficiency of microgrid energy management. Researchers have introduced hybrid model for smart grid electrical energy consumption prediction. This model combines the best features of many forecasting approaches, such as statistical and AI-based technology. Decision-making and Planning for optimizing energy consumption are both enhanced by taking into account a variety of data sources, and variables which allows for more accurate projections. Authors proposed a method to enhance smart grid intrusion detection using deep learning algorithms and ensemble techniques. It employs a plethora of deep learning models to pick up on different data features, making it a more dependable and precise detection system overall. An ensemble technique is used to integrate the results of various models. An approach to detecting smart grid power theft using deep convolutional neural networks (CNNs) has been detailed by authors using data collected from smart meters. Combining deep learning with smart meter data improves the accuracy and efficacy of electricity theft detection, which in turn strengthens the safety of smart grids. Researchers introduced a method for privacy-preserving clustering in smart grid characteristic analysis. It makes advantage of a distributed learning paradigm, wherein devices save their local data and only share aggregated findings. Protecting personal information in this way allows for more thorough analysis of smart grid operations. A deep learning method developed by authors that utilizes recurrent neural networks and attention processes examines smart grid energy usage trends in order to detect anomalies that could indicate power theft. This method shows potential for enhancing grid security and reducing financial losses through the accurate detection and identification of theft episodes. In order to detect and correct inaccurate readings in an intelligent grid AMI in real-time, authors have investigated a combination of DL and ensemble learning methods. This allows for accurate and timely monitoring of energy use, which improves the effectiveness and reliability of energy management. In order to build intelligent systems capable of handling complex and ambiguous input, researchers cover a variety of computational methodologies. To ensure a steady and effective flow of electricity, smart grid fault detection can make use of fuzzy machine learning and additional forms of soft computing to assess digital data and identify issues. A DL construction, namely a Highway Deep Pyramid CNN, has been described by authors for the purpose of predicting smart grid stability. By utilizing the high-level information generated by each convolutional layer and merging them, grid stability may be captured easily. Complex interactions between several parameters can be captured using these combinations. According to researchers, accurate short-term demand forecasting is crucial for smart grids' energy resource organization. When it comes to accurately predicting short-term demands, hybrid deep learning algorithms have shown promising results. Because these models can deal with complex nonlinear interactions as well as time-series data, smart grid load forecasting becomes more accurate and reliable. Intelligent energy networks with fuzzy logic for energy management -based control provide a number of benefits, including improved efficiency as well as stability and capacity to manage the uncertainty of renewable power sources. Because of its superior decision-making capabilities in dynamic circumstances, fuzzy logic is crucial for grid integration of renewables. By integrating anomaly detection and deep learning algorithms, it precisely identifies cyber assaults and responds to them, ensuring the reliability and security of smart grids. The following problems were found as a result of the preceding thorough investigation. Among them are,

- Demand Side Management: By lowering peak loads, demand side management contributes to Smart Grids' maximum energy efficiency.
- Smart Metering: Effective load management requires smart meters. Through the measurement of energy usage, they can assist in identifying trends of electricity consumption and creating efficient plans for energy conservation.
- Transmission lines with little loss reduces the heat loss in the conductor and the waste of energy when creating a magnetic field.
- Power Factor Correction: Minimizing energy losses due by reactive power is achieved through power factor adjustment, which guarantees that the system's current matches its voltage.
- Renewable Integration: Solar, wind, and hydropower are all examples of renewable energy sources that can be integrated into the power grid to decrease emissions and increase efficiency.
- Power Storage: Batteries and flywheels are two examples of power storage devices that can assist keep power flowing consistently, which in turn improves efficiency.
- Communications Infrastructure: The effective functioning of smart grids relies on a strong and protected communications network. It consists of automated meters, sensors, and controllers that gather data for optimization purposes.

An innovative strategy for optimizing Smart Grid energy efficiency, DFN integrate DL with fuzzy logic. This method has the potential to be an effective resource for optimizing Smart Grid energy efficiency. It integrates neural network-based deep learning with fuzzy logic's generalizable decision-making capabilities. FL and deep learning together can optimize energy efficiency with more precision and a more thorough decision-making process. This method can enhance energy efficiency by integrating data from many sources and properly modelling dynamics.

3 Proposed Model

In order to maximize energy efficiency, a grid is a power distribution system that connects numerous power sources, typically from separate ones. The power supply can be made more reliable, emissions of greenhouse gases can be decreased, and chances for cost reductions can be found. Energy efficiency optimization can create better usage of current resources and guarantee the redistribution of excess power back into the main electrical grid by taking advantage of the grid's connectivity. Locations that use a lot of power but have few renewable energy options may benefit the most from this. Along with these essential advantages, optimizing the grid can also aid in determining which electricity sources are causing a decrease in overall efficiency and fixing them. Figure 1 shows the schematic block diagram of the suggested model.

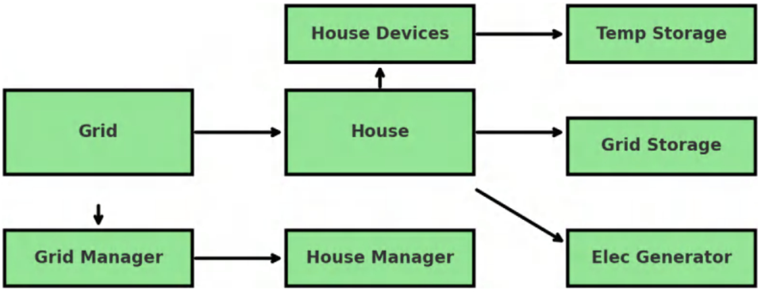

Fig. 1 Schematic block diagram of proposed model

Control software for electricity grids is known as a grid controller, and its purpose is to maximize systems' efficiency. They are utilized for the management of energy systems that are either connected to the grid or not. It is possible to improve energy consumption efficiency and reduce energy waste through the automation of power grid operations that grid controllers do. Another capability of grid controller is the active monitoring and management of DER, which include renewable power sources like wind and solar. By assessing energy consumption and adjusting their control algorithms appropriately, grid controllers aid in the efficient operation of the network. Better and more consistent power can be produced by optimizing the transmission and DER, and grid controllers can make this happen. A household appliance and controller can optimize energy efficiency and help homeowners lower their energy bills and consumption. Everything from air conditioners (AC) to heating and cooling systems, laundry machines, and lighting systems can be linked to single console. Following this, the controller notifies the homeowner of their energy use so that they can maximize efficiency and cut costs by adjusting the settings. Apart from that, it is possible to set the controller to automatically turn off appliances when they are not in use and track energy consumption throughout the house. Because of this, energy waste may be minimized and house efficiency can be maximized.

3.1 Proposed Algorithm

A method developed for machine learning; Deep Fuzzy Net method (DFNA) aims to maximize energy efficiency. Data analysis, pattern recognition, trend identification, and the development of efficient energy solutions are all accomplished through the application of fuzzy logic and neural networks. DFNA's ability to precisely evaluate the present system and propose modifications that may lead to enhanced energy efficiency makes it useful for optimizing energy efficiency. In order to automatically optimize energy consumption, DFNA considers a number of factors, including environmental conditions, system state, and predicted behavior. Because of this, this technique can be used to find the energy systems that are operating at their most efficient. Algorithm 1 demonstrates the features of the suggested model,

Algorithm 1 Deep Fuzzy Net Algorithm

Begin
Initialize the Nets population and Loads
Evaluate the deep fuzzy based on the fitness function
Do
  I = 0
  {
    Calculate the distance of each net from the best using Eq (4)
    Update the Deep Fuzzy Nets position using Eq. (5)
    Evaluate the fitness function
    Sort and rank the Deep Fuzzy Nets according to fitness function values
    If solution improved:
    {
      Go to the start of the loop
    }
    Else:
    {
      Apply GA operators
    }
Increment I by 1
}
While the stopping criteria are not met

When optimizing energy efficiency, data-driven methods like kernel-based fuzzy regression or fuzzy sets for variable approximation can be used to initialize the load of the DFN Algorithm. A functional flow diagram of the suggested model is displayed in Fig. 2. Fuzzification involves converting human-readable data sets from fuzzy sets into the original data sets, such as the energy consumption of the system. The data is then classified into separate sets by applying rules to analyze trends and determine the relationship between input and output values. During the inference stage, the input–output map is calculated by combining the outputs of the rules and the Fuzzification steps.

This map is then utilized as the starting point for the optimization process and as the foundation for the DFN. Before running the DFN Algorithm, ensure that the training data is successfully loaded and correctly formatted, containing the right amount of records. Analyze the algorithm's parameters and structure, verifying that problem-specific hyper parameters, such as the number of hidden layers and neurons, are appropriately set. Once the parameters and structure are confirmed, train and validate the algorithm by splitting the dataset into training and validation sets. Use this dataset to train the Deep Fuzzy Net algorithm (DFNA) and evaluate its precision and effectiveness. Finally, test the trained model using real-world data to assess its practical performance. Following these procedures might help to determine how the

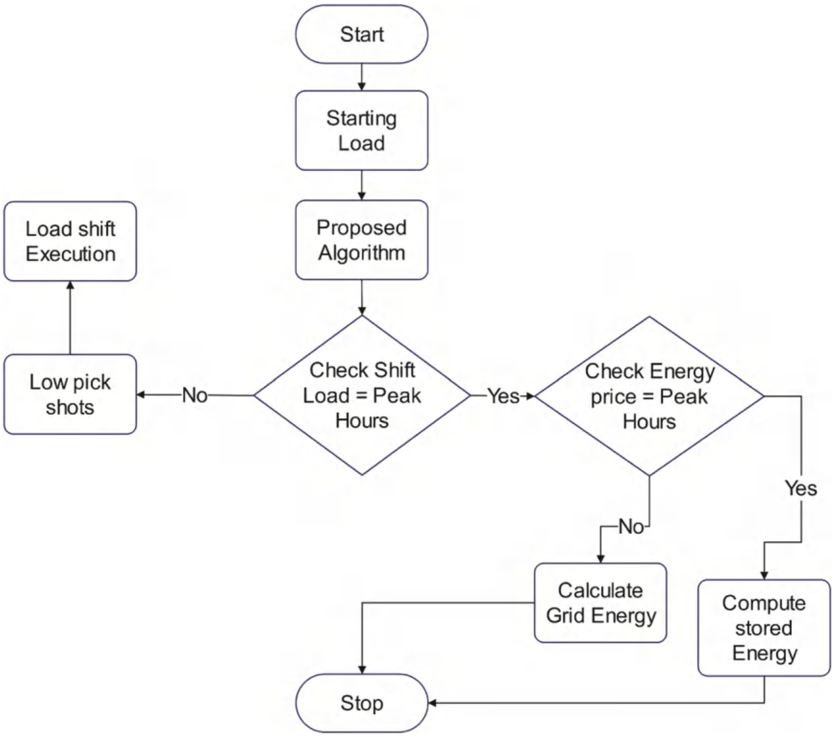

Fig. 2 Functional flow diagram

load on the dataset will impact the results of optimizing energy efficiency utilizing DFNA. The DFNA for Energy Efficiency Optimization may access a variety of online energy market properties, including cost evaluation websites, to research energy prices and highest hours. Examine the rates and peak hours offered by each local energy provider by checking their online price reports. It can use DFNA to assess the data once it knows available energy options and their prices or highest hours. Find the most economical and efficient energy choice by running the data through the algorithm. Get the price and power consumption levels as close to one another as possible.

3.2 Noise Handling and Variable Adjustment

The suggested methodology expertly manages data noise, ensuring precise and trustworthy outcomes. Data noise is made up of insignificant and unrelated information that might skew the real data and make the model underperform. Disruptions in power, bad weather, and human mistake are all potential outside sources of noise in a smart grid setting. The suggested DFN model takes a multi-stage strategy to deal with data noise. The model employs various pre-processing strategies, including data cleaning, feature selection, and normalization, to reduce noise and enhance data quality, ensuring the data used for training is reliable and relevant. It utilizes a deep learning technique with multiple interconnected layers to process the data, allowing the model to detect complex patterns and correlations while mitigating the impact of noise. Additionally, the model leverages Fuzzy Logic (FL) to handle data imprecision and uncertainty, enabling it to make intelligent decisions based on fuzzy rules that effectively model and quantify data noise. It uses regularization and adaptive learning, among other techniques, to withstand noise and prevent overfitting. Researchers need to fine-tune and alter the parameters of the suggested model. Things to remember when adjusting the smart grid model's parameters for deep fuzzy networks are:

1. Training data: Training the model on a big, diverse dataset that accurately represents real-world smart grid system dynamics is optimal. When there is enough data to train the model, it can adapt to different situations and make accurate predictions.
2. Number of layers and neurons: Broadness and depth are two key metrics that measure a network's performance. A deep network, which consists of many layers and a large number of neurons, is able to accurately portray the complex relationships in the data. If add too many layers or neurons to a model, it can overfit, meaning it works fine on training data but struggles to generalize fresh information. Adjusting the amount of layers and neurons and finding the sweet spot are thus of paramount importance.
3. Learning rate: The capacity of the model to update weights during training is determined by this parameter. Although increasing the learning rate makes the model learn faster, it also increases the likelihood of instability and overshooting. A slower learning rate, on the other hand, may result in more consistent convergence throughout training. Finding the best requires playing around with different learning rates.
4. Fuzzy membership function: The membership functions used by DFN model soften the hard input values that are sent into network. Each input variable's range, shape, and number of membership functions can greatly affect the model's performance. This is why, depending on the input data's qualities, it is critical to pick and tweak these functions with care.

A more accurate representation of the complex intelligent grid systems' dynamics can be achieved by fine-tuning these model parameters.

4 Analytical Discussion

An essential part of the method is the analytical conversation that takes place in Deep Fuzzy Nets Approach to Optimizing Smart Grid Energy Efficiency. Creation of accurate models and methodologies to assess the energy effectiveness of various smart grid technologies is made possible by this. Experts can ensure the maximum efficient energy usage in intelligent grid system by discussing various issues and possible solutions analytically. Through critical analysis, these professionals are able to spot any flaws and issues affecting the system's efficiency, and the benefits and drawbacks of other approaches. By doing so, we can be confident that the intelligent grid system is operating at peak efficiency and that the best possible option is selected. The many layering functions are illustrated in Fig. 3.

Data entered into a deep fuzzy net algorithm by the user is processed by the input layer of the algorithm. Normalization, encoding, and feature extraction are just a few of the activities that this layer executes on the supplied raw data. By performing these actions, the data is compressed into a more manageable format, which the DFN may then use to generate optimization rules for energy efficiency. In order to prepare the data for deep fuzzy net usage, the input layer could also make use of various methods for preprocessing and data transformation. First, we have the matrix representation of the grid electric load (A) in Eq. (1)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1p} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{q1} & a_{q1} & a_{q3} & \cdots & a_{qp} \end{bmatrix} \tag{1}$$

Rows and columns have been created in the matrix. Each time stamp is represented by a row in the matrix, while the feature index is shown by a column. The following Eq. (2) shows the format of a time sequence.

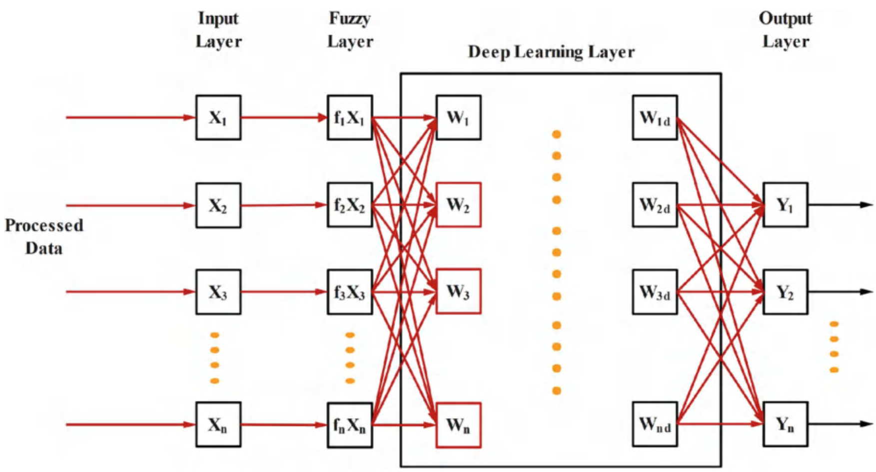

Fig. 3 Different layering functions

$$A = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix} \tag{2}$$

where,

$$l_x = \left[a_{x1}a_{x2}a_{x3}\ldots a_{xp}\right], x \in [1, q] \tag{3}$$

The next step is to calculate the actual data sequence, which can be represented by Eq. (4).

$$\lambda_y^*(x) = \frac{\lambda_y(x) - \min\lambda_y(x)}{\max\lambda_y(x) - \min\lambda_y(x)} \tag{4}$$

The following equation, Eq. (5), represents the Gray correlation coefficient once normalization is complete.

$$F_b\left(\lambda_0^*(x), \lambda_y^*(x)\right) = \frac{\Delta_{\min} + \mu\Delta_{\max}}{\Delta_{y_0}(x) + \mu\Delta_{\max}}, \mu \in (0, 1) \tag{5}$$

$$\Delta_{yo}(x) = \left|\lambda_0^*(x) - \lambda_y^*(x)\right| \tag{6}$$

$$\Delta_{\max} = \max_{y,x}\left|\lambda_0^*(x) - \lambda_y^*(x)\right| \tag{7}$$

$$\Delta_{\min} = \min_{y,x}\left|\lambda_0^*(x) - \lambda_y^*(x)\right| \tag{8}$$

The following is the computed correlation grade in cases where μ is a distinguishing factor:

$$\Delta_{\min} = \min_{y,x}\left|\lambda_0^*(x) - \lambda_y^*(x)\right| \tag{9}$$

$$E_y\left(\lambda_0^*, \lambda_y^*\right) = \frac{\sum_{x=1}^{q} F_b\left(\lambda_0^*(x), \lambda_y^*(x)\right)}{q} \tag{10}$$

An integral part of the method, the Deep Fuzzy Net's fuzzy layer represents approximations and uncertainties in the optimization problem of energy efficiency. Fuzzy neurons are well-suited to tackling complicated issues with a high degree of uncertainty because they mix the benefits of artificial neural networks with FL. By providing a charting from input characteristics to output values, the fuzzy layer models the fundamental patterns in energy efficiency optimization problem and is utilized in this technique. After that, we can utilize this mapping to see how various solutions fare across a variety of input values. As may be seen in the following, the grid features

$$l_y = a_{x1}a_{x2}a_{x3}\dots a_{xp-q} \tag{11}$$

The time slot is represented by (11) and dropped features by q. The following Eq. (12) calculates the feature relevance of a grid based on Algorithm 1.

$$\sigma^Q[\alpha_x] = \sigma^Q[\alpha_x] - \frac{\sum_{y=1}^{x}\text{diff}\big(A, a^*, T_y\big)}{q*y} + \frac{\sum_{F\neq\text{class}(\lambda)}\text{diff}\big(A, a^*, Q_y(F)\big)}{q*y} \tag{12}$$

The randomly chosen grid is located at F and a*. The values of c and G, as shown in the following Eqs. (13) and (14), are the most important factors in feature selection.

$$\hat{\sigma}^c = \frac{\hat{\sigma}^c}{\max(\hat{\sigma}^c)} \tag{13}$$

$$\hat{\sigma}^G = \frac{\hat{\sigma}^G}{\max\big(\hat{\sigma}^G\big)} \tag{14}$$

When designing networks to use less energy, optimization using Deep Fuzzy Net algorithms' DL layers is crucial. Building a network of variables and weights using deep learning layers allows one to assess environmental inputs, make predictions, and decide on energy-efficient strategies to execute.

$$\mu_x = \begin{cases} \text{keep} & \sigma^c[\mu_x] + \sigma^G[\mu_x] > \sigma \\ \text{discard} & \sigma^c[\mu_x] + \sigma^G[\mu_x] \leq \sigma \end{cases} \tag{15}$$

To process and store network rules for decision-making, DFN use DL layers. In particular, the layers learn the environment's complexities and patterns through interaction with external input data, then use that knowledge to decide how to allocate resources most efficiently in terms of energy consumption.

$$B = (b_1, b_2, b_3, \dots, b_P)^V \tag{16}$$

A pth electric load connected to the grid is denoted by b_p. The next step is to calculate the Eigen values using Eq. (17).

$$\beta_r = U^{k*}r, \beta \geq 0\&r \in i* \tag{17}$$

The characteristic space between grids is represented by i*, and β stands for Eigen value. U is matrix covariance of B.

$$U^{l^*r} r = \frac{1}{P} \sum_{y=1}^{p} \psi\left(b_y\right), \left\{r * \psi\left(b_y\right)\right\} \tag{18}$$

$$\sum_{z=1}^{p} \psi(b_z) = 0 \tag{19}$$

In this context, φ represents the feature space mapping of the grid data. These layers are constantly optimizing energy efficiency by adjusting to new data and changing environments.

$$\beta\{\psi(b_z), r\} = \left\{\psi b_z, U^{k*} r\right\} \tag{20}$$

$$r = \sum_{y=1}^{p} \beta_y * \psi\left(b_y\right) \tag{21}$$

$$P_{y,x} = \left\{\psi\left(b_y\right), \psi(b_x)\right\}, \forall y, x \in [1, P] \tag{22}$$

In addition, the layers offer adaptive response, meaning they can modify configuration variables on the fly to react to new circumstances. In addition to discovering patterns and developing new ways to enhance energy effectiveness, the layers of a DFN algorithm also offer a self-learning process.

$$\beta \sum_{y=1}^{P} \beta_y * P_y = \frac{1}{P} \sum_{y=1}^{P} \beta_y \sum_{y=1}^{P} P_{py} * P_{yi} \tag{23}$$

$$(\beta P) * (P\beta) = P^2 \beta^2 \tag{24}$$

$$\left\{r_y, r_x\right\} = 1, \forall y, x \in [1, P] \tag{25}$$

In energy efficiency optimization, the desired output, such reduced energy consumption or better resource utilization, is generated by the deep fuzzy net algorithm's output layer.

$$\left\{\sum_{y=1}^{p} \beta_y^p * \psi\left(b_y\right), \sum_{x=1}^{p} \beta_x^p * \psi(b_x)\right\} = 1 \tag{26}$$

$$\sum_{y=1}^{p}\sum_{x=1}^{p}\beta_y^p * \beta_x^p\{\psi(b_y), \psi(b_x)\} = 1 \quad (27)$$

$$\sum_{y=1}^{P}\sum_{x=1}^{P}\beta_y^p * \beta_x^p * P_{yx} = 1 \quad (28)$$

The output is the product of feeding the inputs from the previous layers into a fuzzy-logic algorithm.

$$\{\beta_p, P * \beta_p\} = 1 \quad (29)$$

$$\beta_p = \{\alpha_p, \beta_p\} = 1 \quad (30)$$

The concealed stratum within the DFN Approach is employed to enhance the precision of the learning process in order to the optimize energy efficiency in Smart Grids. Role of hidden layer is illustrated in Fig. 4.

Better decisions regarding optimizing energy efficiency are made possible by using this hidden layer to record and abstract complicated relations in the data.

$$m_p = \{r_p, \psi(b)\} = \sum_{y=1}^{p}\beta_y^p\{\psi(b_y), \psi(b)\} \quad (31)$$

$$P(b, i) = \exp\left(-\delta\|b - i\|^2\right) \quad (32)$$

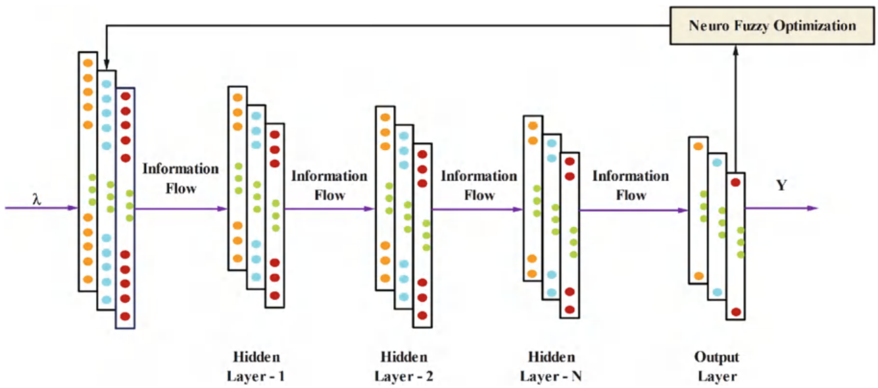

Fig. 4 Hidden layer functions

$$i(b, n) = \sum_{x=1}^{A} n_x * \lambda_x(b) + \psi \tag{33}$$

The optimization process can be guided by the important information that can be extracted by the hidden layer. The network is able to generalize better and learn from previous data due to the hidden layer, which ultimately leads to more efficient optimization methods.

$$W_I(n) = \frac{\sum_{y=1}^{A} \left| G_y^u - i(b, n) \right|_\sigma + \eta n^2}{A} \tag{34}$$

$$b = \begin{cases} 0, & if \left| G_y^u - i(b, n) \right| < \sigma \\ \left| G_y^u - i(b, n) \right| & \text{Otherwise} \end{cases} \tag{35}$$

The process of fuzzification converts an input set of numbers into a less exact collection of numbers, or a fuzzy set. Deep Fuzzy Nets (DFNs) use a process called Fuzzification to convert the smart grid's numerical inputs into fuzzy sets. Using these fuzzy sets, DFN infer fuzzy rules using FL, and finally, smart grid optimization settings are adjusted using these rules to maximize energy efficiency.

$$i\left(b, \delta, \delta^*\right) = \sum_{y=1}^{p} \left(\delta^* - \delta\right) E^*\left(b, b_y\right) + \alpha \tag{36}$$

$$E^*(b, i) = \sum_{y=1}^{A} \lambda_y(b) * \lambda_y(i) \tag{37}$$

An integral part of the suggested method, fuzzy logic gives the system a range of values instead of simply one numerical parameter to work with during optimization.

$$\begin{aligned} E(\delta^*, \delta) = & -\sigma \sum_{y=1}^{p} \left(\delta_y^* + \delta_y\right) + \sum_{y=1}^{p} G_y^u \left(\delta_y^* + \delta_y\right) \\ & -\tfrac{1}{2} \sum_{yx=1}^{p} \left(\delta_y^* + \delta_y\right)\left(\delta_y^* + \delta_y\right) E^*\left(b_y, i_y\right) \end{aligned} \tag{38}$$

A more sophisticated optimization method is made possible, enabling the smart grid to be fine-tuned with greater accuracy in terms of energy efficiency.

$$E_l(y, x) = \begin{cases} c_l(y, x);\ (x) \le II(C_l(y)) \\ j_l(y, x);\ (x) > II(C_l(y)) \end{cases} \tag{39}$$

$$E_{l+1}(y,x) = \begin{cases} E_l(y,x) \text{ ifQ }_C(E_l(y)) \leq II(j_l(y)) \\ x_l(y,x), \text{ Otherwise} \end{cases} \tag{40}$$

Fuzzy nets often use a stack of fuzzy layers as their output layer in order to get to the desired result. This improves the output layer's ability to counsel the user's ultimate decision by considering various inputs and scenarios.

$$E_l(y,x) = \begin{cases} c_l(y,x), \text{ if } \frac{J_l(y)}{J_l(y_{\max})} \leq II(C_l(y)) \\ j_l(y,x), \text{ if } \frac{J_l(y)}{J_l(y_{\max})} > II(C_l(y)) \end{cases} \tag{41}$$

An essential part of the suggested method for optimizing smart grid energy efficiency is defuzzification. Applying it at the end of procedure transforms fuzzy rules—which are not precise or precise enough—into clear, actionable, and interpretable decisions. It entails using fuzzy set and optimization solution's technique to transfer fuzzy association functions to a level of crispness for every regulation.

$$II(C_l(y)) = \frac{\frac{1}{Q_v(C_l(y))}}{\frac{1}{Q_v(C_l(y))} + \frac{1}{Q_v(J_l(y))}} \tag{42}$$

$$II(J_l(y)) = \frac{\frac{1}{Q_V(J_l(x))}}{\frac{1}{Q_v(J_l(x))} + \frac{1}{Q_V(C_l(x))}} \tag{43}$$

In order to increase the smart grid's total energy efficiency, the defuzzification step's choice is utilized to execute the necessary actions. Since the system cannot directly apply decisions taken from fuzzy rules, defuzzification is vital to enable efficient decision-making. Improving the smart grid's energy efficiency relies on this stage.

$$II(C_l(y)) = \frac{Q_V(J_l(y))}{Q_V(C_l(y)) + Q_V(J_l(y))} \tag{44}$$

$$II(J_t(y)) = \frac{Q_V(C_l(y))}{Q_V(J_l(y)) + Q_V(C_l(y))} \tag{45}$$

In addition to processing data as needed and displaying the final results, the output layer is in charge of these tasks. Technology known as Energy Efficiency Optimization in Smart Grids allows for optimization of both conventional and alternative energy sources to achieve the highest possible level of energy efficiency. By controlling peak demand and encouraging more energy efficiency, this device can lessen the environmental effect of energy consumption. It aids in cost reduction by improving management of energy consumption and related expenses, for example peak demand charges. It also helps utilities anticipate and react to shifts in energy demand, which in turn reduces waste and boosts system reliability. Tools and insights for finding, monitoring, and optimizing energy management methods are provided by Energy Efficiency Optimization procedures through dynamic pricing and predictive analytics. In addition to saving money and helping the environment, this technology can increase efficiency and decrease emissions.

5 Comparative Analysis

In order to measure how well the projected Deep Fuzzy Nets Approach (DFNA) works, it was compared to several existing models and algorithms, including the following: DDLM, SLFA, OVCA, ENNM, DLDF, COA, and FLEM, which stands for fuzzy logic-based energy management. This case makes use of the Smart Grid data set and simulation tool MATLAB 2023 to produce the desired outcomes (Table 1).

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5.1 *Calculation of Sensitivity*

Finding the values of the goal function for each combination of input parameters allows one to calculate the sensitivity of DFN Algorithm in energy efficiency optimization. After that, determine the sensitivity score for each parameter by comparing the objective function values with and without variable changed to original value, and then dividing the result by original value.

Figure 5 displays the sensitivity comparison. When comparing, the current DDLM achieved 62.69%, whereas ENNM achieved 64.44%. There was a sensitivity of 76.99% for SLFA, 78.57% for DLDF, 76.55% for COA, 83.55% for OVCA, and 82.85% for FLEM. The suggested DFNA achieved a sensitivity level of 92.20%. When the sensitivity score is high, it indicates that there is a positive correlation between the parameter's value and energy efficiency. Reducing the value of the

Table 1 Calculation of sensitivity (represent in percent)

Number of inputs	DLDF	DFNA	DDLM	COA	ENNM	OVCA	SLFA	FLEM
100	75.58	97.36	65.96	77.11	68.69	88.45	83.38	74.69
200	74.33	94.95	63.98	76.59	67.36	85.09	80.94	77.38
300	75.89	93.78	63.31	75.09	64.78	83.93	78.79	80.05
400	78.57	92.20	62.69	76.55	64.44	83.55	76.99	82.85
500	79.23	91.70	61.88	78.10	63.59	82.22	73.22	83.55
600	79.90	90.60	61.07	78.58	62.80	81.15	72.15	87.30
700	80.53	89.46	60.39	79.98	60.99	80.17	70.19	90.01

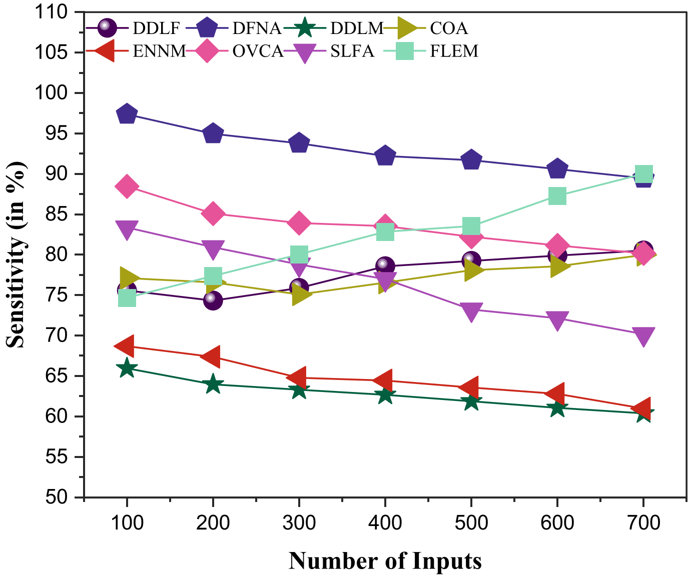

Fig. 5 Sensitivity

parameter is linked to improved energy efficiency when the sensitivity score is negative. Researchers may use the results to find out which parameters have the biggest impact on energy efficiency and how to adjust them for the best savings.

5.2 Calculation of Specificity

In terms of specificity, it refers to the algorithm's capacity to recognize and appropriately represent the characteristics and relationships that impact the energy efficiency of the system. For example, in the event that a system's capabilities encompass photovoltaic, building cooling and wind turbine systems. The efficiency of operations and their characteristics are affected by a complex web of elements. This smart grid inputs comparison is shown in Table 2.

The comparison of specificity is shown in Fig. 6. When comparing, the current DDLM achieved 67.07% and ENNM 65.16%. FLEM achieved 84.05% specificity, OVCA 85.31%, COA 75.63%, DLDF 75.01%, and SLFA 82.15%. The suggested DFNA achieved a specificity level of 95.47%. System operators and designers can optimize energy efficiency through the use of so-called “deep” modelling, which simulates and optimizes energy supply and demand.

Table 2 Calculation of specificity (representin percent)

Number of inputs	DLDF	DFNA	DDLM	COA	ENNM	OVCA	SLFA	FLEM
100	74.86	98.05	71.60	74.19	71.98	90.16	86.72	78.09
200	73.91	99.01	69.09	74.65	70.05	88.90	84.22	79.36
300	73.99	96.86	68.25	75.05	67.85	87.11	83.85	82.69
400	75.01	95.47	67.07	75.63	65.16	85.31	82.15	84.05
500	77.09	94.83	65.84	75.99	63.66	83.85	80.74	86.36
600	78.01	94.28	64.76	74.45	62.08	82.33	79.32	88.62
700	79.16	93.69	62.55	76.85	60.56	80.85	76.86	90.91

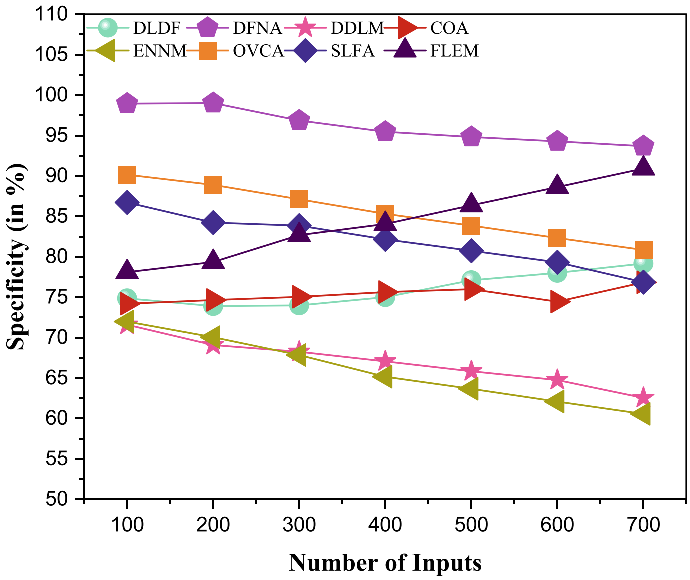

Fig. 6 Specificity

5.3 *Calculating the Prevalence Threshold*

In order to justify its application, the DFNA in Energy Efficiency Optimization must achieve a minimum necessary energy efficiency ratio, which is determined by the predominance threshold. The goal is to make sure the procedure is cost-efficient and produces significant energy savings. Table 3 compares the prevalence threshold in various smart grid inputs. Thresholds for prevalence are compared in Fig. 7. As a point of comparison, the current DDLM hit 56.87%, ENNM 67.36%, SLFA 82.67%, DLDF 78.41%, COA 80.90%, OVCA 87.69%, and FLEM 83.79% prevalence criterion. With a frequency of 94.49%, the suggested DFNA was successful. It is common practice to establish the predominance threshold in relation to the improved system's

Table 3 Calculating the prevalence threshold (represent in percent)

Number of inputs	DLDF	DFNA	DDLM	COA	ENNM	OVCA	SLFA	FLEM
100	74.10	96.09	59.15	76.25	71.85	92.71	86.22	78.73
200	75.18	95.67	58.92	78.88	69.52	90.44	84.60	81.04
300	76.25	95.02	57.22	79.36	68.77	88.89	83.72	81.66
400	78.41	94.49	56.87	80.90	67.36	87.69	82.67	83.79
500	78.98	93.85	55.92	83.49	66.35	86.61	81.88	84.29
600	79.72	93.44	54.27	84.09	64.49	85.95	81.10	84.80
700	82.20	92.33	53.55	85.60	63.70	82.49	80.55	87.19

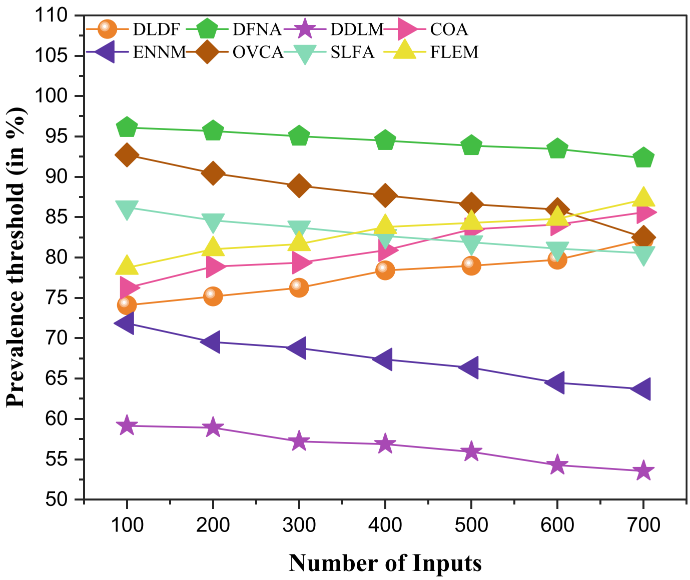

Fig. 7 Threshold for prevalence

energy consumption and the anticipated energy savings from applying the DFN Algorithm. This cut off serves as a filter, limiting algorithm usage to cases when the potential energy savings are sufficient to warrant its implementation.

5.4 Calculating the Critical Success Index

Energy efficiency optimization using the DFNA requires a critical success index (CSI) to assess the efficacy of fuzzy net-based strategy for resolving optimization issues related to energy efficiency. When it comes to optimizing energy efficiency, the CSI for DFNA is a measure of how well the algorithm's optimizer finds the best options. Different smart grid inputs' crucial success indices are compared in Table 4.

Critical Success Index comparison is shown in Fig. 8. From a comparative standpoint, the following were achieved: 62.59% by the existing DDLM, 72.40% by ENNM, 87.52% by SLFA, 72.85% by DLDF, 85.20% by COA, 93.47% by OVCA critical success index, and 87.62% by FLEM. The critical success index of the suggested DFNA was 97.44%. It is determined by comparing system's actual energy efficiency with the optimal energy efficiency result derived from Deep Fuzzy Net Algorithm optimizer. When the CSI score is high, it indicates that the DFNA optimizer was successful in identifying the most energy-efficient alternatives. As a result, maximization of energy efficiency is guaranteed by the DFNA with a high CSI.

Table 4 Calculating the critical success index (represent in percent)

Number of inputs	DLDF	DFNA	DDLM	COA	ENNM	OVCA	SLFA	FLEM
100	68.39	95.81	62.05	79.89	75.54	93.44	89.28	84.61
200	70.20	97.09	62.20	82.11	74.67	93.89	89.50	86.15
300	71.55	97.25	62.44	83.04	72.79	93.21	87.99	86.76
400	72.85	97.44	62.59	85.20	72.40	93.47	87.52	87.62
500	73.90	97.69	62.65	86.16	71.98	93.40	87.25	87.98
600	75.02	97.92	62.81	87.10	71.84	93.39	86.95	89.97
700	75.98	98.95	63.05	88.69	71.12	93.32	86.65	91.05

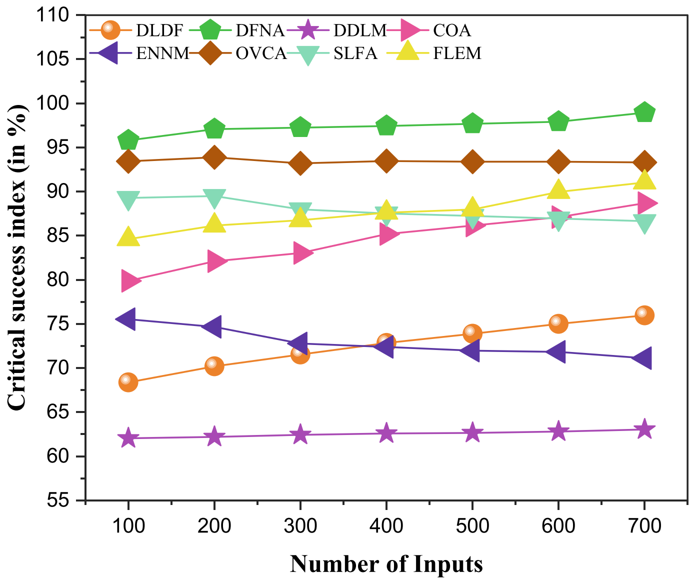

Fig. 8 Critical success index

5.4.1 Real-World Implementation Constraints

For proper training and operation, deep fuzzy networks need massive amounts of data. Data such as this is relevant to smart grids since it contains things like energy consumption, supply, weather, and customer behavior. In addition to being up-to-date and accurate, this data is crucial for the DFN to make sound predictions and judgments. Designing and training these DFN is a difficult process, which adds another challenge. Although deep learning has demonstrated potential in numerous domains, smart grids necessitate particular knowledge and resources for the design and training of deep fuzzy networks. In addition, updating the DFNs in real-time as data streams might be problematic due to the computationally costly and time-consuming training procedure. Another important consideration is how deep fuzzy nets perform in real time. Rapid response to changing conditions and needs is essential in the smart grid context. Because of this, deep fuzzy nets need to be able to process input in near-real-time and make predictions or choices. Thus, it is essential to take the available processing power and speed into account when implementing these algorithms. The current communication and infrastructure of smart grids can be a major obstacle when trying to integrate deep fuzzy nets. There are a lot of moving parts in smart grids, including sensors, devices, and communication protocols. Taking these things into account and making sure deep fuzzy nets are compatible and interoperable is essential for a smooth implementation.

6 Conclusion

To optimize intelligent grid energy efficiency, the Deep Fuzzy Nets method employs a deep learning strategy with fuzzy logic components. Better grid efficiency is possible thanks to this method's enhanced accuracy and adaptability in locating potential areas of energy savings. This method allows for a more complete and accurate analysis of the system since it takes into account multiple variables that are difficult to model using conventional energy optimization techniques. A sensitivity of 91.00%, specificity of 94.45%, prevalence threshold of 92.37%, and critical success score of 96.54% were all achieved by the suggested DFNA. Smart grid energy efficiency optimization might be reshaped by additional developments in Deep Fuzzy Nets. Fuzzy nets' deep learning skills will let them monitor and identify patterns, spot unusual behaviour or changes that can affect power use, and automate its management. To maximize the efficiency of the intelligent grid's power consumption, it will also pave the way for the creation of autonomous systems. Possible future use cases include using operational data like prices, electricity demand, and grid health to automate the analysis of load forecasts and control interventions in real-time. The following are a few of the planned improvements to the suggested model in the future:

- Optimization of energy storage and demand response: Smart grids can use it to maximize the use of energy storage systems, manage demand response, and make predictions about the future. System efficiency can be improved, and intermittent renewable energy sources can be better managed. Peak demand can also be reduced.
- Grid fault prediction and mitigation: It may be programmed to detect and predict potential grid faults, including voltage dips or line breaks, using historical data and real-time observations. This information can be used to proactively prevent power outages and lessen the impact of these problems.
- Real-time demand forecasting and load balancing: A growing amount of real-time data is being added to the smart grid through the installation of smart meters and updated metering equipment. To enhance load balancing and system reliability, accurate demand forecasting is essential, as is adjusting power generation and distribution correspondingly.

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Making Sense of the Messy: How Fuzzy Logic Can Help Us Solve Real-Life Problems

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Abstract The world we live in is rarely black and white. Real-life problems often involve ambiguity, uncertainty, and degrees of truth. Traditional logic, with its crisp boundaries and absolute values, can struggle in these messy situations. Fuzzy logic, however, offers a powerful tool for navigating the complexities of everyday life. This chapter explores the application of fuzzy logic in real-world problem solving. The book chapter begins by introducing the core concepts of fuzzy logic, including fuzzy sets and membership degrees. It then delves into various practical applications of fuzzy logic, showcasing its effectiveness in diverse domains such as AI, Pattern Recognition, Control Systems, Risk Assessment, and Medical Diagnosis. Through a chosen case study, the book chapter illustrates the problem-solving capabilities of fuzzy logic in a specific scenario. The chapter concludes by highlighting the benefits of fuzzy logic and its potential to become an even more valuable tool in tackling the messy problems we encounter in daily life.

Keywords Fuzzy logic · Real-world problems · Decision making · Uncertainty · Ambiguity

1 Introduction

While human beings perform an array of tasks with relative ease (e.g., think creatively, see patterns, make certain kinds of decisions), writing programs and designing machines to perform those same tasks tends to be much more difficult. Traditionally, work in artificial intelligence has sought to make tasks that are very difficult for computers—like playing chess—easier for computers. However, human cognition

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and natural language tend to be very fuzzy, and AI methods designed to capture that essential fuzziness would no doubt be useful as well. Fuzzy logic is a framework for representing knowledge that is both imprecise and uncertain (as opposed to the precise, yes/no statements of classical logical systems). This book uses research carried out in a wide variety of settings—air conditioners, washing machines, railway engineering and physical sciences, to name a few—to demonstrate how the theory of Zadeh's abstracted crisp and fuzzy relations can be used to model and simulate real-world problem solving. removeItem will discuss how decision support is needed when a system is managing a removable data set and how fuzzy logic can be integrated in providing intelligent suggestions. This work makes sense of the messy, providing readers with guidance and concrete examples of how to solve specific problems, a wealth of exercises for testing and developing skills, and a unifying theory of fuzzy logic and its applications [1].

2 Theoretical Framework

2.1 Understanding Fuzzy Logic

Linguistic variables have a very wide range of applications, such as customer preference, doctor's diagnosis, and weather forecasting [2, 3].

A decision must be made, realizing that no matter what the decision, it is based on incomplete, uncertain information. A unique feature of fuzzy logic is the use of linguistic variables, which makes it very close to human decision making. A person will select a general category instead of a precise number, e.g. cool, warm, hot instead of temperature in degrees centigrade. Fuzzy logic defines these variables in a precise manner, for example cool could be 10 °C and warm 30 °C, and thereby enables effective use of the above methods [4].

An inspector rates a particular location according to how suitable it is for termites. This can be called a truth value. If the rating is high, it is to be expected that there is a high number of termites present. Now instead of a precise yes/no decision procedure, he merely adds up the ratings over the entire area to obtain an overall suitability rating [5].

Traditional logic would require a precise definition of what constitutes a termite-prone area—something impossible in this case—and a precise decision procedure, an unsatisfactory method because we really have only a rough idea of what we are seeking to decide. Fuzzy logic provides a simple way to arrive at a decision, without obtaining an exact answer [1].

To understand this better, consider a termite inspector. Most people have only a vague idea of what constitutes a termite-prone area. In fact, if you ask a dozen people to mark it on a map, you will get a poor consensus as to its boundaries. These are the conditions for which fuzzy logic is ideally suited [6].

Fuzzy logic is not as fuzzy as it sounds; it is a way of formalizing approximate reasoning. Its main strength is that it allows the modeling of the type of problems people solve effectively, without requiring an accurate mathematical analysis of the situation [7].

2.2 Benefits of Using Fuzzy Logic

In general, fuzzy systems are often much simpler to design than their traditional counterparts. The focus is shifted from precise modeling of a system to developing a working system for which only a vague model is available. This simplification is due to the abstract nature of fuzzy systems and their ability to directly utilize human knowledge. Often the system can be developed by a knowledge engineer, without the need for expert input, by gathering information from books, papers, and interviews with experts in the field. The system is then tested against data from the expert and modified until it can adequately model the expert's decisions. This is in contrast to the precise modeling of a system from data using techniques from mathematical optimization, which can often prove to be a complex and arduous task [8].

Solving problems and making decisions in the real world, where data is often imprecise and uncertain, can be a difficult task. Developing systems based on fuzzy logic has the potential to make this task much easier. As seen in the previous example, systems can be created which directly mirror the way a human makes decisions, freeing them from having to first quantify the decision-making process in the form of a mathematical model [9].

The previous section outlined the application of fuzzy logic to a problem from the real world. This section concentrates on the advantages of using fuzzy logic to solve such real-world problems. Traditional (non-fuzzy) methods of solving the problem are then used as a benchmark for comparison.

2.3 Limitations of Fuzzy Logic

Unfortunately, when a new system or a new process is designed, it is necessary to test and implement it. Often, the case is that there may be too many rules and too much data for a system to be put into place, then tested to draw new facts from it. This may be due to the fact that the system could be incomplete or it may be that a partially designed system is being tested at each stage. The problem with not having a complete set of data or having to change input data rules is that it will not be consistent. The idea of changing certain rules or certain facts and having to go back and modify other derived facts can be a very awkward reality in expert system maintenance. With the use of truth tables and logical derivations, it is very difficult to represent data consistency and change. This is where fuzzy logic can be a more useful tool, even in its attempt to solve real-life problems [10].

Classical logic is known to be deterministic. The idea is that in a given situation something will either be true or not true, there is no in between and lambda is not used. This type of logic has been a useful tool in the progression of artificial intelligence, especially in the production of expert systems. Expert systems are designed to hold a large amount of specialist knowledge with a human-like problem-solving ability. They have been used in and are applicable to a wide range of areas. However, the main purpose for the use of classical logic is to derive new facts from an existing set of facts. With the given rule or rules, then a 'modus ponens' type of inference rule is used to derive new facts from the rule and the given facts [11].

More than one type of logic can be taken into consideration when dealing with a logic-based problem. A comparison can be made between classical and fuzzy concepts of logic, and it serves us right to purport that the key disparity between these is that classical logic is a special case of fuzzy logic. Classical logic has had a deep and far-reaching influence on the development of computing which has been in effect since its conception. Another notably older type of logic is Boolean and digital logic. These mappings of binary logic are associated with control system applications; they are a special case of multivalued logic and switching theory [1].

2.4 Fuzzy Logic Versus Traditional Logic

Traditional logic provides a rigorous and systematic method for deducing the truth of a statement. The steps of traditional logic are to form a statement, then form a rule, and finally make a logical deduction to prove the truth or untruth of the statement. This style of reasoning breaks down for statements that are not simply true or false, for the Golden Gate fallacy is eminently true and false. Fuzzy logic was designed to deal with this sort of statement, where binary conditions are not adequate to express the truth value of the statement. In the same way, the query "failed due to low memory" is too general a statement, and computationally we have no recourse to a simple boolean rule [12].

Fuzzy logic has moved from the position of academic curiosity to an important field to be dealt with. This discipline of thought is a leading paradigm for dealing with systems which are ambiguous, and its methods are enabling technologies with wide applicability. Fuzzy logic (FL) is a method of reasoning that resembles human reasoning. It makes use of approximate information in an intuitive way. As a distinct alternative to traditional Boolean logic, FL has become accepted in a range of areas where symbolic logic has hitherto been unable to provide viable methods of reasoning. As heuristics become better understood, and as computers became faster, the use of fuzzy logic enjoyed tremendous growth. This shift from early skepticism was primarily due to the collapse of the symbolic logic paradigm under the weight of its own hubris and its lack of practical utility [13].

3 Applications of Fuzzy Logic in Real-Life Problems

This section will give some insights into the applications of fuzzy logic in real-life problems that can be categorized into several broad areas. A number of applications from various real-life problems will be discussed throughout these categories. Due to the nature of the characteristics of fuzzy logic, most of the applications offer a high potential using it, and some have eventually replaced the conventional methods and become the standard for tackling the associated problem [14].

Fuzzy systems have gained widespread popularity in various fields of research and applications, especially in real-life problems. The universality of fuzzy logic offers a broad philosophy to the theory and practical development in real-life applications. In addition, the unique characteristic of fuzzy logic that generally differentiates it from traditional crisp logic is the capability to provide a smooth transition between true and false. It is known that real-life situations are always dynamic and tend to be ambiguous. That is why the robustness of fuzzy logic, providing a graceful degradation or the ability of the system to provide the exact same control strategy despite changes in environmental conditions, has made it very appealing for real-life systems [15].

3.1 Fuzzy Logic in Decision Making

The full potential of fuzzy logic comes out in its ability to process vague information. Emotion, vague information, and sometimes wrong information frequently play a significant role in the decision-making process. Classical logic cannot cope with this kind of input. Much effort has been made in trying to get computers to behave intelligently in decision-making situations. Consider the field of expert systems which began as an attempt to create an AI program that could mimic the decision-making ability of a human expert in a certain specialized field. The energy and resources poured into the expert system field have not yielded very impressive results. In hindsight, workers in the field have realized that the traditional binary logic is not always sufficient to model human decision processes because expert human decision making often involves vague information, anecdotes, and other intuitive processes. Experiments of expert systems show that often a simple decision rule has many exceptions but an expert in that area can still come up with a decision that is usually acceptable if slightly off the wall. The unmatched success of expert humans, while not always logical, is the sort of thing we would like to model and reproduce in computers. Fuzzy logic provides a much better tool for modeling these kinds of processes than traditional binary logic [16].

3.2 Fuzzy Logic in Pattern Recognition

Pure pattern recognition is a topic that many traditionalists use to try to refute the value of fuzzy logic. Although the idea of a set containing the elements that belong to the set in some absolute sense fits a simple pattern recognizer well, real-world patterns are rarely that simple. Often, the same pattern will need to be classified differently based on the context of the situation. For example, think of the throttle control from an automatic washing machine compared to that of a racing car. A fuzzy logic system can naturally represent and deal with this sort of ambiguity. For instance, the membership of the word "fast" representing motor speed is high for the racing car and low for the washing machine. Traditional systems to cater for this would require intricate state machines and/or complex coding of context classifiers. Static pattern recognition that uses the measured variables (as opposed to linguistic variables) can also benefit from fuzzy logic techniques. High-level pattern recognition in causal data can be achieved through fuzzy clustering, which is a means of splitting a data set into groups with varying degrees of membership and relevance to each other. This is a top-down approach, so let's start at the top with an example of recognizing patterns of winning and losing [17, 18].

3.3 Fuzzy Logic in Control Systems

Central to a control system is the concept of an error. An error is the difference between the actual result and the desired result. For instance, we set our thermostat to a desired temperature. The error is the difference between the actual temperature and the desired temperature. If the actual temperature is equal to the desired temperature, then there is no error and the system is said to be in equilibrium. The ultimate goal is to minimize this error. This will result in a cooling system turning off when it reaches the desired temperature or a car changing gears so that the engine RPM matches the desired RPM. The problem with classical boolean logic is that it is not well equipped to handle the concept of error. A classic example would be the implementation of an automatic transmission gear change. This can be highly subjective with many possible answers. The necessity of a precise definite position for each answer makes it hard to model this kind of problem using classical logic [19].

Control systems are present in every aspect of modern life. Ever since the industrial revolution resulted in the automation of previously manual tasks, men have sought to automate other tasks. Control systems manage your microwave oven, your car's engine, the anti-lock brakes on that car, and the cruise control. They maintain the temperature of your refrigerator and the altitude of a plane. The evolution of control systems has continued in a steady progression to the present day where we are now seeking to automate systems to a higher and higher degree with the ultimate goal being a completely autonomous system [12].

3.4 Fuzzy Logic in Artificial Intelligence

Spinner has pursued an approach to encoding fuzzy systems in Prolog. This choice was made for two main reasons, the first being that despite the lack of a precise semantics for fuzzy Prolog, the idea of a logic programming language with built-in fuzziness is very appealing. The second reason is that Prolog is often used in AI teaching, and it would be useful to have a fuzzy Prolog system to use for teaching purposes, particularly in the context of teaching about expert systems. In the short term, we hope that this work will be useful for experimenting with expert systems and approximate reasoning. In the longer term, we would like to make use of this implementation work to carry out further research on the relationship between logic programming and fuzzy logic and to use this to enhance other work on logic programming and AI [20].

Various methods have been used for implementing fuzzy logic in artificial intelligence. Most of these are still in the experimental stage. Some researchers have used fuzzy logic as a more natural way to represent rules and representing a system of fuzzy logic rules as a fuzzy constraint network. This allows the use of fuzzy rules for constraint satisfaction. Others have applied standard techniques from symbolic artificial intelligence to the fuzzy domain, such as automatic theorem proving, in an attempt to implement a true fuzzy logic inference engine. There has been some success in encoding fuzzy systems as constraint logic programs and using finite model theory to reason about the fuzziness. Recently, more work has been done in the implementation of fuzzy systems with neural networks or genetic algorithms. This may turn out to be the most practical way to use fuzzy logic in AI, as both of these learning systems make few assumptions about the information to be learned and can in theory adapt to any system given the right encoding. In general, the best method of implementing fuzzy logic in AI has been a subject of much debate. Considerable research has gone into a general theory of approximate reasoning, using many valued logics, non-monotonic logics, and other extensions of classical logic. Fuzzy logic is just one of many formalisms for dealing with imprecision and uncertainty, and the best ways of using it and other fuzzy techniques to enhance AI are yet to be discovered [21].

3.5 Fuzzy Logic in Risk Assessment

Fuzzy methods can also be easily adapted to complex time-dependent risk models and to decision support systems that make use of the risk assessment to choose the best course of action. Overall, fuzzy logic is a very strong candidate for risk assessment in terms of both representing the knowledge pulled in from other disciplines and providing and integrating the method with tools for actually using the risk assessment to make decisions [22].

Fuzzy logic offers an alternative method, which, while employing much of the formalism of probability theory, provides a system to represent and manipulate knowledge in a way that reflects the natural language evaluation done by experts. This is usually the only evidence available in real-life situations. Fuzzy methods also provide a more natural way to model the risk factor itself, with a fuzzy set whose membership function represents the uncertainty concerning the probability of the undesirable event actually occurring. This compares very favorably with the traditional risk assessment method, where an event's probability is multiplied by its consequences valuation, often leading to rather arbitrary classifications of low, medium, and high risk [22].

Risk assessment is a topic of considerable importance in any field where decisions are made. The consequences of unanticipated results, especially disastrous ones, are to be minimized. Traditional risk assessment methods are based largely on probability theory, with its well-known problems of translating uncertain evidence into the binary terms of true or false [23].

3.6 Fuzzy Logic in Medical Diagnosis

Fuzzy logic has been extended into medical diagnosis due to its availability to deal with imprecise knowledge. In a situation where symptoms rarely point to a single disease and tests are often inconclusive, a fuzzy system which can take account of degrees of possibility can be of great help. A system has been designed for the diagnosis of otitis media, based on aural symptoms and examination of the eardrum. These are entered as fuzzy data with membership functions to represent the degrees to which they indicate certain diagnoses. The system is made radically simple to facilitate use by doctors who are not familiar with the concept of fuzzy logic. It gives a ranked list of the possible diagnoses, which the doctor can take account of along with other evidence and the patient's history in reaching a conclusion. This is a good example of a fuzzy system not giving a definite answer but helping the decision maker to draw his own conclusions more effectively. Similar systems have been developed for diagnosis of various other diseases using a wide range of data, including blood tests and medical images. Fuzzy systems offer a major development in providing decision support for doctors, in some cases coming close to the use of artificial intelligence to model the expert physician's diagnostic methods. With advancing technology and the vast amount of resources in medical data today, these systems are likely to become more complex and powerful in the future [24].

3.7 Fuzzy Logic in Financial Analysis

Results can be applied to highlight definite positions and conditions, though we see that for general market trend following, the movement is not too smooth between short and long positions and would best be treated as a priority list for specific action at any given time.

Fuzzy logic is an extension of Boolean logic by Lotfi Zadeh in 1965 based on the mathematical theory of evidence or degree of belief. Patterned after the human brain, the control unit connected to the car's engine (hence the system's name, Mamdani-type system) interprets the input variables through a series of fuzzy logic functions on a provided range. Output dependent on pertinent average crossover strength is reflected through more fuzzy language processing with ceiling values of the successive moving averages for the given time interval. Linguistic variables and hedges allow for statements with qualifiers that hold degrees of truth [25].

Fuzzy logic has been applied with great effect to systems analysis and decision-support environments. Fuzzy logic simplifies some of the complex mathematical modeling required for decision-making, resulting in models that make more sense to traders. It is our contention that fuzzy logic is a natural language for expressing trading heuristics. We present here an example using moving averages, with the final goal being a complete system for moving average crossovers that is to be implemented in any programming environment [26].

3.8 Fuzzy Logic in Traffic Control

The conventional methods of automatic transmission control can be divided into three categories. The first is the open-loop control system, which is determined by the amount of engine torque and lead/lag shift scheduling with a transition rule chart but does not have shift timing feedback. The second is the closed-loop control system, which has feedback shift timing control using RPM and throttle position but still uses the same transition rule from the open-loop system. The third is to take a hybrid approach of the first two methods. Although these methods have improved shift quality and have been more adaptable to computerized control vs. older purely hydraulic methods, there are still issues in the compromise between shift quality and fuel economy, as well as the real driver's intentions. This is where fuzzy logic steps in [27].

Automatic Transmission Control

In the context of real-life applications, one of the most successful uses of fuzzy logic has been in the control of consumer products, more specifically the automatic gearshift in passenger cars. The introduction of fuzzy control has shown a great degree of success and rapid growth during the past five years, improving shift quality, ratio transition scheduling, and direct throttle to torque converter lock-up. This information will compare the conventional methods of automatic transmission control with the fuzzy logic approach, as well as discuss the design and advantages of fuzzy control [28].

3.9 Fuzzy Logic in Natural Language Processing

Another source of difficulty in natural language understanding is the presence of vague words and rules. All of this makes NLP a formidable example of the real-world reasoning and knowledge applications mentioned earlier.

Fuzzy Logic in Natural Language Processing: The primary mission of natural language processing (NLP) is to improve the interaction between computers and human languages. Essentially, the aim is for the computer to understand the command or question given to it. This understanding is an extremely difficult process and is crucially dependent on the context. Most words have several different meanings; thus, the meaning of a word is dependent on the context. For example, the word "bank" has a different meaning in the sentences "he swam from the bank" and "he cashed a check at the bank" [29].

A most significant development in fuzzy logic has been the provision of a mathematical theory for dealing with vagueness. This has been achieved through the introduction of many-valued logics, the best known of which is the logic of continuous t-norms and t-conorms on the unit interval. Fuzzy logic provides an obvious home for the sort of reasoning done with linguistic rules such as "in most cases," "usually," or "probably not" [30].

Fuzzy logic is essentially a multi-valued logic, in direct contrast to the two-valued binary logic. In place of the crisp sets of classical mathematics, fuzzy logic uses fuzzy sets. A fuzzy set is a class of objects with a continuum of grades of membership. An idea is to a mental concept what a fuzzy set is to an explicit definition [31].

4 Discussion

A fundamentally new and groundbreaking way of thinking about reasoning and the pivotal role of logic has emerged from extensive investigations into the intricate nature of real-world inference. The concept of fuzzy logic, which has been introduced as a profound solution to the challenging question of how we are to effectively handle and navigate the multitude of subtleties and nuances that intricately govern the relentless pursuit of knowledge, has revolutionized the field. In stark contrast to the traditional mainstream symbolic logic and artificial intelligence communities, where the focus has been rather narrow and confined, the realm of fuzzy logic has brought forth a refreshing perspective. By embracing a significantly broader array of problems and scenarios, researchers and experts in the field have undertaken the arduous task of formalizing an incredibly diverse range of reasoning techniques. These techniques, previously only discussed informally, have now been meticulously constructed within a comprehensive framework. Through their tireless efforts, these pioneering individuals have not only shed new light on the immense complexities of human reasoning, but have also catalyzed incredible progress in a plethora of domains. By introducing fuzzy logic, they have paved the way for a profound transformation in how we approach and address the inherently uncertain and imprecise aspects of knowledge and decision-making. Indeed, the impact of fuzzy logic extends far beyond its theoretical implications. The practical applications of this revolutionary paradigm are vast and wide-ranging. Fields such as engineering, mathematics, economics, and medicine have already been deeply influenced by the powerful tools and methodologies that have emerged from the integration of fuzzy logic. Its ability to effectively handle ambiguity and uncertainty has empowered analysts, experts, and decision-makers to tackle complex problems with a level of precision and flexibility that was previously unattainable. As we continue to delve further into the intricacies of reasoning and delve deeper into the remarkable potential of fuzzy logic, there is no doubt that this innovative approach will continue to shape our understanding of the world and revolutionize the way we think and reason. With each new advancement and breakthrough, the boundaries of knowledge are pushed further, unveiling countless possibilities and opening doors to uncharted territories. The journey towards a more comprehensive and nuanced comprehension of the world around us has only just begun, and fuzzy logic stands as a beacon of progress, guiding us towards a brighter future.

5 Conclusion

The hybrid systems, with their combined strengths from both the feed forward systems and other fuzzy systems, significantly surpass them in terms of power when it comes to decision making. In fact, they stand head and shoulders above the rest, as they possess the best architecture to seamlessly convert intricate, multi-dimensional decision rule sets—whether they are ambiguous and uncertain or clear and precise—into a real-time model. Consequently, this denotes that the model itself is either (1) so exceedingly challenging to quantify through the capabilities of a human being or a deterministic algorithm, or (2) lacks any existing information that would enable solving the problem in a definite and predictable manner. Fuzzy logic, without a doubt, has established itself as an exceptionally effective approach that adeptly navigates the complexities of assigning values to ill-defined decision rules formulated by human experts. By doing so, it successfully translates these rules into a highly intricate and sophisticated system, capable of tackling even the most perplexing challenges [32].

This chapter presents the concept of "Fuzzy logic", a logical system that closely emulates human thinking and natural language. The main motivation behind utilizing fuzzy logic is to develop expert systems that replicate human decision-making by incorporating linguistic hedges and multivalued logics. In fuzzy logic, truth values are expressed linguistically, such as "very poor," "rich," "large," and "not very good," and are represented by a value between 0 and 1. Fuzzy logic offers a highly adaptable and intuitive approach to uncertainty representation, serving as a seamless expansion of traditional logic and probability theory [33].

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Application of Fuzzy Logic to Model and Control Rotavirus Spread Among Vaccinated and Unvaccinated Children

Vinita Dwivedi and Subrata Jana

Abstract This study addresses the World Health Organization's recommendation for vaccination, It is firmly supported by studies demonstrating its important effect in lowering childhood diseases and mortality. Specifically, we focus on rotavirus, aiming to deepen our understanding of its transmission dynamics through the development and analysis of a specialized SIR epidemic model. This model divides the child populations into three key compartments are susceptible, infected and recovered. To improve the model's accuracy, we further distinguish between vaccinated and unvaccinated susceptible children, as well as infectious and non-infectious infected individuals. This refined categorization allows for a more detailed exploration of rotavirus dynamics transmission. The model looks at both endemic and disease-free equilibria. It determines local and global stability conditions that depend on the fuzzy fundamental reproduction number (R_0). The centroid approach is used to carry out the defuzzification procedure. Additionally, we perform a local stability study for the endemic equilibrium for $R_0 > 1$ and a global stability evaluation using Lyapunov theory under specific conditions. Numerical simulations using Python software verify our analytical results and provide a thorough evaluation of the model's effectiveness in lowering child rotavirus spread.

Keywords Vaccination · Fuzzy sets · Rotavirus disease

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1 Introduction

Rotavirus is recognized by the World Health Organization (WHO) as the leading cause of severe diarrheal illness in newborns and young children globally, posing a major threat to child health and contributing significantly to childhood mortality, with an estimated 525,000 deaths annually among children under five [1, 2]. After an incubation period of from 24 to 72 h, this highly contagious virus manifests signs such as fever, vomiting, diarrhoea, and abdominal pain. It usually spreads through person-to-person contact and the ingestion of infected food or water [3]. The majority of severe cases occur between the ages of 4 and 23 months, and almost all children will have at least one rotavirus infection before they turn five [3, 4]. There are about 215,000 infant deaths annually due to rotavirus-induced diarrhoea, more than half of which take place in Sub-Saharan Africa [1, 5, 6]. Among the various rotavirus species and child mortality, especially in Sub-Saharan Africa, making it a significant global health concern [7–9].

In 2013, the World Health Organisation advised that all childhood immunisation programs incorporate rotavirus vaccinations in order to counteract the risk of rotavirus outbreaks [10]. This recommendation has led to a decline in rotavirus incidence, particularly in low-mortality countries where the vaccines have shown higher efficacy. However, clinical trials consistently reveal that the vaccine's effectiveness is lower in high-mortality countries [11], prompting various hypotheses to explain the disparity between vaccine performance in low- and high-income settings.

Understanding the rotavirus dynamics transmission, evaluating the efficacy of therapies, and developing public health initiatives have all benefited greatly from the use of mathematical models [4, 6]. By classifying people into distinct vaccination-related states and taking into account a variety of influencing factors, these models also aid in explaining the behaviours of populations that have received vaccinations as opposed to those that have not [12, 13]. In order to test and validate these models, investigate alternative scenarios, and predict the possible effects of different intervention techniques on rotavirus transmission, researchers use estimates from clinical trials and epidemiological data.

1.1 *Initial Considerations for Defuzzification Techniques and Fuzzy Numbers*

Discussing decision-making problems using a fuzzy approach is often more advantageous. According to [14], employing fuzzy numbers offers benefits over a probabilistic approach.

Fuzzy Number: Fuzzy numbers are fuzzy sets that satisfy convexity and normality.

A fuzzy number $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ with $\alpha_1 < \alpha_2 < \alpha_3$ is triangular if the definition of its membership function is

$$\mu_{\tilde{\alpha}} = \begin{cases} \frac{x-\alpha_1}{\alpha_2-\alpha_1}, \alpha_1 \leq x \leq \alpha_2 \\ \frac{\alpha_3-x}{\alpha_3-\alpha_2}, \alpha_2 \leq x \leq \alpha_3 \\ 0, \textit{otherwise} \end{cases}$$

Similarly, trapezoidal, parabolic, pentagonal, and hexagonal fuzzy numbers can also be defined. There are various methods available for defuzzifying fuzzy numbers, with the centroid method being one of the most commonly used. For a triangular fuzzy number, the centroid method is expressed as $d_F(\tilde{\alpha}) = \frac{\alpha_1+\alpha_2+\alpha_3}{3}$.

2 Fuzzy Model Development

Given the inherent uncertainty in the environment, it is often challenging to define certain parameters with precision, which is why we choose to fuzzify some of these parameters. In this context, we fuzzify the parameters π, β_1, σ, representing them as triangular fuzzy numbers: $\tilde{\pi} = (\pi_1, \pi_2, \pi_3)$, $\widetilde{\beta_1} = (\beta_{1_1}, \beta_{1_2}, \beta_{1_3})$, $\widetilde{\beta_2} = (\beta_{2_1}, \beta_{2_2}, \beta_{2_3})$, $\tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$.

In this section, we develop an $S_1S_2I_1I_2\ R$ compartmental model, inspired by the traditional SIR model [15]. $S_2(t)$ indicates the density of vaccinated susceptible people in this model, while $S_1(t)$ reflects the population density of non-vaccinated susceptible individuals. The density of afflicted people who are not vaccinated but are deemed non-infectious that is, they do not aid in the spread of the illness is represented by the symbol $I_1(t)$. R(t) denotes the density of recovered peoples, whereas $I_2(t)$ shows the density of rotavirus-infected and contagious individuals.

The following fundamental presumptions form the basis of this model's dynamics:

i. The peoples birth rate is shown by b. A fuzzy fractional $\tilde{\pi}$ of the newborn people joins the recovered compartment, while the remaining fuzzy fraction $(1 - \tilde{\pi})$ enters the non-vaccinated susceptible compartment.
ii. Individuals that are vulnerable but have not received vaccinations exhibit a migration rate of towards the non-infectious infected compartment and a fuzzy transmission rate of $\widetilde{\beta_1}$ towards the infected (rotavirus) compartment.
iii. At a fuzzy transmission rate of $\widetilde{\beta_2}$, breastfed vulnerable individuals move to the infected (rotavirus) compartment. A fuzzy rate of $\tilde{\sigma}$ is also used to describe the movement of non-infectious infected persons from the non-vaccinated compartment to the infected compartment.
iv. People infected with rotavirus that are not infectious move at a rate of γ_1 to the recovering compartment, whereas infected people recover at a rate of γ_2.
v. The population's overall natural mortality rate is μ, with distinct death rates for the non-infectious infected population (ν_1) and the infectious (rotavirus-infected) population (ν_2).

These presumptions serve as the foundation for the disease transmission modelling, which results in a set of ordinary differential equations that characterise the mathematical model that is suggested and incorporates the previously mentioned variables.

$$\begin{aligned}
\frac{\widetilde{\mathrm{dS_1}}}{\mathrm{dt}} &= (1-\tilde{\pi})\mathrm{b} - \alpha S_1 - \widetilde{\upbeta_1}\mathrm{S_1 I_2} - \mu S_1 \\
\frac{\widetilde{\mathrm{dS_1}}}{\mathrm{dt}} &= (1-\tilde{\pi})\mathrm{b} - \alpha S_1 - \widetilde{\upbeta_1}\mathrm{S_1 I_2} - \mu S_1 \\
\frac{\widetilde{\mathrm{dI_1}}}{\mathrm{dt}} &= \alpha S_1 - \gamma_1 \mathrm{I_1} - \mu I_1 - \nu_1 \mathrm{I_1} - \tilde{\sigma}\mathrm{I_1 I_2} \\
\frac{\widetilde{\mathrm{dI_2}}}{\mathrm{dt}} &= \widetilde{\upbeta_1}\mathrm{S_1 I_2} + \widetilde{\upbeta_2}\mathrm{S_2 I_2} - \gamma_2 \mathrm{I_2} - \mu I_2 - \nu_2 \mathrm{I_2} + \tilde{\sigma}\mathrm{I_1 I_2} \\
\frac{\mathrm{dR}}{\mathrm{dt}} &= \gamma_1 \mathrm{I_1} + \gamma_2 \mathrm{I_2} - \mu R
\end{aligned} \tag{1}$$

with $S_1(0) > 0, S_2(0) > 0, I_1(0) > 0, I_2(0) > 0, R(0) \geq 0$.

Let the entire population size N(t) be, i.e. $N(t) = S_1(t)+S_2(t)+I_1(t)+I_2(t)+R(t)$. Clearly, we have

$$\frac{\mathrm{dN}}{\mathrm{dt}} = \mathrm{b} - \mu N - \nu_1 \mathrm{I_1} - \nu_2 \mathrm{I_2} \tag{2}$$

This reduced system is what it offers:

$$\begin{aligned}
\frac{\widetilde{\mathrm{dS_1}}}{\mathrm{dt}} &= (1-\tilde{\pi})\mathrm{b} - \alpha S_1 - \widetilde{\upbeta_1}\mathrm{S_1 I_2} - \mu S_1 \\
\frac{\widetilde{\mathrm{dS_2}}}{\mathrm{dt}} &= \tilde{\pi}\mathrm{b} - \widetilde{\upbeta_2}\mathrm{S_2 I_2} - \mu S_2 \\
\frac{\widetilde{\mathrm{dI_1}}}{\mathrm{dt}} &= \alpha S_1 - \gamma_1 \mathrm{I_1} - \mu I_1 - \nu_1 \mathrm{I_1} - \tilde{\sigma}\mathrm{I_1 I_2} \\
\frac{\widetilde{\mathrm{dI_2}}}{\mathrm{dt}} &= \widetilde{\upbeta_1}\mathrm{S_1 I_2} + \widetilde{\upbeta_2}\mathrm{S_2 I_2} - \gamma_2 \mathrm{I_2} - \mu I_2 - \nu_2 \mathrm{I_2} + \tilde{\sigma}\mathrm{I_1 I_2}
\end{aligned} \tag{3}$$

under the following preliminary circumstances:
$S_1(0) > 0, S_2(0) > 0, I_1(0) > 0, I_2(0) > 0, N(0) > 0$.

3 Equilibrium States and Basic Reproduction Number Existence

In this section, we identify all biologically and feasibly relevant equilibria allowed by the system described in Eq. 3. Equation 3 system obviously has a disease-free equilibrium (DFE), given by $E^0 = \left(S_1^0, S_2^0, I_1^0, 0\right)$, where

$$S_1^0 = \frac{(1-\tilde{\pi})b}{(\alpha+\mu)};\ S_2^0 = \frac{\tilde{\pi} b}{\mu};\ I_1^0 = \frac{\alpha(1-\tilde{\pi})b}{(\alpha+\mu)(\gamma_1+\mu+\nu_1)};\ I_2^0 = 0.$$

Further, the endemic equilibrium $E^* = \left(S_1^*, S_2^*, I_1^*, I_2^*\right)$ of the system of equations is solved by Eq. 3:

$$(1-\tilde{\pi})b - \alpha S_1 - \widetilde{\beta}_1 S_1 I_2 - \mu S_1 = 0,$$

$$\tilde{\pi} b - \widetilde{\beta}_2 S_2 I_2 - \mu S_2 = 0,$$

$$\alpha S_1 - \gamma_1 I_1 - \mu I_1 - \nu_1 I_1 - \tilde{\sigma} I_1 I_2 = 0,$$

$$\widetilde{\beta}_1 S_1 I_2 + \widetilde{\beta}_2 S_2 I_2 - \gamma_2 I_2 - \mu I_2 - \nu_2 I_2 + \tilde{\sigma} I_1 I_2 = 0,$$

$$\gamma_1 I_1 + \gamma_2 I_2 - \mu R = 0.$$

which gives,

$$S_1^* = \frac{(1-\tilde{\pi})b}{\left(\alpha+\widetilde{\beta}_1 I_2+\mu\right)};\ S_2^* = \frac{\tilde{\pi} b}{\mu+\widetilde{\beta}_2 I_2};\ I_1^* = \frac{\alpha(1-\tilde{\pi})b}{\left(\alpha+\widetilde{\beta}_1 I_2+\mu\right)(\gamma_1+\mu+\nu_1+\tilde{\sigma} I_2)},$$

and I_2^* satisfies the following equation:

$$A I_2^{*3} + B I_2^{*2} + C I_2^* + D = 0, \tag{4}$$

where

$$\begin{aligned}
A &= \tilde{\sigma}\widetilde{\beta_1}\widetilde{\beta_2}(\gamma_2+\mu+\nu_2) > 0,\\
B &= \tilde{\sigma}(\gamma_2+\mu+\nu_2)\left(\widetilde{\beta_1}\mu+\widetilde{\beta_2}(\alpha+\mu)\right) + \widetilde{\beta}_1\widetilde{\beta_2}(\gamma_1+\mu+\nu_1)(\gamma_2+\mu+\nu_2) - \tilde{\sigma}\widetilde{\beta_1}\widetilde{\beta_2}b > 0\,,\\
C &= -\Big[\tilde{\sigma}b(1-\tilde{\pi})\left(\mu\widetilde{\beta}_1+\alpha\widetilde{\beta}_2\right) + \widetilde{\beta}_1\widetilde{\beta}_2 b(\gamma_1+\mu+\nu_1) + \tilde{\sigma}(\alpha+\mu)\left(\widetilde{\beta}_2\tilde{\pi}b - \mu(\gamma_2+\mu+\nu_2)\right)\\
&\quad -(\gamma_1+\mu+\nu_1)(\gamma_2+\mu+\nu_2)\left(\widetilde{\beta}_1\mu+\widetilde{\beta}_2(\alpha+\mu)\right)\Big]\\
D &= -\Big[\mu(1-\tilde{\pi})b\left(\widetilde{\beta_1}(\gamma_1+\mu+\nu_1)+\tilde{\sigma}\alpha\right) + (\alpha+\mu)(\gamma_1+\mu+\nu_1)\left(\widetilde{\beta_2}\tilde{\pi}b - \mu(\gamma_2\mu+\nu_2)\right)\Big]
\end{aligned}$$

According to "Descartes' Rule of Signs," Eq. 4 will have at least one positive root if the fuzzy $\widetilde{\mathcal{R}_0} > 1$.

$$\widetilde{\mathcal{R}_0} = \frac{\tilde{\sigma}\mathrm{b}(1-\tilde{\pi})\big(\upmu\widetilde{\upbeta_1} + \widetilde{\upbeta_2}\upalpha\big) + \widetilde{\beta_1}\widetilde{\beta_2}\mathrm{b}(\gamma_1 + \upmu + \upnu_1) + \tilde{\sigma}(\alpha+\mu)\Big(\widetilde{\beta_2\tilde{\pi}}b - \mu(\gamma_2+\mu+\nu_2)\Big)}{(\upgamma_1 + \upmu + \upnu_1)(\upgamma_2 + \upmu + \upnu_2)\big(\upmu\widetilde{\upbeta_1} + \widetilde{\upbeta_2}(\upalpha + \upmu)\big)}$$

$$\widetilde{\mathcal{R}_0} = \mathcal{R}_{0_1}, \mathcal{R}_{0_2}, \mathcal{R}_{0_3}$$

We now determine the defuzzified value of $\widetilde{\mathcal{R}_0}$ using the centroid method.

$$\mathrm{d_F}\widetilde{\mathcal{R}_0} = \frac{1}{3}\big(\mathcal{R}_{0_1} + \mathcal{R}_{0_1} + \mathcal{R}_{0_1}\big)$$

4 Stability Analysis

Using the following process, we acquire the Jacobian matrix for every equilibrium so that we may assess the local stability of system represented by Eq. 3.

$$\begin{bmatrix} -\alpha - \widetilde{\beta_1}I_2 - \mu & 0 & 0 & -\widetilde{\beta_1}S_1 \\ 0 & -\widetilde{\beta_2}I_2 - \mu & 0 & -\widetilde{\beta_2}S_2 \\ \alpha & 0 & -\upgamma_1 - \upmu - \upnu_1 - \tilde{\sigma}\mathrm{I}_2 & -\tilde{\sigma}\mathrm{I}_1 \\ \widetilde{\beta_1}I_2 & \widetilde{\beta_2}I_2 & \tilde{\sigma}\mathrm{I}_2 & \widetilde{\upbeta_1}\mathrm{S}_1 + \widetilde{\upbeta_2}\mathrm{S}_2 - \upgamma_2 - \upmu - \upnu_2 + \tilde{\sigma}\mathrm{I}_1 \end{bmatrix}$$

4.1 Local Stability of DFE

The disease-free equilibrium (DFE) $E^0 = (S_1^0, S_2^0, I_1^0, 0)$ is first examined for local stability. The following is the characteristic equation linked to this DFE:

$$\left(\frac{\widetilde{\beta_1}(1-\widetilde{\pi})b}{(\alpha+\mu)} + \frac{\widetilde{\beta_2}\widetilde{\pi}b}{\mu} - (\gamma_2+\mu+\nu_2) - \lambda + \frac{\widetilde{\sigma}\alpha(1-\widetilde{\pi})b}{(\alpha+\mu)(\gamma_1+\mu+\nu_1)}\right)$$
$$(\gamma_1+\mu+\nu_1+\lambda)(\mu+\lambda)(\alpha+\mu+\lambda) = 0 \tag{5}$$

This indicates that$\lambda_1 = -(\gamma_1+\mu+\nu_1)$, $\lambda_2 = -\mu$, $\lambda_3 = -(\alpha+\mu)$ are negative eigenvalues of the characteristic Eq. 5, while the remaining eigenvalue is:

$$\lambda_4 = \frac{\widetilde{\beta_1}(1-\tilde{\pi})b}{(\alpha+\mu)} + \frac{\tilde{\sigma}\alpha(1-\tilde{\pi})b}{(\alpha+\mu)(\gamma_1+\mu+\nu_1)} + \frac{\widetilde{\beta_2}\tilde{\pi}b}{\mu} - (\gamma_2+\mu+\nu_2)$$

Of Eq. 5 will be negative if.

$$\begin{aligned}&\big(\widetilde{R}_0 - 1\big)\big(\widetilde{\beta}_1 b\mu(1-\widetilde{\pi})(\gamma_1+\mu+v_1) + \tilde{\sigma}\alpha\mu(1-\widetilde{\pi})b \\ &\quad +(\alpha+\mu)(\gamma_1+\mu+v_1)\big(\tilde{\beta}_1 S_2^0\mu - \mu + \tilde{\beta}_1 I_2^0\big)\big) < 0\end{aligned} \quad \text{i.e., } \widetilde{\mathcal{R}_0} < 1.$$

Then we have following theorem:

Theorem 1 The system's disease free equilibrium When fuzzy $\widetilde{\mathcal{R}_0} < 1$ and unstable when fuzzy $\widetilde{\mathcal{R}_0} > 1$, Eq. 3 is locally asymptotically stable.

4.2 Local Stability of EE

We will now calculate endemic equilibrium's local stability analysis, which is $E^* = \big(S_1^*, S_2^*, I_1^*, I_2^*, R^*\big)$. The endemic equilibrium's characteristic equation has the following form:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

where,

$$\begin{aligned}
a_1 &= \frac{(1-\tilde{\pi})b}{S_1^*} + \frac{\tilde{\pi}b}{S_2^*} + \frac{\alpha S_1^*}{I_1^*},\\
a_2 &= \frac{(1-\tilde{\pi})b}{S_1^*}\left(\frac{\tilde{\pi}b}{S_2^*} + \frac{\alpha S_1^*}{I_1^*}\right) + \frac{\alpha\tilde{\pi}bS_1^*}{S_2^*I_1^*} + \widetilde{\sigma^2}I_1^*I_2^* + \widetilde{\beta_2^2}S_2^*I_2^* + \widetilde{\beta_1^2}S_1^*I_2^*,\\
a_3 &= \frac{(1-\tilde{\pi})b}{S_1^*}\left(\frac{\alpha\tilde{\pi}bS_1^*}{S_2^*I_1^*} + \widetilde{\sigma^2}I_1^*I_2^* + \widetilde{\beta_2^2}S_2^*I_2^*\right) + \frac{\widetilde{\sigma^2}I_1^*I_2^*\tilde{\pi}b}{S_2^*} + \frac{\alpha\widetilde{\beta_2^2}S_2^*I_2^*S_1^*}{I_1^*}\\
&\quad + \frac{\widetilde{\beta_1^2}S_1^*I_2^*\tilde{\pi}b}{S_2^*} + \alpha\widetilde{\beta_1}S_1^*\tilde{\sigma}I_2^* + \frac{\widetilde{\beta_1^2}\big(S_1^*\big)^2\alpha I_2^*}{I_1^*},\\
a_4 &= \frac{(1-\tilde{\pi})b}{S_1^*}\left(\frac{\widetilde{\sigma^2}I_1^*I_2^*\tilde{\pi}b}{S_2^*} + \frac{\alpha\widetilde{\beta_2^2}S_2^*I_2^*S_1^*}{I_1^*}\right) + \frac{\alpha\widetilde{\beta_1}S_1^*\tilde{\pi}b\tilde{\sigma}I_2^*}{S_2^*} + \frac{\alpha\widetilde{\beta_1^2}\big(S_1^*\big)^2\tilde{\pi}bI_2^*}{S_2^*I_1^*}.
\end{aligned}$$

It is clear that $a_1 > 0$, $a_2 > 0$, $a_3 > 0$ and $a_4 > 0$. If the Routh-Hurwitz criterion requirement, i.e., $a_1a_2a_3 > a_3^2 + a_1^2a_4$, is met, then all roots of characteristic equation of J(E^*) will either have negative real parts or be negative. The following theorem provides a conclusion on this result:

Theorem 2 Assuming that fuzzy $\widetilde{\mathcal{R}_0} > 1$, Eq. 3 system has an endemic equilibrium E^*. In this instance, E^* is LAS as long as the subsequent criteria are met: $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $a_4 > 0$ and $a_1a_2a_3 > a_3^2 + a_1^2a_4$.

4.3 Global Stability of DFE

Theorem 3 Equation 3 represents the globally asymptotically stable DFE of the system.

Proof Using the technique described in [16], we have demonstrated global stability of DFE in this case. Equation 3 for the model system in question Assume that $Z = (I_2)$ and $X = (S_1, S_2, I_1))$. In this case, $U_0 = (X_0, Z_0))$.

$X_0 = \left(S_1^0, S_2^0, I_1^0\right)$ and $Z_0 = (0)$. At $Z = Z_0$, $G(X, 0) = \left(S_1^0, S_2^0, I_1^0\right)$. We have,

$$\begin{aligned}
\frac{d\widetilde{S_1}}{dt} &= (1-\tilde{\pi})b - \alpha S_1 - \mu S_1, \\
\frac{d\widetilde{S_2}}{dt} &= \tilde{\pi}b - \mu S_2, \\
\frac{d\widetilde{I_1}}{dt} &= \alpha S_1 - \gamma_1 I_1 - \mu I_1 - \nu_1 I_1
\end{aligned} \tag{6}$$

Using the Lyapunov function, it is simple to demonstrate that, under certain assumptions, this simplified system is globally stable. We locate the non-zero equilibrium, which is provided as, in order to demonstrate the global stability of reduced system Eq. 6.

$$S_1^0 = \frac{(1-\tilde{\pi})b}{(\alpha+\mu)};\ S_2^0 = \frac{\tilde{\pi}b}{\mu};\ I_1^0 = \frac{\alpha(1-\tilde{\pi})b}{(\alpha+\mu)(\gamma_1+\mu+\nu_1)}$$

Now let's look at the positive definite function,

$$V(t) = \frac{K_1}{2}\left(S_1(t) - S_1^0\right)^2 + \frac{1}{2}\left(S_2(t) - S_2^0\right)^2 + \frac{K_2}{2}\left(I_1(t) - I_1^0\right)^2$$

where K_1 and K_2 are arbitrary constants that will be appropriately selected at a later time. Utilising the value of the system Eq. 6 and differentiating V(t) with respect to t, we obtain that

$$\begin{aligned}
\dot{V} = &-K_1(\alpha+\mu)\left(S_1 - S_1^0\right)^2 - \frac{\tilde{\pi}b}{\mu}\left(S_2 - S_2^0\right)^2 \\
&+ K_2\alpha\left(I_1 - I_1^0\right)\left(S_1 - S_1^0\right) - K_2(\gamma_1+\mu+\nu_1)\left(I_1 - I_1^0\right)^2
\end{aligned} \tag{7}$$

Let us assume that.

$$K_1 = \frac{\alpha^2}{4(\alpha+\mu)(\gamma_1+\mu+\nu_1)} \text{ and } K_2 = \frac{1}{(\gamma_1+\mu+\nu_1)}.$$

Then, we get from Eq. 7 that

$$\dot{\mathrm{V}} = -\left(\frac{\alpha}{2(\gamma_1 + \mu + \nu_1)}\left(\mathrm{S}_1 - \mathrm{S}_1^0\right) - \left(\mathrm{I}_1 - \mathrm{I}_1^0\right)\right)^2 - \frac{\tilde{\pi}\mathrm{b}}{\mu}\left(\mathrm{S}_2 - \mathrm{S}_2^0\right)^2 < 0$$

$X = X_0(= S_1, S_2, I_1))$ is globally asymptotically stable since $\dot{\mathrm{V}}$ is negative definite, which means that the system Eq. 6 is globally stable. As a result, [16] condition (i) is met. Currently, we can get that from system Eq. 3 fourth equation.

$$\frac{\mathrm{dZ}}{\mathrm{dt}} = \mathrm{G(X, Z)} = \mathcal{A}\mathrm{Z} - \mathrm{Q(X, Z)}$$

where,

$$\mathcal{A} = -(\gamma_2 + \mu + \nu_2) \text{ and } \mathrm{Q(X, Z)} = \widetilde{\beta}_1\mathrm{S}_1\mathrm{I}_2 + \widetilde{\beta}_2\mathrm{S}_2\mathrm{I}_2 + \tilde{\sigma}\mathrm{I}_1\mathrm{I}_2 > 0$$

Therefore, since $\mathcal{A}$ off-diagonal elements are non-negative, $\mathcal{A}$ is an M-matrix. As a result, it is clear that the DFE is globally asymptotically stable according to criteria (i) and (ii) of [16].

Note: It is evident from Theorem 2 that $R_0 > 1$ means that the endemic equilibrium E^*, if it exists, is unique (as an interior equilibrium) and that it is locally asymptotically stable. E^0 is unstable when $\widetilde{\mathcal{R}_0} > 1$, according to Theorem 1. The uniform persistence is implied by the instability of E^0 and $E^0 \in \partial\gamma$ [4], meaning that there is a constant $\xi > 0$ such that:

$$\operatorname{liminf}_{\mathrm{t}\to\infty} X(t) > \xi,\ \mathrm{X} = \mathrm{S}_1, \mathrm{S}_2, \mathrm{I}_1, \mathrm{I}_2.$$

4.4 Global Stability of Endemic Equilibrium

We analyse the global stability point of the endemic equilibrium point by taking into account the following positive definite function.

$$\mathrm{W} = \frac{\mathrm{P}_1}{2}\left(\mathrm{S}_1 - \mathrm{S}_1^*\right)^2 + \frac{\mathrm{P}_2}{2}\left(\mathrm{S}_2 - \mathrm{S}_2^*\right)^2 + \frac{1}{2}\left(\mathrm{I}_1 - \mathrm{I}_1^* - \mathrm{I}_1^* \log\frac{\mathrm{I}_1}{\mathrm{I}_1^*}\right) + \frac{1}{2}\left(\mathrm{I}_2 - \mathrm{I}_2^* - \mathrm{I}_2^* \log\frac{\mathrm{I}_2}{\mathrm{I}_2^*}\right)$$

where P_1 and P_2 are arbitrary constants, will be chosen suitably later. On differentiating W w.r.t. 't', we get

$$\dot{\mathrm{W}} = \mathrm{P}_1\left(\mathrm{S}_1 - \mathrm{S}_1^*\right)\dot{\mathrm{S}}_1 + \mathrm{P}_2\left(\mathrm{S}_2 - \mathrm{S}_2^*\right)\dot{\mathrm{S}}_2 + \left(\mathrm{I}_1 - \mathrm{I}_1^*\right)\frac{\dot{\mathrm{I}}_1}{\mathrm{I}_1} + \left(\mathrm{I}_2 - \mathrm{I}_2^*\right)\frac{\dot{\mathrm{I}}_2}{\mathrm{I}_2}$$

Evaluating $\dot{\mathrm{W}}$ along the solutions of model system Eq. 3, we get

$$\dot{W} = -P_1(\alpha + \mu)\big(S_1 - S_1^*\big)^2 - \mu P_2\big(S_2 - S_2^*\big)^2 - P_1\widetilde{\beta}_1\big(S_1 - S_1^*\big)^2 I_2$$
$$- P_2\widetilde{\beta}_2\big(S_2 - S_2^*\big)^2 I_2 - \frac{\alpha}{I_1 I_1^*}S_1^*\big(I_1 - I_1^*\big)^2 + \widetilde{\beta}_1\big(S_1 - S_1^*\big)\big(I_1 - I_1^*\big)\big(1 - P_1 S_1^*\big)$$
$$+_2\,\big(S_2 - S_2^*\big)\big(I_2 - I_2^*\big)\big(1 - P_2 S_2^*\big) + \frac{\alpha}{I_1}\big(I_1 - I_1^*\big)\big(S_1 - S_1^*\big).$$

After deciding on $P_1 = \frac{1}{S_1^*}$, $P_2 = \frac{1}{S_2^*}$, and appropriately determining the limits of the state variables $\xi < I(t) \leq \frac{\mu}{b}$.

$$\dot{W} = -\left(\frac{(\alpha + \mu)}{S_1^*}\big(S_1 - S_1^*\big)^2 + \frac{\alpha\mu}{bI_1^*}S_1^*\big(I_1 - I_1^*\big)^2 - 2\frac{\alpha}{2\xi}\big(I_1 - I_1^*\big)\big(S_1 - S_1^*\big)\right)$$
$$- \frac{\mu}{S_2^*}\big(S_2 - S_2^*\big)^2 - \frac{\widetilde{\beta}_1}{S_1^*}\big(S_1 - S_1^*\big)^2 I_2 - \frac{\widetilde{\beta}_2}{S_2^*}\big(S_2 - S_2^*\big)^2 I_2.$$

In the feasible region γ, $\dot{W}$ will now be negative definite as long as the following inequalities are met: $4(\alpha + \mu)\mu\xi^2 > \alpha b I_1^*$.

Theorem 4 The EE is globally stable in region γ. If $4(\alpha + \mu)\mu\xi^2 > \alpha b I_1^*$ is hold.

5 Numerical Simulations

We looked at the optimal control problem that accompanied the compartmental model of childhood illness dynamics. We will use Python to do the numerical simulations in this part in order to validate these analytical findings.

5.1 Stability Analysis

To begin, we confirm the validity of the theoretical results outlined in Theorems 2 and 4.

Further, the system Eq. 1 has a basic reproduction number $d_F\widetilde{\mathcal{R}_0} = 2.39302 > 1$ for the parametric values: ($b = 0.07$, $\pi = (0.5, 0.8, 1)$, $\alpha = 0.04$, $\mu = 0.0102$, $\beta_2 = (0.005, 0.01, 0.015)$, $\gamma_1 = 0.3$, $\gamma_2 = 0.09$, $\sigma = (0.04, 0.05, 0.06)$, $\nu_1 = 0.2$, $\nu_2 = 0.17$, $\beta_1 = (0.4, 0.5, 0.6)$). Here, $E^* = (0.8083, 1.2501, 0.1120, 0.5432, 5.9016)$ exists and is locally asymptotically stable, as can be seen in Fig. 1. Furthermore, we have presented the system Eq. 3 solution trajectories for the endemic equilibrium's nonlinear stability behaviour (refer to Figs. 2 and 3).

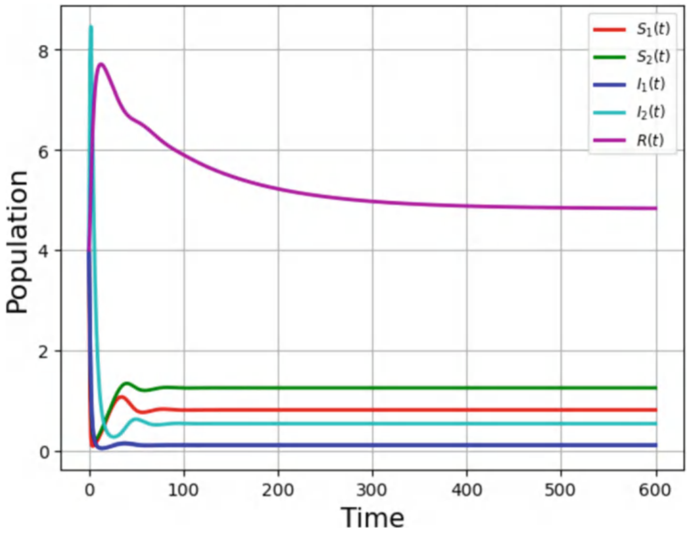

Fig. 1 Local stability of E^* for $d_F\widetilde{\mathcal{R}_0} > 1$ of system Eq. 1

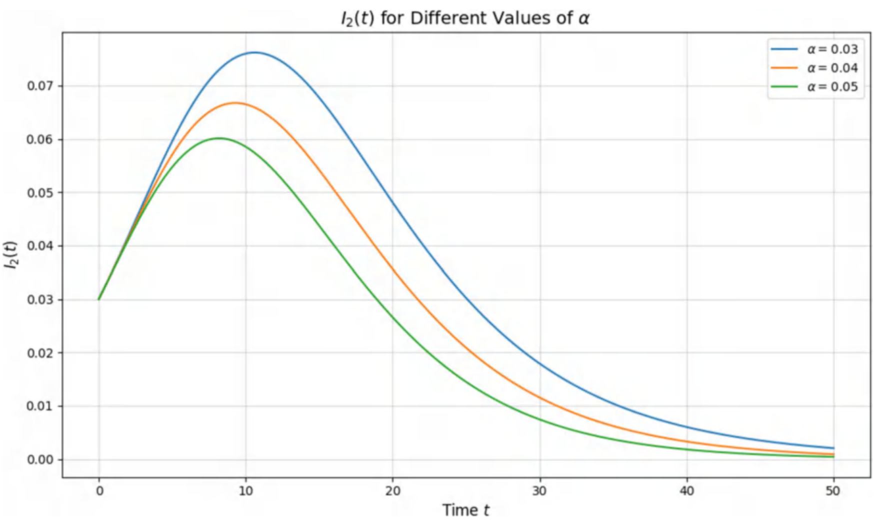

Fig. 2 Graph of $I_2(t)$ for different value of parameter α of system Eq. 1

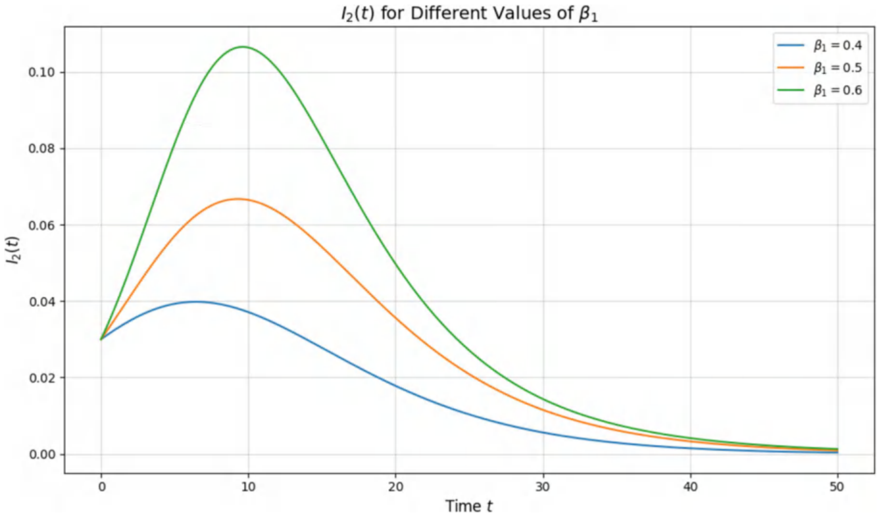

Fig. 3 Graph of $I_2(t)$ for different value of parameter β_1 of system Eq. 1

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Synthesis of Fuzzy Sets and Their Practical Applications

Shiv K. Sharma and Shweta Singh

Abstract This chapter delves into the practical uses of fuzzy sets, which have significantly contributed to the development of numerous technologies over time. The serious exploration and implementation of fuzzy sets began in the early 1970s or perhaps the early 1980s. Fuzzy set theory provides a mathematical framework that, despite its flexibility, now aids in redefining logic through algorithms and graphical representations. The chapter covers various operators and applications available for fuzzy sets, which are used not only in handling uncertainty but also in predicting outcomes in scientific activities, natural phenomena, aerospace [1], networks, and beyond [2]. Recent developments have highlighted the evolving applications of fuzzy logic, which has seen a surge in both scientific and non-scientific fields. The importance and practical relevance of fuzzy logic in real-world applications are indisputable and pivotal to modern advancements.

Keywords Fuzzy set · Vagueness · Elusiveness · Network theory

1 Introduction

Fuzzy sets and Fuzzy logics have an evident role in the modern world. Fuzzy sets and fuzzy logic left us with astonishing success. Behind the systematic presentation of fuzzy sets and fuzzy logic is a profound phenomenon say, any object A and any theory B can easily be fuzzified by substituting the notion of Crisp set in A and B with that of a fuzzy set [3].

Fuzzy sets and Fuzzy logics are very effective implementation, it provides enormous coverage with exemplary visualization which expands its uses widely. In application to the basic algorithm, topology, graphs, probability theory steer to the fuzzy algorithm, fuzzy geopolitics, fuzzy graphs, and fuzzy plausibility theory. The same applications appear in the applied field too, fuzzification happened such as neural

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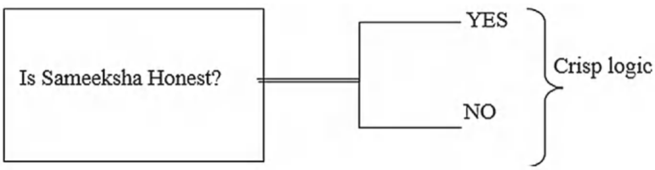
network theory steering us to fuzzy auditory network theory, solidity theory leads to fuzzy stability theory, and pattern recognition leads to fuzzy pattern acknowledgment. Fuzzy sets and fuzzy logic gave us a sense of expanded and enhanced opportunity to face the real-world problem and most importantly it gave us the methodology to work vaguely and in approximation.

2 Crisp Sets

A crisp set is defined as under distinct elements, which are derived from the universal set. The universal set is the set containing all objects or elements, also called classical sets. The crisp set provides a Boolean system in the form of 1 or 0 [4].

Example



3 Fuzzy Sets

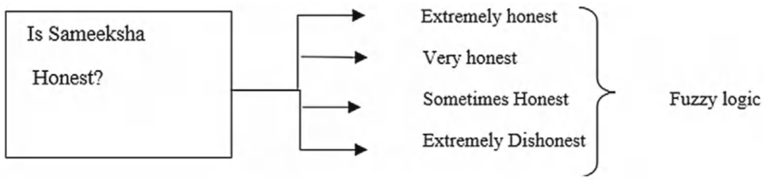
Lofti Zadeh, a professor at the prestigious University of California, was the first to propose the fuzzy set theory in 1965 [5]. Zadeh presented a radical shift that was first recognized in the East, then spread throughout the world as it proved to be successful and beneficial [6].

3.1 Definition

Fuzzy set is a mathematical model of vaguely [7] providing qualitative or quantitative data which is usually produced in day-to-day language. Basically, this model depends upon the classical concept of set, and its characteristic function. These sets mainly state if an object is related to that, not related to that set, or partially related to that set.

Example 1 If a person wants to describe the class of clothing line as being expensive considering Gucci, Chanel, Prada, Armani, Zara, and H&M. Hence some brands like Gucci and Chanel are obviously highly priced and some other brands like Zara and H&M are not that pricy in comparison to other brands. The fuzzy set of pricey brands may be defined as follows using a fuzzy set and membership value. The membership values of (Gucci, 1), (Prada, 0.8), (Armani, 0.7), (Zara, 0.5), and (H&M, 0.4), here Gucci has 1 as its membership value, although the membership value of "Zara," which is inexpensive, is 0.5.

Example 2



4 Operations of Fuzzy Sets

The proverbial operations such as algebraical product, algebraical sum, union, intersection, and complement are all metaphorical operations that may be performed on fuzzy sets. Then came research into fuzzy sets and their applications [8] in a variety of domains, including automata theory, topology, logic control, game integral, pattern acknowledgment, lingual, decision making, codification, structure, information restoration, and so on [9]. Moreover, to this addition, certain new operations were added named "bounded-sum" and "bounded-difference" [4] which were presented by Zadeh in the year 1975 to etamine fuzzy reasoning phenomena which give us a method of dealing with difficult reasoning issues in order to make a quick choice. Fuzzy matrix games with intuitionistic fuzzy goals and intuitionistic fuzzy linear programming duality were discussed [10].

4.1 Equivalent Fuzzy Sets

Two Fuzzy sets are called Equivalent fuzzy sets if $A_1(t)$ and $A_2(t)$ are equal and if $\mu A_1(t) = \mu A_2(t)$ for all $t \in T$. It can be stated like

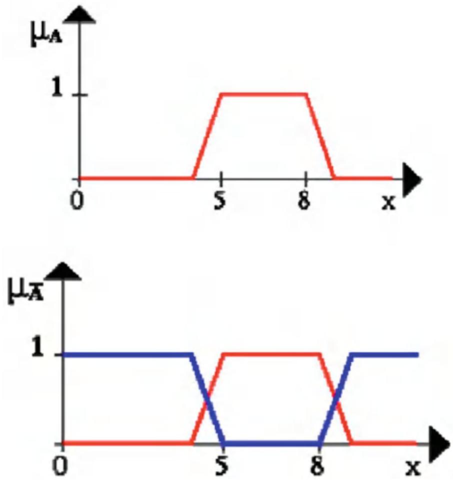
$$A_1(t) = A_2(t),\ \text{if}\, \mu A_1(t) = \mu A_2(t),$$

Its opposite non-equal fuzzy sets in which $\mu A_1(t) \neq \mu A_2(t)$ for at least $t \in T$.

Example

$$A_1(t) = \{(t_1, 1.11), (t_2, 2.23), (t_3, 3.36), (t_4, 1.47)\}$$
$$A_2(t) = \{(t_1, 4.15), (t_2, 2.35), (t_3, 1.31), (t_4, 1.72)\}$$

Fig. 1 Complement of fuzzy set



Above is non-equal fuzzy set as $\mu A_1(t) \neq \mu A_2(t)$ for different $t \in T$, $A_1(t) \neq A_2(t)$.

4.2 Complement of Fuzzy Set

Any set's complement is evaluated in relation to the universal set. Any compliment to a set is in direct opposition to that set. The complement of the set A(t) is $\bar{A}(t)$.

Example Figure 1 is the graphical representation of complement of fuzzy set.

4.3 Intersection of Fuzzy Set

Let $A_1(t)$ and $A_2(t)$ be the two sets then the intersection of two fuzzy sets can be stated as the similar elements belonging to both the sets i.e. common in both sets. It is denoted by $(A_1 \cap A_2)$ (t). It can belong to distinct sets to varying degrees. Though the degree of membership in both sets of each element is smaller. The value of the membership function is shown as

$$\mu(A_1 \cap A_2)(t) = \min\{\mu A_1(t),\ \mu A_2(t)\}$$

Example

$$A_1(t) = \{(t_1, 1.8), (t_2, 4.5), (t_3, 2.93), (t_4, 1.11)\}$$
$$A_2(t) = \{(t_1, 1.1), (t_2, 2.2), (t_3, 2.4), (t_4, 1.8)\}$$

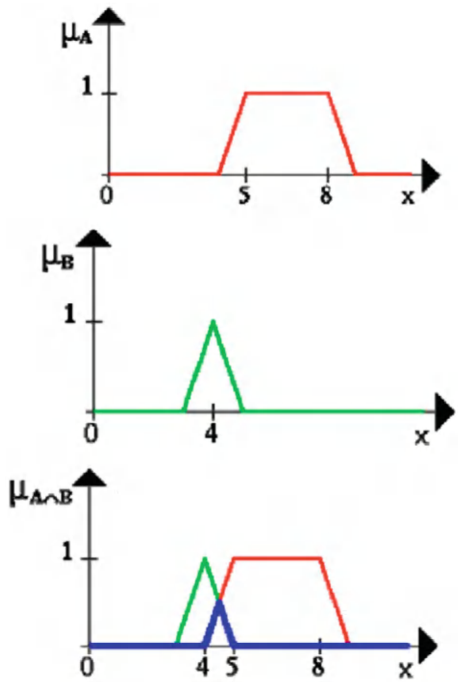


Fig. 2 Intersection of two sets

$$\mu(A_1 \cap A_2)(t_1) = \min\{\mu A_1(t_1), \mu A_2(t_1)\} = \min\{1.8, 1.1\} = 1.1$$
$$\mu(A_1 \cap A_2)(t_2) = \min\{\mu A_1(t_2), \mu A_2(t_2)\} = \min\{4.5, 2.2\} = 2.2$$
$$\mu(A_1 \cap A_2)(t_3) = \min\{\mu A_1(t_3), \mu A_2(t_3)\} = \min\{2.93, 2.4\} = 2.4$$
$$\mu(A_1 \cap A_2)(t_4) = \min\{\mu A_1(t_4), \mu A_2(t_4)\} = \min\{1.11, 1.8\} = 1.11$$

A graphical example is given in Fig. 2.

4.4 Union of Fuzzy Set

Let $A_1(t)$ and $A_2(t)$ be the two sets then the elements can be expressed as the union of fuzzy sets and defuzzification techniques for fuzzy output [11]. belonging to both the sets i.e. all the elements of $A_1(t)$ and $A_2(t)$. It is denoted by (A_1UA_2) (t). In each set, it obtains varying degrees of membership value. Despite the fact that the element's degree of membership is the highest, the element's membership value is either set.

Example

$$A_1(t) = \{(t_1, 1.8), (t_2, 4.5), (t_3, 2.93), (t_4, 1.11)\}$$
$$A_2(t) = \{(t_1, 1.1), (t_2, 2.2), (t_3, 2.4), (t_4, 1.8)\}$$
$$\mu(A_1UA_2)(t_1) = \min\{\mu A_1(t_1), \mu A_2(t_1)\} = \min\{1.8, 1.1\} = 1.8$$

Fig. 3 A(t) and B(t) sets

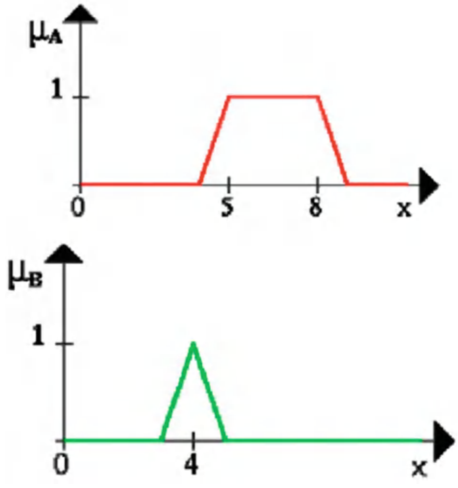
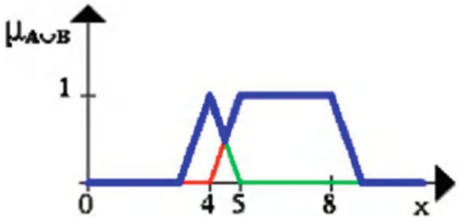


Fig. 4 Union of two sets A and B



$$\mu(A_1UA_2)(t_2) = \min\{\mu A_1(t_2), \mu A_2(t_2)\} = \min\{4.5, 2.2\} = 4.5$$
$$\mu(A_1UA_2)(t_3) = \min\{\mu A_1(t_3), \mu A_2(t_3)\} = \min\{2.93, 2.4\} = 2.93$$
$$\mu(A_1UA_2)(t_4) = \min\{\mu A_1(t_4), \mu A_2(t_4)\} = \min\{1.11, 1.8\} = 1.8$$

Example A Graphical example is given in Figs. 3 and 4

4.5 Algebraic Product of Fuzzy Set

Consider two sets, $A_1(t)$ and $A_2(t)$, and the algebraic product of fuzzy sets, $A_1(t)$ and $A_2(t)$ for every $t \in T$, is denoted by $A_1(t)*A_2(t)$ and defined as follows.

$$A_1(t) *_1 A_2(t) = \{(t, \mu A_1(t) * \mu A_2(t)), t\ T\}$$

Example

$$A_1(t) = \{(t_1, 2.8), (t_2, 5.5), (t_3, 3.9), (t_4, 1.1)\}$$
$$A_2(t) = \{(t_1, 5.1), (t_2, 2.2), (t_3, 2.4), (t_4, 1.8)\}$$
$$A_1(t) * A_2(t) = \{(t_1, 14.28), (t_2, 12.1), (t3, 9.36), (t_4, 1.98)$$

4.6 Multiplying Fuzzy Sets by a Crisp Number

Let A(s) be some given fuzzy set and 's' be any crisp set then the multiplication of two fuzzy sets can be stated as follows

$$A(s) * (\text{crisp set}) = \{(s,\ s * \mu A(s)),\ s\ \in\ S\}$$

Example

$$A(s) = \{(s_1, 3.8), (s_2, 2.5), (s_3, 2.9), (s_4, 1.1)\}$$

Let us consider if $s = 0.4$;

then $A(s).\, s = \{(s_1, 1.52),\ (s_2, 1.0), (s_3, 1.16), (s_4, 0.44)\}$.

4.7 Power of Fuzzy Sets

Let A(t) be any fuzzy set and 'p' be p-the power of the set then the following is a list of its membership value.

$$\mu A^P(t) = \{\mu A(t)\}^P,\ t \in T\}$$

$p >= 1 \rightarrow A^{\,P}(t)$ is called concentration.
$p < 1 \rightarrow A^{\,P}(t)$ is called dilation.

Example

$$K(t) = \{(t_1, 2.8), (t_2, 11.5), (t_3, 2.9), (t_4, 1.1)\}$$

Let us consider if $p = 2$;

then $[K(t)]^2 = \{(t_1, 5.6), (t_2, 23), (t_3, 5.8), (t_4, 2.2)\}$.

4.8 Algebraic Sum of Two Fuzzy Sets

Consider two fuzzy sets, $T_1(s)$ and $T_2(s)$. The algebraical sum of fuzzy sets, $T_1(s)$ and $T_2(s)$, maybe expressed as $T_1(s)$ and $T_2(s)$ for any $s \in S$, and is denoted by $T_1(s) + T_2(s)$ and defined as follows.

$$T_1(s) + T_2(s) = \{(s, \mu(T_1(s) + T_2(s)), s\ \ S\}$$

where $\mu(T_1(s) + T_2(s)) = \mu T_1(s) + \mu T_2(s) - \mu T_1(s).\, \mu T_2(s)$

Example

$$T_1(s) = \{(s_1, 1.1), (s_2, 5.7), (s_3, 5.5), (s_4, 2.1)\}$$
$$T_2(s) = \{(s_1, 3.6), (s_2, 1.5), (s_3, 1.8), (s_4, 6.3)\}$$

Then, $T_1(s) + T_2(s) = \left\{(s_1, 4.7), (s_2, 7.2), \left(s_{\bar{3}}, 7.3\right), (s_4, 0.37)\right\}$.

4.9 Bounded Sum of Two Fuzzy Sets

If $T_1(s)$ and $T_2(s)$ are two fuzzy sets, then the bounded sum of two sets may be defined as $T_1(s)$ and $T_2(s)$ for every $s \in S$, which is represented by $T_1(s) \oplus T_2(s)$ and defined as follows.

$T_1(s) \oplus T_2(s) = \{s, \theta^*[T_1 \oplus T_2(s)], s \in S\}$.

Where $\theta^*(T_1 \oplus T_2(s)) = \min\{1, \mu T_1(s) + \mu T_2(s)\}$.

Example

$$T_1(s) = \{(s_1, 0.8), (s_2, 0.5), (s_3, 0.9), (s_4, 0.1)\}$$
$$T_2(s) = \{(s_1, 0.1), (s_2, 0.2), (s_3, 0.4), (s_4, 0.8)\}$$
$$T_1(s) \oplus T_2(s) = \{(s_1, 0.9), (s_2, 0.7), (s_3, 1.0), (s_4, 0.9)$$

4.10 Algebraic Difference of Two Fuzzy Sets

Let $B_1(t)$ and $B_2(k)$ be two fuzzy sets then algebraic deference can be stated as $B_1(k)$ and $B_2(k)$ for all $k \in K$, is represented by $B_1(k) - B_2(k)$ and determined as follows

$$B_1(k) - B_2(k) = \{(k, \beta * [B_1(k) - B_2(k)],\ k \in K\}$$

where $\mu B_1 - A_1(k) = \beta * (B_1 \cap A_1(k))$

Example

$$B_1(k) = \{(k_1, 0.8), (k_2, 0.5), (k_3, 0.9), (k_4, 0.1)\}$$
$$A_1(k) = \{(k_1, 0.1), (k_2, 0.2), (k_3, 0.4), (k_4, 0.8)\}$$
$$\overline{A}_1(k) = \{(k_1, 0.9), (k_2, 0.8), (k_3, 0.6), (k_4, 0.2)\}$$
$$B_1(k) - A_1(k) = \{(k_1, 0.8), (k_2, 0.5), (k_3, 0.6), (k_4, 0.1)\}$$

4.11 Bounded Difference of Two Fuzzy Sets

Let $B_1(t)$ and $B_2(t)$ be two fuzzy sets (t). For each $t \in T$, the bounded difference between these two sets may be specified as $B_1(t)$ and $B_2(t)$ and is represented as given below

$$B_1(t) \ominus B_2(t) = \{(t, \mu * [B_1(t) \ominus B_2(t)],\ t \in T\}$$

where $B_1(t) \ominus B_2(t) = \max\{0, \mu * [A_1(t) + \mu A_2(t) - 1]\}$

Example

$$B_1(t) = \{(t_1, 0.9), (t_2, 0.8), (t_3, 0.7), (t_4, 0.6)\}$$
$$B_2(t) = \{(t_1, 0.2), (t_2, 0.4), (t_3, 0.1), (t_4, 0.3)\}$$
$$B_1(t) \ominus B_2(t) = \{(t_1, 0.1), (t_2, 0.2), (t_3, 0), (t_4, 0)\}$$

4.12 Cartesian Product of Two Fuzzy Sets

If $A_1(t)$ and $A_2(y)$ are two fuzzy sets defined upon universal sets T and Y, then $A_1(t)$ and $A_2(y)$ are the orthogonal product of two fuzzy sets, with t T and y Y. The following is how A_1 (t) $*A_2(y)$ is defined:

$$\mu * [(A_1(t) * A_2(t)] = \min\{\mu A_1(t), \mu A_2(y)\}$$

Example

$$A_1(t) = \{(t_1, 1.1), (t_2, 2.8), (t_3, 1.4), (t_4, 2.8)\}$$
$$A_2(y) = \left\{(y, 1.3), \left(y_2, 1.7\right), \left(y_3, 2.6\right)\right\}$$

$$\min\left\{\mu A_1(t_1), \mu A_2\left(y_1\right)\right\} = \min\{1.1, 1.3\} = 1.1, \min\left\{\mu A_1(t_1), \mu A_2\left(y_2\right)\right\} = \min \{1.1, 0.7\} = 1.1 \min\left\{\mu A_1(t_1), \mu A_2\left(y_2\right)\right\} = \min\{1.1, 2.6\} = 1.1$$

$$\min\left\{\mu A_1(t_2), \mu A_2\left(y_1\right)\right\} = \min\{2.8, 1.3\} = 1.3, \min\left\{\mu A_1(t_2), \mu A_2\left(y_2\right)\right\} = \min \{2.8, 1.7\} = 1.7 \min\left\{\mu A_1(t_2), \mu A_2\left(y_3\right)\right\} = \min\{2.8, 2.6\} = 2.6$$

$$\min\left\{\mu A_1(t_3), \mu A_2\left(y_1\right)\right\} = \min\{1.4, 1.3\} = 1.3, \min\left\{\mu A_1(t_3), \mu A_2\left(y_2\right)\right\} = \min\{1.4, 1.7\} = 1.4 \min\left\{\mu A_1(t_3), \mu A_2\left(y_3\right)\right\} = \min\{1.4, 2.6\} = 1.4$$

$$\min\left\{\mu A_1(t_4), \mu A_2\left(y_1\right)\right\} = \min\{2.8, 1.3\} = 1.3, \min\left\{\mu A_1(t_4), \mu A_2\left(y_2\right)\right\} = \min\{2.8, 1.7\} = 1.7 \min\left\{\mu A_1(t_4), \mu A_2\left(y_3\right)\right\} = \min\{2.8, 2.6\} = 2.6$$

5 Application of Fuzzy Sets in Real Life

There are various applications of fuzzy sets in different fields.

5.1 *To Provide an Optimized Wireless Network*

In recent years, the use of connectivity is required globally to provide different benefits on every occasion and time. The newest technology provides connectivity without any interruption separating from the ongoing location, device type, and convenience of the network. A wireless mesh network (WMN) [12] is a type of communication network that includes radio buttons held in the grid link structure. Wireless networks often include customer mesh and grid routers. The grid router contains additional routing features due to the presence of wireless interface cards [13]. The fuzzy logic is executed through MATLAB using the Mamdani system. Model executes the whole operation on the basis of significant input, output, and setting certain rules.

5.2 *To Provide an Estimation of Intellectual Capital*

A Nowadays the economy is actually the development and knowledge within that position. This growth is represented by its digital and global applications. To help managers comprehend the performance of intellectual capital, Tai and Chen suggested a viable business a two-tuple fuzzy linguistic approach [14, 15] is used to evaluate intellectual capital. It uses three components to explain the whole system which has management, market, and knowledge capital as inputs. It uses 27 rules [14] to obtain intellectual capital. Though intellectual capital can also be calculated by using human knowledge but utilizing computer knowledge we get swift results with accuracy by providing it specific inputs and rules. Opinion mining based on fuzzy-based system were explained [16].

5.3 *To Provide Predictions of Rainfall*

There are so many ways to predict rainfall these days through computers or through human computation. One such way is through a fuzzy rainfall prediction system that uses the Mamdani system [15] to forecast the rainfall [17] and a few rules to predict it. Initially, the number of inputs modifies in different ranges *say* low, moderate, and high along with the triangular membership function and trapezoidal membership function

(Figs. 5 and 6). However, as the number of rules grows, so does the complexity of the output.

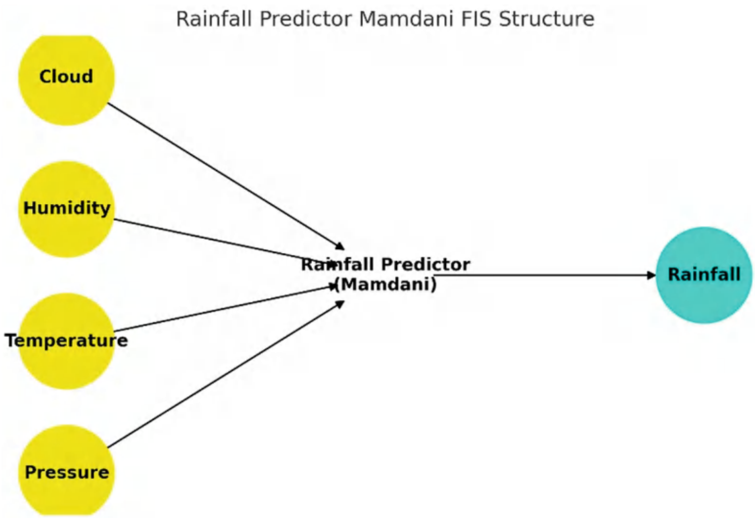


Fig. 5 Mamdani system requires choosing factors affecting rainfall as cloud, humidity, temperature and pressure

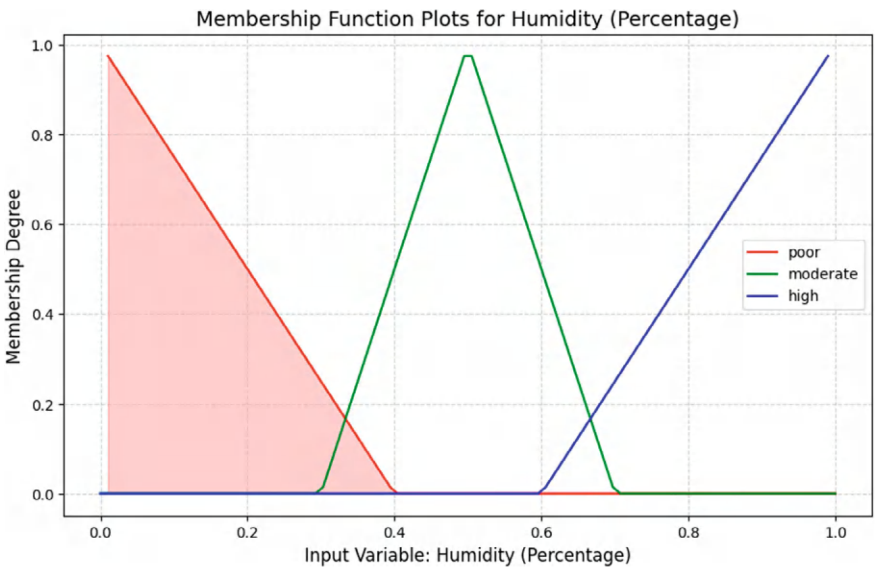


Fig. 6 Here we take the measure of clouds in octant, humidity in percentage, the temperature in degree Celsius, and Pressure in bars

Below measurements are taken of cloud in octant, humidity in percentage, the temperature in degrees Celsius, and pressure in bars [18]. Then it will go through the Mamdani system [15] and predict rainfall. Here we have included eight rules to predict rainfall. The number of rules is determined by the elements that influence the system (Figs. 7 and 8).

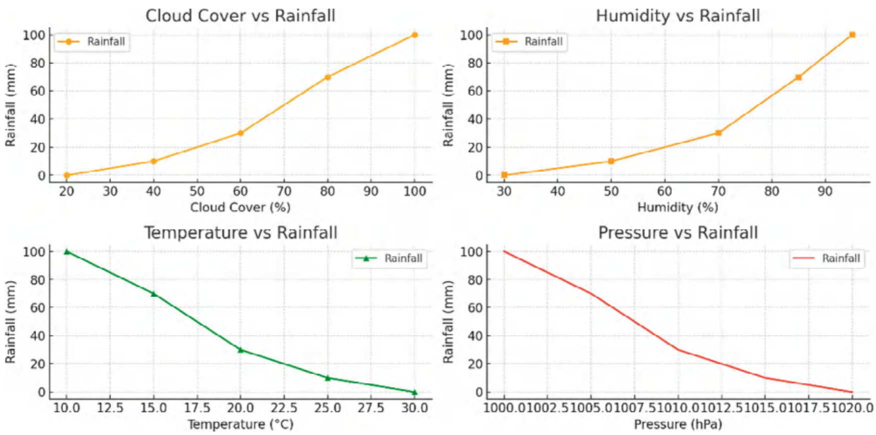


Fig. 7 Here we have to add rules on which the whole system will work by setting the factors affecting rain

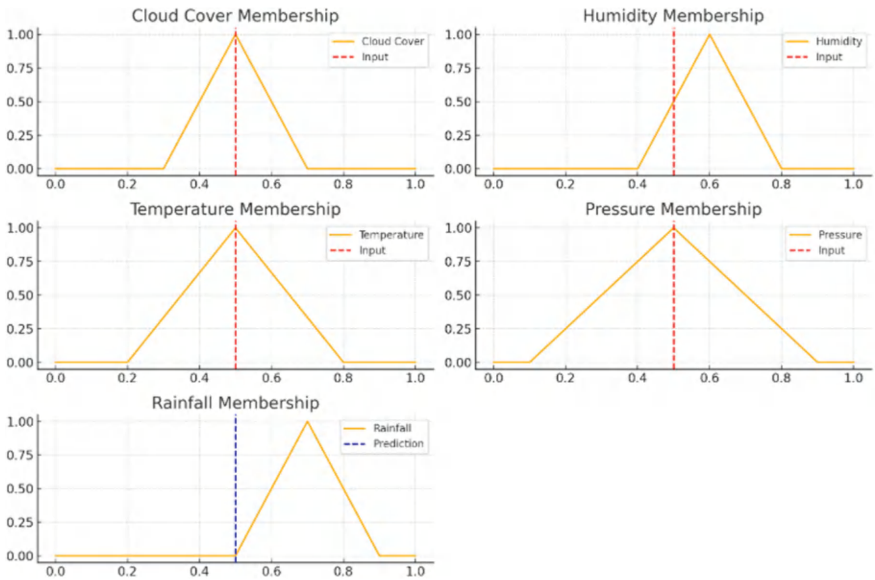


Fig. 8 Here we can see the rain prediction when all factors and rules and compiled together

6 Conclusion

The fuzzy set theory proposed ambiguity and vagueness [7] which help us in multiple ways. Fuzzy sets give a tool for the human brain which helps in attempting various problems. Motivated by the uses of fuzzy logic we have found different applications practically with the help of the Mamdani system using MATLAB for analysing the rainfall.

The nature of fuzzy logic applications has changed dramatically over the last two decades. The quantity, visibility, and importance of non-engineering applications have increased. Medicinal science, social sciences, policies, scam detection systems, credit-worthiness assessment systems, and economics are just a few examples.

Fuzzy logic plays a crucial role in mathematical disciplines, including algebraic operation, analysis, consolidation, restrain theory, graphical theory, gauge theory, maximization, operations research, and geopolitics, to name a few [19].

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Optimizing Inventory Levels in Retail: Fuzzy Logic-Based Decision Support Systems for Adaptive Inventory Management

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Abstract Inventory management is a critical component of supply chain operations, impacting a company's ability to meet customer demand while controlling costs. Traditional inventory management methods, such as Economic Order Quantity (EOQ) and Reorder Point, offer foundational strategies for optimizing inventory levels. However, these methods often fall short in dynamic and unpredictable market conditions. This chapter introduces an innovative fuzzy decision-making algorithm designed to enhance inventory management by incorporating real-time data and fuzzy logic principles. By adapting to fluctuating demand and supply chain variables, the algorithm aims to reduce the risks of stock outs and overstocking. Through a detailed examination of the algorithm's steps-fuzzification, rule base construction, fuzzy inference, and defuzzification we demonstrate how businesses can achieve more responsive and efficient inventory control. Additionally, we provide a numerical example and a practical application scenario for a retail chain, illustrating the algorithm's effectiveness in optimizing inventory levels. The chapter concludes with a discussion on the benefits of integrating fuzzy logic into inventory management and suggestions for future research and implementation.

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1 Introduction

Inventory management plays a crucial role in the success of retail operations, influencing costs, customer satisfaction, and overall business performance. Traditional inventory models often struggle to adapt to the dynamic and uncertain nature of retail environments, leading to inefficiencies and suboptimal outcomes [1–3]. Fuzzy logic has emerged as a valuable tool for decision-making in inventory management, particularly in the context of retail supply chains. The inherent uncertainty and imprecision in retail environments make traditional decision-making models less effective, leading to suboptimal inventory strategies and increased costs [4, 5]. By incorporating fuzzy logic principles, decision support systems (DSS) can dynamically adjust inventory strategies based on real-time data inputs. These inputs include sales rates, inventory levels, supplier lead times, and demand trends, among others. Fuzzy logic enables DSS to handle the complexity and variability of these data inputs, leading to more accurate demand forecasting, optimal inventory replenishment, and improved supply chain performance [6, 7]. Furthermore, fuzzy logic-based DSS can integrate data from multiple sources, such as point-of-sale systems, inventory management software, and market analytics platforms. This integration allows for a holistic view of the supply chain, enabling retailers to make informed decisions that align with business goals and customer demand patterns. The adaptive nature of fuzzy logic-based DSS ensures that inventory strategies can be dynamically adjusted in response to changing market conditions, ensuring operational efficiency and customer satisfaction.

Effective operations and supply chain management play a pivotal role in the success of modern businesses. Jacobs and Chase [8] delve into the essential principles of operations and supply chain management, emphasizing the importance of strategic planning and execution. In the realm of operations management and supply chains, [9] provide insights into the intricate processes involved, highlighting the need for efficient coordination and optimization across various stages. Chopra and Meindl [10] present strategic approaches, planning methodologies, and operational tactics necessary for achieving competitive advantages in the marketplace. Stevenson [11] elaborates on fundamental concepts, focusing on productivity enhancement, quality management, and cost control strategies. Heizer et al. [5] explore different facets of operations management, including process analysis, layout strategies, and capacity planning, essential for streamlining operations in diverse industries. Slack et al. [12] present a comprehensive view of operations management, covering topics such as service design, project management, and performance improvement methodologies. Chase et al. [13] offer a holistic perspective on operations and supply chain management, integrating key concepts with real-world case studies and practical applications.

Several studies have delved into the realm of fuzzy logic's application within inventory control systems, particularly focusing on diverse sectors like retail chains, online retailing, and fast fashion retailers. Liang et al. [14] highlighted how fuzzy logic contributes to optimizing inventory levels in retail chains, enabling more efficient resource utilization. Lim et al. [15] emphasized fuzzy logic's role in enhancing operational efficiency, particularly in online retailing contexts where demand fluctuations are common. Additionally, Liu et al. [16] showcased how fuzzy logic aids in addressing challenges specific to fast fashion retailers, such as rapid product turnover and seasonal demand variations. On the decision support system (DSS) front, Patel and Patel [17] demonstrated the potential of fuzzy logic-based DSS in demand forecasting, providing retailers with valuable insights into future market trends. Sharma and Mishra [18] discussed how fuzzy logic contributes to inventory replenishment strategies, ensuring optimal stock levels while minimizing costs. Smith et al. [19] outlined the advantages of using fuzzy logic in adaptive inventory management, allowing retailers to dynamically adjust their inventory strategies in response to changing market conditions. Moreover, Tan et al. [20] explored fuzzy logic's role in developing intelligent DSS for inventory optimization in retail chains, facilitating data-driven decision-making processes. Wang et al. [21] highlighted the effectiveness of fuzzy logic-based DSS in inventory replenishment for online grocery retailing, ensuring timely and accurate stock replenishment. Wang et al. [22] further emphasized the benefits of fuzzy logic-based DSS in addressing challenges related to demand variability and inventory optimization in e-commerce retailing. Lastly, Wu et al. [23] showcased how fuzzy logic-based DSS can improve demand forecasting accuracy, aiding retailers in making informed inventory management decisions. Zhang et al. [24] developed a dynamic inventory management model for e-commerce retailers using fuzzy logic to optimize stock levels and reduce uncertainty. Zhou et al. [25] explored the application of fuzzy logic in inventory control systems for fast fashion retailers, enhancing demand forecasting and supply chain efficiency.

This chapter will delve into the fundamentals of inventory management, exploring concepts such as inventory, inventory management, inventory control, and related terms with examples. It will then discuss the limitations of traditional inventory management methods and introduce the innovative approach of fuzzy decision-making algorithms tailored for inventory optimization in retail settings. Through case studies, numerical examples, and practical applications, we will demonstrate how fuzzy logic based decision support systems can revolutionize inventory management practices, leading to improved efficiency, reduced costs, and enhanced customer satisfaction in the retail industry. The remainder of the chapter is organized as follows: Sect. 2 provides a literature review of inventory management and fuzzy logic applications in retail. Section 3 delves into various methods and algorithms for optimizing inventory levels in retail settings, and we introduce an innovative fuzzy decision-making algorithm designed to address the shortcomings of traditional inventory management approaches. This algorithm utilizes real-time data and fuzzy logic principles to dynamically adjust inventory levels, enhancing accuracy and efficiency in inventory management. Section 4 presents another fuzzy decision-making algorithm for inventory management, offering further enhancements by incorporating real-time data and fuzzy logic principles. This alternative algorithm aims to provide more adaptable and responsive inventory control, thereby reducing risks associated with stockouts and overstocking. Finally, Sect. 5 concludes the chapter by summarizing key insights and contributions to the field of inventory optimization in retail, emphasizing the significance of adopting fuzzy logic-based decision support systems for adaptive inventory management.

2 Preliminaries

In this section, we present some basic concepts such as inventory, inventory management, inventory control, and related terms with examples, which are fundamental for our study.

2.1 Inventory [8]

Inventory refers to the stock of goods and materials that a business holds for the purpose of resale or production.

Inventory Example: A retail store's inventory includes items such as clothing, electronics, groceries, and other products available for sale to customers.

2.2 Inventory Management [9]

Inventory management involves the planning, controlling, and optimization of inventory levels to ensure efficient operations.

Inventory Management Example: A manufacturing company uses inventory management techniques to maintain optimal levels of raw materials.

2.3 Inventory Control [10]

Inventory control focuses on maintaining accurate records of inventory levels and implementing strategies to prevent stockouts or overstocking.

Inventory Control Example: A warehouse uses barcode scanning and inventory software to track incoming and outgoing inventory.

2.4 Stockout [11]

A stockout occurs when an item is out of stock or unavailable for purchase when demanded by customers.

Stockout Example: A customer visits an online store to buy a specific product but finds it is out of stock.

2.5 Reorder Point [5]

The reorder point is the inventory level at which a company should reorder a product to avoid stockouts.

Reorder Point Example: A retailer sets a reorder point of 100 units for a popular item with a lead time of 7 days.

2.6 Safety Stock [12]

Safety stock is the extra inventory held to mitigate the risk of stockouts due to unexpected fluctuations in demand.

Safety Stock Example: A distributor maintains safety stock of essential items during peak seasons.

2.7 Economic Order Quantity (EOQ) [13]

EOQ is the optimal order quantity that minimizes total inventory costs.

EOQ Example: A retailer calculates the EOQ for a product based on factors like demand rate and ordering cost.

3 Optimizing Inventory Levels

In this section, we have studied various methods and algorithms for optimizing inventory levels in retail settings. The goal is to strike a balance between having enough stock to meet customer demand without incurring excessive holding costs. Traditional methods such as Economic Order Quantity (EOQ) and Reorder Point have been widely used, but they often fall short in dynamic market conditions.

We introduce an innovative fuzzy decision-making algorithm designed to address the limitations of traditional inventory management approaches. This algorithm utilizes real-time data and fuzzy logic principles to adjust inventory levels dynamically, improving the accuracy and efficiency of inventory management.

3.1 Traditional Inventory Management Methods

Traditional methods like EOQ and Reorder Point calculations provide a foundational understanding of inventory optimization. These methods use historical data to predict future demand and determine optimal order quantities and timing. However, they may not adapt well to fluctuating market conditions or unexpected changes in demand.

3.2 Fuzzy Decision-Making Algorithm

The fuzzy decision-making algorithm is designed to enhance inventory management by incorporating real-time data and fuzzy logic. This approach allows for more adaptable and responsive inventory control, reducing the risks of stockouts and overstocking. The algorithm's steps include fuzzification, rule base construction, fuzzy inference, and defuzzification, creating a dynamic system that adjusts inventory levels based on current conditions.

3.3 Problem Scenario

ABC Retail is a clothing store chain with multiple outlets. They struggle with maintaining optimal inventory levels across their stores. Overstocking leads to increased storage costs and risks of unsold items becoming obsolete. Under stocking results in lost sales opportunities and dissatisfied customers.

Data Inputs:

(1) **Sales Data**: Historical sales data for each clothing item (daily/weekly/monthly).
(2) **Inventory Levels**: Current stock levels for each item in each store.
(3) **Supplier Lead Time**: Time taken for suppliers to replenish inventory.
(4) **Customer Demand Trends**: Seasonal variations, trend analysis of customer preferences.

Challenges:

(1) **Demand Variability**: Fluctuating customer demand makes it challenging to predict inventory needs accurately.

(2) **Lead Time Uncertainty**: Supplier lead times vary, affecting the timing of inventory replenishment.
(3) **Space Constraints**: Limited storage space in stores necessitates efficient inventory allocation.
(4) **Cost Considerations**: Balancing inventory costs (storage, holding, and ordering) with sales revenue.
(5) **Traditional Approach**: ABC Retail currently uses basic inventory models (e.g., Economic Order Quantity, Reorder Point) but struggles with adaptability to dynamic market conditions and demand fluctuations.

3.4 Proposed Innovative Fuzzy Decision-Making Algorithm

Develop an innovative fuzzy decision-making algorithm to dynamically adjust inventory levels based on real-time data and market conditions.

Steps:

(1) **Fuzzification**: Convert crisp data into fuzzy sets using linguistic variables.
(2) **Rule Base Construction**: Define rules based on fuzzy logic principles and inventory management objectives.
(3) **Fuzzy Inference System (FIS)**: Develop a Fuzzy Inference System integrating linguistic variables and rule base.
(4) **Defuzzification**: Convert fuzzy outputs into actionable inventory decisions using defuzzification methods.
(5) **Real-time Monitoring and Feedback Loop**: Continuously monitor real-time data and incorporate feedback to refine the algorithm.

Benefits of the Fuzzy Decision-Making Algorithm

(1) **Adaptability**: The algorithm can adapt to dynamic demand patterns and market changes, improving inventory accuracy.
(2) **Risk Mitigation**: Reduces the risk of stockouts and overstocking by dynamically adjusting inventory levels.
(3) **Cost Efficiency**: Optimizes inventory costs by aligning ordering quantities with actual demand and lead times.
(4) **Customer Satisfaction**: Ensures product availability, reducing instances of out-of-stock items and enhancing customer experience.

By implementing this innovative fuzzy decision-making algorithm, ABC Retail can improve its inventory management processes, reduce costs, and enhance customer satisfaction, thereby gaining a competitive advantage in the retail market.

3.5 Numerical Example: Inventory Optimization Using Fuzzy Decision-Making Algorithm

To provide a numerical example for the inventory optimization problem using a fuzzy decision-making algorithm, let's consider some hypothetical data and scenarios for ABC Retail:

Data Inputs

(1) **Sales Data**: Historical sales data for a specific clothing item in one store over the past 30 days:
- Average Daily Sales: 50 units
- Maximum Daily Sales: 100 units
- Minimum Daily Sales: 20 units

(2) **Inventory Levels**: Current stock level for the same clothing item in the store:
- Current Inventory: 300 units

(3) **Supplier Lead Time**: Average lead time for inventory replenishment:
- Lead Time: 7 days

(4) **Customer Demand Trends**: Seasonal trend analysis indicates an expected 20% increase in demand over the next month.

Fuzzy Variables and Membership Functions

- **Sales Rate**: Low (0–30 units/day), Medium (20–70 units/day), High (50–100 units/day)
- **Inventory Level**: Low (0–200 units), Medium (150–400 units), High (350–600 units)
- **Lead Time**: Short (0–5 days), Medium (3–10 days), Long (7–15 days)
- **Demand Increase**: Low (0–10% increase), Medium (5–20% increase), High (1530% increase)

Rules

(1) IF Sales Rate is High AND Inventory Level is Low, THEN Increase Reorder Frequency.
(2) IF Lead Time is Long AND Sales Rate is Low, THEN Reduce Inventory Order Quantity.
(3) IF Demand Increase is High AND Inventory Level is Medium, THEN Increase Order Quantity.

Fuzzy Decision-Making Process

(1) **Fuzzification**:
- Sales Rate: Medium (0.5)
- Inventory Level: High (0.8)

- Lead Time: Medium (0.6)
- Demand Increase: Medium (0.6)

(2) **Rule Activation**:

- Rule 1 Activation Strength: $0.5 \times 0.8 = 0.4$
- Rule 2 Activation Strength: No Activation
- Rule 3 Activation Strength: $0.6 \times 0.8 = 0.48$

(3) **Defuzzification**:

- Increase Reorder Frequency: Medium (0.4)
- Increase Order Quantity: High (0.48)

Decision Outcome

Based on the fuzzy decision-making algorithm, the system suggests a Medium level increase in reorder frequency and a High level increase in order quantity for the clothing item in question. This decision is made considering the current sales rate, inventory level, lead time, and expected demand increase.

Outcome Evaluation

ABC Retail can implement the recommended actions (increase reorder frequency and order quantity) and monitor the impact on inventory management efficiency, sales performance, and customer satisfaction. The fuzzy decision-making algorithm can be adjusted over time based on actual outcomes and feedback from store operations.

4 Another Fuzzy Decision-Making Algorithm for Inventory Management

To further enhance inventory management, we propose an alternative fuzzy decision-making algorithm. This algorithm incorporates real-time data and fuzzy logic principles to provide more adaptable and responsive inventory control, thereby reducing the risks of stock outs and overstocking. The key steps of this algorithm include fuzzification, rule base construction, fuzzy inference, and defuzzification. The result is a dynamic system that continuously adjusts inventory levels based on current market conditions.

4.1 Steps of the Alternative Fuzzy Decision-Making Algorithm

4.1.1 Fuzzification

The first step is to convert crisp input data into fuzzy sets. This involves defining the membership functions for various input variables. The input variables may include sales rate, inventory level, supplier lead time, and demand forecast. For example:

- **Sales Rate**: Low, Medium, High
- **Inventory Level**: Low, Medium, High
- **Lead Time**: Short, Medium, Long
- **Demand Forecast**: Low, Medium, High

4.1.2 Rule Base Construction

Construct a set of if–then rules based on fuzzy logic principles and inventory management objectives. Each rule combines the fuzzy input variables to produce a fuzzy output. For example:

- **Rule 1**: IF Sales Rate is High AND Inventory Level is Low, THEN Increase Order Quantity.
- **Rule 2**: IF Lead Time is Long AND Sales Rate is Low, THEN Decrease Order Quantity.
- **Rule 3**: IF Demand Forecast is High AND Inventory Level is Medium, THEN Increase Safety Stock.
- **Rule 4**: IF Inventory Level is High AND Sales Rate is Medium, THEN Maintain Current Order Quantity.

4.1.3 Fuzzy Inference System (FIS)

Develop a Fuzzy Inference System that integrates the fuzzified inputs with the rule base. The FIS evaluates all the applicable rules for a given set of inputs and determines the degree to which each rule applies.

4.1.4 Defuzzification

Convert the fuzzy output of the FIS into crisp actionable decisions using defuzzification methods. This step translates the fuzzy recommendations into specific order quantities, reorder points, and safety stock levels.

4.2 Example of the Alternative Fuzzy Decision-Making Algorithm in Action

To illustrate the application of this alternative algorithm, consider the following hypothetical data for a specific product at ABC Retail:

- **Sales Data**: Average daily sales of 60 units, with maximum and minimum sales of 90 and 30 units, respectively.
- **Current Inventory**: 250 units
- **Supplier Lead Time**: Average lead time of 10 days
- **Demand Forecast**: Expected 15% increase over the next month

Fuzzy Variables and Membership Functions:

- **Sales Rate**: Low (0–40 units/day), Medium (30–70 units/day), High (60–100 units/day)
- **Inventory Level**: Low (0–200 units), Medium (150–300 units), High (250–400 units)
- **Lead Time**: Short (0–7 days), Medium (5–15 days), Long (10–20 days)
- **Demand Forecast**: Low (0–10% increase), Medium (5–20% increase), High (1530% increase)

Example Rule Activation:

- Sales Rate: Medium (0.7)
- Inventory Level: Medium (0.6)
- Lead Time: Medium (0.8)
- Demand Forecast: High (0.6)

Rule Activation Strengths:

- Rule 1: $0.7 \times 0.4 = 0.28$
- Rule 2: $0.8 \times 0.3 = 0.24$
- Rule 3: $0.6 \times 0.6 = 0.36$
- Rule 4: $0.6 \times 0.7 = 0.42$

Defuzzification: Using the centroid method, calculate the weighted average of the outputs to determine the final decision.

Decision Outcome:

Based on the fuzzy decision-making algorithm, the system suggests a moderate increase in order quantity and a slight increase in safety stock to account for the expected rise in demand.

By incorporating this alternative fuzzy decision-making algorithm, ABC Retail can further optimize its inventory management, ensuring a more responsive and adaptable approach to dynamic market conditions.

5 Conclusion

In this chapter, we explored various fundamental concepts related to inventory management, including inventory, inventory management, inventory control, stock outs; reorder points, safety stock, and Economic Order Quantity (EOQ). We highlighted the importance of these concepts through practical examples and examined traditional methods for optimizing inventory levels. While these methods provide a solid foundation, they often lack the flexibility to adapt to dynamic market conditions. To address these limitations, we introduced an innovative fuzzy decision-making algorithm designed to enhance inventory management by incorporating real-time data and fuzzy logic principles. This algorithm improves inventory control by making more adaptable and responsive decisions, thereby reducing the risks of stock outs and overstocking. We detailed the steps of this algorithm, including fuzzification, rule base construction, fuzzy inference, and defuzzification. Furthermore, we presented an alternative fuzzy decision-making algorithm, demonstrating its application through a hypothetical scenario involving ABC Retail. By implementing these advanced algorithms, businesses can dynamically adjust inventory levels based on current market conditions, leading to improved inventory accuracy, cost efficiency, and customer satisfaction.

Overall, the integration of fuzzy logic into inventory management offers a promising approach for businesses seeking to optimize their inventory levels in an ever-changing market environment. Future work could involve testing these algorithms in real-world settings and refining them based on empirical data to further enhance their effectiveness.

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Neutrosophic Numbers in Identifying Best Teacher Awardee Using SWOT Analysis

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Abstract In this competitive world, each and every individual run for their livelihood. It ends up without any breathing time also sometimes. In that busy pathway, each field requires recognition to prove uniqueness in their relevant field. Awards are the specified recognition presented as a token, to recognize the excellency of the receiver in any fields. It also serves as a encouraging element to make this work involved fruitfully in future. Now, it is the responsibility of the organization which honours the awardee, to see that it reaches the suitable person. Various factors are involved in the recognition process of the awardee. Neutrosophic numbers finds its application in many real life situations in finding out a suitable solution. This provides a solution to situation with uncertainty. This chapter aims at defining out a methodology in identifying the best teacher, to provide recognition with an award. In this process, it is proposed to make use of neutrosophic numbers and perform SWOT analysis.

Keywords Neutrosophic numbers · SWOT analysis · Award · Best teacher · Decision making

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1 Introduction

Out of various professions, teaching is a profession, which is considered as a most respectful and noble profession. In every field, people think of their individual development. But only in the teaching field, teachers show their involvement and dedication in the development of the students rather than themself. The real happiness of the teachers lies in seeing their students in a higher position than them. They encourage their students in every aspect and they help them mold themselves to prove themselves the best in society.

These types of encouragement are needed for teachers to do their responsibilities without any hurdles. The encouragement appears in the form of awards. Awards are recognition given in almost all professions which boosts up energy in further development of individual. In the teaching field, it is provided considering various aspects of teaching such as experience, results, publications, projects, funding, many such criteria.

The organization which felicitates with the award come across various profiles and applicants. Selecting an efficient profile suitable for the award is a tedious process. They fix grading for each criterion and according to the scoring, they filter the profile in each level and finalize the awardee. It involves multiple levels of selection. In this technological world, there are various methods of criteria selection. The method of Multi Criteria Decision Making method has attracted most researchers, since it serves as an efficient method in identifying the selection of finalists. This decision identifying method comes with different forms of analysis. A few of them are AHP, ANP, TOPSIS, CBA, ranking and correlation. The major focus of this chapter is to identify the best teacher awardee making use of MCDM method using SWOT analysis.

N. Halouani et al. in the year 2009, made use of promethee method for identifying the suitable project [1]. Vijay Manikrao Athawale et al. have made their selection of location as per required facilities applying promethee method in the year 2010 [2]. Laurence Turcksina in the year 2011 used AHP promethee method in identifying a suitable policy for energizing vehicle fleet [3]. Mark Velasquez et al. in the year 2013 have made a study on various multi criteria decision making models [4]. Danielle Costa Morais et al. in the year 2015 made their work in energy generation with the help of promethee method in the assession of technology [5]. Reverse logistics has been made as study in the work done by Jafar Rezaei in the year 2015 with the help of multi criteria decision making [6]. Dilber Uzun Ozsahi et al. in the year 2017 made an analysis in cancer treatment applying fuzzy promethee method [7]. In the year 2017 Riska Hanifah et al. presented their decision making work on banking sector [8].

Ade Parlaungan Nasution et al. in the year 2019 various application of promethee method in decision making [9]. Lazim Abdullah et al. in the same year has made a comparative analysis on green supplier selection making use of preference function [10]. Akmaludin, Erene Gernaria Sihombing et al. in the year 2020 made selection of credit loan during the covid pandemic period using multi criteria decision making method [11]. Bettina Zargini in the same year applied alternative preference function in decision making problems [12]. In the same year Chao Tian et al. applied AHP and fuzzy picture method in identifying suitable tourism environment [13]. Ikuobase Emovon and Okpako Stephen Oghenenyerovwho in the year 2020 applied multi criteria decision making model in selecting material which gives optimal output [14]. Qiang Zhang, Junhua Hu et al. in the year 2020 applied multi criteria decision making method in the selection of material for GGPFWA operator [15]. S. Gomathy et al. in 2020 presented a discussion on the application of plithogenic process [16]. Nivetha Martin and Florentin Smarandache in 2020 discussed about the decision making procedure using plithogenic coginitive maps [17].

Fernando Castro Sánchez et al. in the year 2021 performed an analysis of statistics in the development of education [18]. Laila Oubahman et al. in the year 2021 made a review on transportation using promethee method [19]. Selection of supplier using AHP promethee method has been done by Muhammad Akmal Md Isa in the year 2021 [20]. Noel Batista Hernandez et al. in 2021 proposed a plithogenic approach to identify the competence among the university students with respect to entrepreneurship [21].

The selection of elective courses making use of promethee method has been done by Haryono Setiadi in the year 2022 [22]. Martina Bertoncini et al. in the year 2022 made a comparative study on energy requalification applying promethee method [23]. Fuzzy promethee method in the selection of media platform has been done by Melike Kubra Ekiz Bozdemir et al. in the year 2022 [24]. Musavarah Sarwar et al. in the year 2022 made use of fuzzy promethee method in selecting optimal supplier for industrial management [25]. Ömer Ramadan et al. in the year 2022 performed fuzzy promethee method in selection of rail projects which involves priority [26]. Hamed Taherdoost and Mitra Madanchian applied multi criteria decision making method in applications together with its procedures in the year 2023 [27]. Kala Raja Mohan et al. in the year 2023 made an analysis on gold rate using statistics [28]. Mayang Sari et al. in the year 2023 made use of promethee method in identifying the priority in road improvement system [29]. S. Sudha et al. in 2023 introduced an extension to plithogenic process on sociagram [30]. With these study as base, the neutrosophic swot analysis to identify the best teacher awardee has been proposed in this chapter.

2 Standard Definitions

Various definitions involved in the decision-making algorithm are listed below for reference.

2.1 Neutrosophic Sets

A set which is characterized by a truth membership function, an indeterminacy membership function and a falsity membership function is termed as neutrosophic set.

2.2 Neutrosophic Intersection

Given two neutrosophic sets (a_1, a_2, a_3) and (b_1, b_2, b_3) the neutrosophic intersection is defined as $(a_1, a_2, a_3) \bigcap (b_1, b_2, b_3) = \left(\frac{a_1+a_2}{2}, \max(b_1, b_2),\ \min\,(c_{1,}c_2)\right)$

2.3 Neutrosophic Union

Given two neutrosophic sets (a_1, a_2, a_3) and (b_1, b_2, b_3) the neutrosophic union is defined as $(a_1, a_2, a_3) \bigcup (b_1, b_2, b_3) = \left(\frac{a_1+a_2}{2}, \min\,(b_1, b_2),\ \max\,(c_{1,}c_2)\right)$

2.4 Median

Median. refers to the middle value in the set of given numbers arranged in either ascending or descending order of occurrence. In case of an even number of terms, the average of two middle terms is termed as the median.

With respect to neutrosophic sets, the median of the sets are referred to the set of median of true membership function, median of indeterminacy membership function and a falsity membership function.

2.5 Rank

The order of the values which are arranged either in ascending or descending order of preference is termed as rank.

2.6 Attributes

Attributes refer to the objective responsible for the selection of the decision. The criteria are fixed based on the requirement of the solution required.

3 SWOT Analysis

The decision making problem based on the attributes can be categorized into four types namely Strength, Weakness, Opportunities and Threads. Each of these aspects are tested for each alternatives and the final results are identified in SWOT analysis.

3.1 Methodology

The various steps involved in the process of outranking algorithm are listed below.

- Attributes
- SWOT criteria
- Linguistic values
- Median SWOT values
- Combined SWOT values
- Crisp values
- Rank
- Decision

4 Selection of Awardee

Teachers are the building pillars of students. Hence, they keep on developing themselves to keep them updated. For recent update in their field, they involve themselves in various factors. They keep on studying throughout their life. They publish journals and patents. They give guidance to other scholars in research work. They involve themselves in training programs. Based on the factors involved in teachers, four different criteria are fixed for the decision-making process.

Recognition of teachers in the name of awards brings happiness and confidence for their grooming work. The selection of awardees from teachers for an award ceremony is considered for decision-making in this chapter. For this process nomination of candidates from the Department of Mathematics has been taken into consideration. For this decision-making process four attributes are chosen namely excellent, good, average and poor and four criteria are considered. Four applicants are termed as A1, A2, A3, A4.

Four aspects in SWOT analysis are defined as mentioned below. Education, experience, publication, funding projects, feedback, guidance are chosen as strengths termed as S1, S2, S3, S4, S5, S6. The weakness termed as W1, W2 are considered as punctuality and lack of responsibility. O1, O2, O3 are termed as the opportunities which are considered as higher studies, resource persons, research experience. With respect to threads, health issues such as stress, competence to overcome colleagues

Table 1 Neutrosophic score values of attributes

Criteria	Symbol	Neutrosophic values
Excellent	E	(0.8, 0.3, 0.1)
Good	G	(0.6, 0.3, 0.2)
Poor	P	(0.2, 0.5, 0.1)
Average	A	(0.4, 0.1, 0.2)

are considered as threads termed as T1, T2. The four attributes are assigned score value as given in Table 1.

Four applicants for the award are considered for SWOT analysis. The applicants score in linguistic form for each of the aspects in SWOT analysis is presented in Table 2.

For each applicant, a single neutrosophic number corresponding to the aspects of strengths, weakness, opportunities, threads are found. It is done by calculating the median of the sets corresponding the divisions considered in each aspect. The values obtained are presented in Table 3.

Using the values obtained in Table 3, the combined aspects such as SO, ST, WO and WT are found. These are calculated using the definition of neutrosophic intersection and neutrosophic union. Table 4 shows the values obtained using intersection and union. The definition of union is considered for SO, ST and the definition of intersection is considered for WO and WT.

Table 5 shows the crisp values obtained to each of the combined SWOT values. The crisp value is obtained using the function $\frac{2T+I-F}{3}$

In order to decide on the best teacher awardee, ranking is used. The values of SO + ST and WO + WT are found to each of the applicants. The ranks are fixed based on the difference of the values obtained, whose values are presented in Table 6.

Table 2 Linguistic values in SWOT analysis for 4 applicants

	S1	S2	S3	S4	S5	S6	W1	W2	O1	O2	O3	T1	T2
A1	E	E	E	G	G	E	E	G	E	G	E	A	G
A2	G	E	G	P	G	P	G	A	A	G	G	P	G
A3	E	P	G	P	A	P	A	G	E	P	P	G	G
A4	E	P	P	P	P	P	P	P	G	P	G	P	P

Table 3 Median SWOT values

	S	W	O	T
A1	(0.8, 0.3, 0.1)	(0.7, 0.3, 0.15)	(0.8, 0.3, 0.1)	(0.5, 0.2, 0.2)
A2	(0.6, 0.3, 0.15)	(0.6, 0.2, 0.15)	(0.8, 0.3, 0.1)	(0.4, 0.4, 0.15)
A3	(0.5, 0.5, 0.1)	(0.5, 0.15, 0.2)	(0.2, 0.5, 0.1)	(0.6, 0.3, 0.2)
A4	(0.2, 0.5, 0.1)	(0.2, 0.5, 0.1)	(0.3, 0.3, 0.2)	(0.2, 0.5, 0.1)

Table 4 Combined SWOT values

	SO	ST	WO	WT
A1	(0.8, 0.3, 0.1)	(0.65, 0.2, 0.2)	(0.75, 0.3, 0.1)	(0.6, 0.3, 0.15)
A2	(0.7, 0.3, 0.15)	(0.5, 0.3, 0.15)	(0.7, 0.3, 0.1)	(0.5, 0.4, 0.15)
A3	(0.35, 0.5, 0.1)	(0.55, 0.3, 0.2)	(0.35, 0.5, 0.1)	(0.35, 0.3, 0.2)
A4	(0.25, 0.3, 0.2)	(0.2, 0.5, 0.1)	(0.25, 0.5, 0.1)	(0.2, 0.5, 0.1)

Table 5 Crisp values of combined SWOT values

	SO	ST	WO	WT
A1	0.6	0.143	0.57	0.45
A2	0.517	0.383	0.533	0.417
A3	0.367	0.4	0.367	0.267
A4	0.2	0.3	0.3	0.267

Table 6 Rank values

	SO + ST	WO + WT	Difference	Rank
A1	0.743	1.02	0.277	1
A2	0.9	0.95	0.05	4
A3	0.767	0.634	0.133	2
A4	0.5	0.567	0.067	3

From the ranks obtained as shown in Table 6, it is clear that A1 has the first rank making the applicant suitable for the award among the four contestants. A3 occupies the second place, A4 occupies the third place and A2 occupies the fourth place.

5 Conclusion

Multi Criteria Decision Making methods find their applications in various daily life situations. The selection process provides an eminent solution. Among various MCDM methods, SWOT analysis is applied to identify the awardee for an award ceremony. The various criteria and attributes related to the study have been identified. The scoring system has been framed. The identification of the awardee has been done by applying neutrosophic sets. This method provides a decision in identifying the awardee with perfect evaluation. The method can be applied in finding the best student of an institution, best employee of an organization and many more.

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Fuzzy Logics in Multi-criteria Decision Making Problems

Archana Parashar and Renu Negi

Abstract This chapter delves into the mutually beneficial link that exists between a multi-criteria decision-making (MCDM) process and fuzzy logic, which essentially handles the intricacies of determining actions. The evaluation of MCDM describes its complexities, core ideas, and wide range of industrial applicability. The chapter talks about fuzzy logic and explains how it may be used to embrace ambiguity and uncertainty, providing a more nuanced method of making decisions. It illustrates how the use of fuzzy logic improves the process of determining the course of action, enabling the ones making decisions to confidently traverse complex landscapes through a thorough assessment of the Fuzzy Technique for Order Preference by Similarity to the Ideal Solution (F-TOPSIS) and Fuzzy Analytical Hierarchy (F-AHP) methods. Ultimately, it emphasises the utility of such approaches in real-life scenarios, highlighting their effectiveness in tackling current decision-making problems. This chapter is a thorough manual for comprehending and utilising the complementary strengths of multi-criteria decision-making and fuzzy logic to make wise selections in a variety of fields.

Keywords Fuzzy logic · Decision-making · Multi-criteria

1 Introduction

Making decisions is a fundamental aspect of our day-to-day lives. In industries, organisations, and institutions, where a single decision can wield tremendous influence, learning the art of making the right choice becomes utterly indispensable. Think about the decisions you make every day—from what to eat for breakfast to big

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choices at work. Sometimes, these decisions are easy, like picking between cereal and toast. But other times, they're tough, especially when there are too many factors to consider. Imagine you're a manager at a company. You have to decide upon something that affects the lives of many people (like whom to lay off or who deserves an appraisal) that isn't just a simple yes or no choice. They're more like puzzles with many pieces to fit together. That's where multi-criteria decision-making (MCDM) comes in handy. It's like a tool that helps you make sense of all the factors and choose the best option.

In June 1965, Zadeh [8] in his article 'Fuzzy Sets' in the Journal, 'Information and Control' introduced Fuzzy Logic. It is a mathematical concept that tries to be as close as possible to human imagination and perception. Fuzzy logics are based on the fact that real-life problems can be presented with some degree of truth and falsity. They are primarily employed in expert systems and artificial intelligence applications, offering broad utility in selection processes, system evaluations, and ranking of alternatives across banking, industrial, and diverse contexts.

The foundational concept of fuzzy logic stems from set theory. According to classical set theory, elements are either members of a set or not. The fuzzy set theory determines the degree of membership of an element in a set, which allows values within the range of 0–1, demonstrating the level of belongingness. The membership of an element x is denoted by $\mu_A(x)$, values of which lie between [0, 1], where 0 represents the non-membership of the given set and 1 indicates the complete membership of the set. In this way, fuzzy logic plays a prominent role in modelling impreciseness and vagueness while making decisions.

Now, problems in real life aren't always straightforward. They're often messy and complicated and that's where fuzzy logic comes into play. Instead of thinking in black and white, fuzzy logic lets us consider all the shades of grey. It helps us make decisions even when things aren't crystal clear. Such approaches become super important in businesses, where decisions can have big consequences. Fuzzy logic facilitates conclusions whenever there is uncertainty [7], whereas various MCDM methods help us make wise decisions keeping all the criteria and alternatives in consideration. Integration of both concepts helps decision-makers make more informed choices considering all the ambiguities.

To facilitate a deeper understanding of these concepts, this chapter will offer an in-depth discussion on the decision-making process while dealing with multiple criteria (MCDM), its characteristics, and its diverse types. It will then talk about the incorporation of fuzzy logic with various MCDM methods and offer a comprehensive insight into Fuzzy-AHP and Fuzzy-TOPSIS methods with a final concise overview of applications of these methods in real-life situations.

2 Understanding Multi-criteria Decision Process

Multi-Attribute Utility theory [6] popularly known as Multi-Criteria Decision-Making (MCDM), is a sub-discipline that comes under Operational Research where multiple conflicting criteria are explicitly evaluated to make a sound decision. Decisions that are made on a daily basis usually implicitly weigh multiple criteria and can be taken solely based on intuition. Whereas in businesses, institutions or government setting stakes are high and thereby makes structuring a problem and explicitly evaluating the multiple criteria more significant. Well structured complex problems take multi-facets of a problem into account and lead to better and informed decisions. Such problems are graphically represented through hierarchical structures. For instance, you need to decide whether a nuclear plant should be built or not. This involves deciding the ideal place for construction, what factors to consider, etc. This decision not only contains complexities under multiple criteria but also has a deeper impact on various individuals. Such decisions are not as trivial as deciding what to wear or what to eat.

In problems involving multiple criteria, one must account for diverse attributes, inherent constraints, and potential conflicts that require optimization. Such problems consist of four essential elements: Goals, Objectives, Criteria, and Alternatives. The characteristics of a multiple-criteria decision-making situation are as follows:

- Different Objectives/Attributes
- Conflict Among Criteria
- Incommensurable Units (different scales of Measurement)
- Design/Selection (process which search the most attractive of all criteria (dimension)).

A multi criteria decision making has two sub fields which are based upon the type of multiple elements a decision-making problem carries [6]. These are:

(i) **Multi-Attribute Decision-Making (MADM)**: In problems related to MADM, the given alternatives are evaluated based on different attributes of the object. They are also called ‘discrete problems’. Such problems concentrate on situations with explicitly known alternatives with fixed numbers. In problems involving such assessments, solutions between a discrete number of alternatives are chosen. In MADM, the limitations are unclear but aim, attributes (various criteria) and alternate options are clearer making the interaction level between decision-makers limited.

(ii) **Multi-Objective Decision-Making (MODM)**: These types of decision-making problems are often referred to as ‘continuous decision-making problems’ that are characterised by decision spaces with an infinite number of potential solutions across multiple, often conflicting, objectives. In such cases, alternatives located in the feasibility space are accepted as the answer to the given problem that requires a final clear conclusion. It is a problem for maximising efficiency for which there hasn’t been a clear-cut solution found. Such decision-making problems use implicit qualities and criteria as goals. Here, there is a significant degree of interaction among decision-makers despite the lack of clear objectives and noticeable restrictions (Table 1).

In a MCDM problem, identification of characteristics is crucial before initiating any further operations. With this, let us understand these aspects of a MCDM problem from the example presented below.

Problem 1 You are a hiring manager in a company and you're given the charge of recruiting a General Manager. There are 7 candidates to be interviewed. The desired criteria for the selection of an ideal candidate includes:

i. Must possess a Masters in Business Administration (MBA).
ii. Minimum experience of eight years.
iii. Salary expectation not exceeding 15 LPA.
iv. Availability for immediate joining (Fig. 1).

How would you set up the interviews to choose the best candidate?

Sol.: Let's first break this problem element-wise.

Table 1 Distinguishing between MADM and MODM

	Criteria	Goal	Attribute	Constraint	Alternative	Usage
Multi-Attribute Decision-Making Problem (MADM)	Attribute	Inherent	Explicit	Inherent	Countable	Selection/ evaluation
Multi-Objective Decision-Making Problem (MODM)	Goal	Explicit	Inherent	Direct mathematical representation	Uncountable	Design

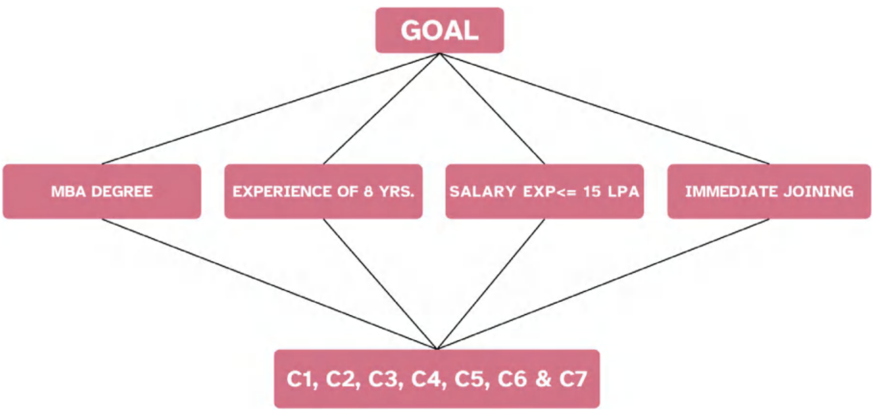

Fig. 1 Hierarchical representation of Problem 1

First, the goal is, 'recruiting a General Manager' and the objective is to 'shortlist the best candidate out of the given seven alternatives'.

Criteria involved are: Educational Qualification (MBA), Relevant Experience, Expected Salary and Transition Period. And here, we have 7 candidates as Alternatives of each other.

Let's determine if the provided problem fits into the criteria for decision making involving multiple criteria (MCDM) by examining its characteristics. The given problem has multiple criteria/attributes as stated earlier.

Now, an ideal candidate should have Maximum Education qualification and Experience whereas we need to minimise the Salary Expected and the Transition Period. Since, there are few attributes that need maximisation and other needs minimization, there is a conflict among criteria.

In order to evaluate a person's education we generally have a count of degrees a person holds, relevant experience is given in years, salary in Indian Rupees (INR) and Transition Period is in days or months. All of these are the units of evaluation for their respective attributes. Such units are called Incommensurable Units. Lastly, hiring/recruiting is a selection process. Therefore, it is verified that the given problem is a MCDM problem.

There are various methods to approach a multi-criteria decision-making problem. Some of the most effective ones are Analytical Network Process (**ANP**), Analytical Hierarchy Process (**AHP**), Technique for Order Preference by Similarity to the Ideal solution (**TOPSIS**), Elimination et choice Translating Reality (**ELECTRE**) and Preference Ranking Organization Method for Enrichment Evaluations (**PROMETHEE**) [4]. Among these, Analytical Hierarchy Process and Technique for Order Preference by Similarity to the Ideal solution are one of the widely used methods. Therefore, in the upcoming section, a detailed discussion on AHP and TOPSIS methods with an amalgamation of fuzzy logic is presented.

3 Integration of Fuzzy Logics with MCDM

Interweaving the logics of fuzzy set theory with the process of deciding when multiple criteria are involved enhances decision-making by accommodating imprecise information and uncertainty. This method substitutes crisp values with fuzzy values, enabling the consideration of degrees of truth and facilitating the handling of vague or subjective criteria. MCDM, thus equipped with the theory of fuzzy sets, can effectively address the complex process of making decisions in real-world scenarios with several conflicting conditions. Approaches like Fuzzy-Analytical Hierarchy Process (F-AHP) and Fuzzy-Technique for Order of Preference by Similarity to Ideal Solution (F-TOPSIS) adeptly manage uncertainty, subjective judgments, and decision modelling complexity when it comes to solving a MCDM problem. Familiarity with these methodologies empowers decision makers to navigate such problems and make well-informed decisions in uncertain environments.

3.1 The F-AHP Method

The Analytical Hierarchy Process (AHP) was developed by Saaty [1] in 1980. It cleverly integrates the assessment scores and opinions of experts into an uncomplicated elementary hierarchy system through the decomposition of complicated problems from higher degree to lower ones. The AHP method for solving MCDM problems is the best while dealing with value measurement models. However, impreciseness and ambiguity in the expert's judgement is a crucial aspect of human decision-making that can easily be dealt with the help of the fuzzy sets theory introduced by Zadeh in 1965 [8].

Fuzzy-AHP systematically integrates the principles of the Analytical Hierarchy Process (AHP) with the theory of fuzzy sets. This combination aims to incorporate uncertainty and vagueness of data in decision-making processes through fuzzy values, thereby offering a clear picture and more reliable solutions. In this method, pairwise comparisons of different alternatives based on specified criteria are conducted using linguistic variables, represented by triangular numbers, to enhance decision support for MCDM problems.

The triangular membership function was defined by Van Laarhoven and Pedrycz in 1983 [1] specifically for pairwise comparison of elements and it is believed to be one the first F-AHP experiment ever performed. Later, Buckley established the priorities of fuzzy ratios for comparison using triangular membership functions and introduced additional methods to evaluate the relative significance of each criterion and option. The procedure to determine the best alternative using the fuzzy-AHP technique is given below [1]:

Step 1: Comparing the stated criteria and alternatives by means of linguistic terminologies as per the table given below:

Using this scale comparison can be done and opinions can be drawn. For eg., if a criteria B1 holds weak importance as compared to B2, it will have a fuzzy triangular number (2, 3, 4). Whereas, when B2 is compared with B1, it will have a reverse fuzzy triangular scale as (1/4, 1/2, 1/2).

[**Note**: Inverse of (r, s, t) = (1/t, 1/s, 1/r)].

Represent the above pair wise comparison in matrix form.

Step 2: When a number of decision makers are involved, the average of choices made by each decision maker $(\tilde{d}_{ij}^{k})$is estimated using:

$$\tilde{d}_{ij} = \sum_{k=1}^{K} \frac{\tilde{d}_{ij}^{k}}{K}$$

Here, $\tilde{d}_{ij}^{k}$is the preference of ith criteria over jth criteria by kth decision maker.

Step 3: Next, the geometric mean (GM) of comparable fuzzy values of individual criterion will be calculated with the aid of the formula given below:

$$\tilde{r}_i = \left(\prod_{j=1}^{n} d_{ij}\right)^{1/n}, \ i = 1, 2, 3, \ldots n$$

Step 4: Calculating fuzzy weights of each criteria by first doing vector wise summation of $\tilde{r}_i$.

Then, find the reciprocal of the summation vector. Arrange it in increasing order and replace the fuzzy triangular number.

Compute the fuzzy weights of criterion $i(w_i)$ by multiplying each $(\tilde{r}_i)$ with the reverse vector.

$$w_i = \tilde{r}_i \otimes (\tilde{r}_1 \otimes \tilde{r}_2 \otimes \ldots \tilde{r}_n)^{-1} = \cdot(rw_i, sw_i, tw_i)$$

Step 5: As, w_i still has fuzzy triangular number value, they require defuzzification by applying the 'Center of Area Method' introduced by Chou and Chang [3].

$$M_i = \frac{(rw_i + sw_i + tw_i)}{3}$$

Step 6: Now, the non fuzzy weights (M_i) needs to be normalised.

$$N_i = \frac{M_i}{\sum_{i=1}^{n} M_i}$$

Perform these steps for all the criteria and alternatives in order to derive the normalised weights. After this, multiplying the weight of each alternative with the relative parameters (criteria) and getting the final calculation done. The option having the highest score is considered to be the decision maker.

Problem 2 Raj wanted to buy a smartphone. So far, he was able to narrow down his options to three models: B1, B2 and B3. However, each phone has different features and specifications that are important to him. He decided to use Multi-criteria decision-making technique (MCDM) in order to make an informed decision based on the following criteria:

1. Performance (P1).
2. Camera Quality (P2).
3. Battery Life (P3).
4. Price (P4) (Fig. 2).

Consider the following pairwise evaluation table (formed utilising the scale of comparative significance (ref. Table 2) and through comparison among various criteria as per Raj's preference) (Table 3).

Based on the provided criteria, which smartphone would you choose: B1, B2 or B3?

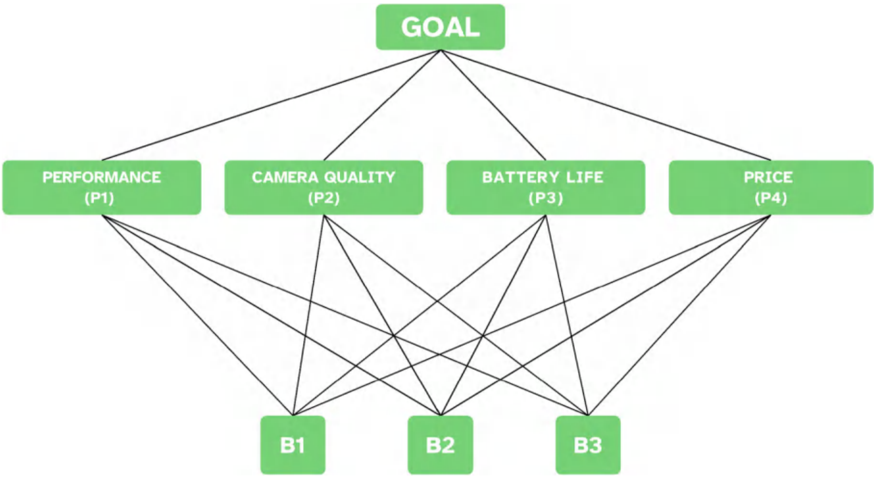

Fig. 2 Hierarchical representation of Problem 2

Table 2 Fuzzy values for linguistic terms as per scale proposed by Saaty [5]

Saaty scale Ref. values	1	3	5	7	9	2, 4, 6, 8	1/3, 1/5, 1/7, 1/9
Linguistic terminologies (as per preference)	Equal	Weak	Fair	Strong	Absolutely significant	Intermediate/mediary values	Inverse comparison
Fuzzy triangular numbers	(1, 1, 1)	(2, 3, 4)	(4, 5, 6)	(6, 7, 8)	(9, 9, 9)	(1, 2, 3), (3, 4, 5), (5, 6, 7), (7, 8, 9)	–

Table 3 Pairwise evaluation table for Problem 2

	Performance (P1)	Camera quality (P2)	Battery life (P3)	Price (P4)
Performance (P1)	1	3	2	4
Camera quality (P2)	1/3	1	6	5
Battery life (P3)	1/2	1/6	1	7
Price (P4)	1/4	1/5	1/7	1

Using the Fuzzy Analytical Hierarchy Method (F-AHP) evaluates his options and selects the best model.

Solving by F-AHP Method:

Step 1: Refer to the pairwise comparison matrix given in the problem.

Step 2: Formation of **Fuzzified pairwise comparison matrix** by converting the linguistic (qualitative) terms into triangular fuzzy values.

Observe that, we can convert the values of crisp from table given above into fuzzy numbers but for reciprocals we are going to apply (Table 4):

$$\tilde{A}^{-1} = (r, s, t)^{-1} = \left(\frac{1}{t}, \frac{1}{s}, \frac{1}{r}\right)$$

Step 3: Compute the fuzzy GM value by applying the formula:

$$\tilde{r}_i = \left(\prod_{j=1}^{n} d_{ij}\right)^{1/n} = [(r_1 * r_2 * r_3 * r_4)^{1/n}; (s_1 * s_2 * s_3 * s_4)^{1/n}; (t_1 * t_2 * t_3 * t_4)^{1/n}],$$

where n is the number of criteria (Table 5).

Step 4: Find the reciprocal of geometric mean summation.

Geometric mean summation = (1.56 + 1.49 + 0.73 + 0.25; 2.21 + 1.78 + 0.87 + 0.29; 2.78 + 2.14 + 1.12 + 0.34) = (4.04; 5.15; 6.38).

Reciprocal of geometric mean summation = $\left(\frac{1}{4.04}; \frac{1}{5.15}; \frac{1}{6.38}\right)$ (Table 6).

Step 5: Calculating fuzzy weight for different criteria by multiplying each fuzzy GM value with the inverse of GM summation we will get fuzzy weights (Table 7).

Table 4 Fuzzified pairwise comparison matrix

	Performance (P1)	Camera quality (P2)	Battery life (P3)	Price (P4)
Performance (P1)	(1, 1, 1)	(2, 3, 4)	(1, 2, 3)	(3, 4, 5)
Camera quality (P2)	(1/4, 1/3, 1/2)	(1, 1, 1)	(5, 6, 7)	(4, 5, 6)
Battery life (P3)	(1/3, 1/2, 1)	(1/7, 1/6, 1/5)	(1, 1, 1)	(6, 7, 8)
Price (P4)	(1/5, 1/4, 1/6)	(1/6, 1/5, 1/4)	(1/8, 1/7, 1/6)	(1, 1, 1)

Table 5 Comparison matrix for various criteria

	Performance (P1)	Camera quality (P2)	Battery life (P3)	Price (P4)	Fuzzy geometric mean value (r_i)		
Performance (P1)	(1, 1, 1)	(2, 3, 4)	(1, 2, 3)	(3, 4, 5)	1.57	2.21	2.78
Camera quality (P2)	(1/4. 1/3, 1/2)	(1, 1, 1)	(5, 6, 7)	(4, 5, 6)	1.49	1.78	2.14
Battery life (P3)	(1/3, 1/2, 1)	(1/7, 1/6, 1/5)	(1, 1, 1)	(6, 7, 8)	0.73	0.87	1.12
Price (P4)	(1/5, 1/4, 1/3)	(1/6, 1/5, 1/4)	(1/8, 1/7, 1/6)	(1, 1, 1)	0.25	0.29	0.34

Table 6 GMs of fuzzy values of comparison

	Fuzzy geometric mean value (r_i)		
Performance (P1)	1.56	2.21	2.78
Camera quality (P2)	1.49	1.78	2.14
Battery life (P3)	0.73	0.87	1.12
Price (P4)	0.25	0.29	0.34
Total	4.04	5.15	6.38
Reciprocal of geometric mean summation	0.25	0.19	0.16
Increasing order	0.16	0.19	0.25

Table 7 Relative fuzzy weights for P1, P2 and P3

	Fuzzy geometric mean value			Fuzzy weights		
Performance (P1)	1.56	2.21	2.78	0.25	0.42	0.70
Camera quality (P2)	1.49	1.78	2.14	0.24	0.34	0.54
Battery life (P3)	0.73	0.87	1.12	0.12	0.17	0.28
Price (P4)	0.25	0.29	0.34	0.04	0.06	0.09

Step 6: Calculating the non-fuzzy (averaged) weight of each criteria using the mean of fuzzy numbers for each criterion and subsequently, determining the normalised weights for each criterion based on the non-fuzzy weights (Table 8).

Step 7: Computing the weightage of each criterion w.r.t. the alternative.

(**Note**: It might already be given or assumed as per the information provided.) (Tables 9, 10, 11 and 12)

Step 8: Computing the geometric Mean and fuzzy weights of alternatives w.r.t. the criteria individually (Tables 13, 14, 15 and 16).

Table 8 Non-fuzzy (averaged) and normalised weights for P1, P2 and P3

	Non-fuzzy weight	Normalised weights
Performance (P1)	0.46	0.43
Camera quality (P2)	0.37	0.34
Battery life (P3)	0.19	0.18
Price (P4)	0.06	0.06

Table 9 Comparison matrix w.r.t. P1

	B1	B2	B3
B1	(1, 1, 1)	(4, 5, 6)	(6, 7, 8)
B2	(1/6, 1/5, 1/4)	(1, 1, 1)	(2, 3, 4)
B3	(1/8, 1/7, 1/6)	(1/4, 1/3, 1/2)	(1, 1, 1)

Table 10 Comparison matrix w.r.t. P2

	B1	B2	B3
B1	(1, 1, 1)	(1/7, 1/6, 1/5)	(1/4, 1/3, 1/2)
B2	(5, 6, 7)	(1, 1, 1)	(1/8, 1/7, 1/6)
B3	(2, 3, 4)	(6, 7, 8)	(1, 1, 1)

Table 11 Comparison matrix w.r.t. P3

	B1	B2	B3
B1	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/9, 1/8, 1/7)
B2	(2, 3, 4)	(1, 1, 1)	(5, 6, 7)
B3	(7, 8, 9)	(1/7, 1/6, 1/5)	(1, 1, 1)

Table 12 Comparison matrix w.r.t. P4

	B1	B2	B3
B1	(1, 1, 1)	(2, 3, 4)	(1/6, 1/5, 1/4)
B2	(1/4, 1/3, 1/2)	(1, 1, 1)	(1, 2, 3)
B3	(4, 5, 6)	(1/3, 1/2, 1)	(1, 1, 1)

Table 13 GM and fuzzy weight w.r.t. P1

	Geometric mean			Fuzzy weight		
B1	2.88	3.27	3.63	0.58	0.72	0.94
B2	0.69	0.84	1	0.14	0.19	0.26
B3	0.31	0.36	0.44	0.06	0.08	0.11
Total	3.88	4.47	5.07			
Reverse	0.26	0.22	0.20			
↑ing order	0.20	0.22	0.26			

Table 14 GM and fuzzy weight w.r.t. P2

	Geometric mean			Fuzzy weight		
B1	0.33	0.38	0.46	0.07	0.09	0.13
B2	0.85	0.95	1.05	0.18	0.23	0.30
B3	2.29	2.76	3.17	0.48	0.66	0.92
Total	3.47	4.09	4.68			
Reverse	0.29	0.24	0.21			
↑ing order	0.21	0.24	0.29			

Table 15 GM and fuzzy weight w.r.t. P3

	Geometric mean			Fuzzy weight		
B1	0.3	0.35	0.41	0.06	0.09	0.12
B2	2.15	2.62	3.03	0.45	0.65	0.88
B3	1	1.1	1.22	0.21	0.28	0.35
Total	3.45	4.07	4.67			
Inverse	0.29	0.25	0.21			
↑ing Order	0.21	0.25	0.29			

Table 16 GM and fuzzy weight w.r.t. P4

	Geometric mean			Fuzzy weight		
B1	0.69	0.84	1	0.17	0.28	0.41
B2	0.63	0.87	1.14	0.16	0.29	0.47
B3	1.1	1.35	1.81	0.28	0.45	0.74
Total	2.42	3.06	3.95			
Inverse	0.41	0.33	0.25			
↑ing Order	0.25	0.33	0.41			

Table 17 Non-fuzzy weight and NW of P1

	Non-fuzzy weight	Normalised weight
B1	0.75	0.73
B2	0.2	0.19
B3	0.08	0.08

Table 18 Non-fuzzy weight and NW of P2

	Non-fuzzy weight	Normalised weight
B1	0.10	0.10
B2	0.24	0.23
B3	0.69	0.67

Step 9: Computing the non fuzzy and normalised weights (NW) of each alternative for each criterion (Tables 17, 18, 19 and 20).

Step 10: Compiling the normalised weights of each criterion in a table (Table 21).

Step 11: Combining the outcomes for every alternative based on individual criteria (Table 22).

Table 19 Non-fuzzy weight and NW of P3

	Non-fuzzy weight	Normalised weight
B1	0.09	0.09
B2	0.66	0.64
B3	0.28	0.27

Table 20 Non-fuzzy weight and NW of P4

	Non-fuzzy weight	Normalised weight
B1	0.29	0.27
B2	0.31	0.28
B3	0.49	0.45

Table 21 Normalised weights of P1, P2, P3 and P4 w.r.t. alternatives

Alternatives	Performance (P1)	Camera quality (P2)	Battery life (P3)	Price (P4)
B1	0.73	0.10	0.09	0.27
B2	0.19	0.23	0.64	0.28
B3	0.08	0.67	0.27	0.45

Table 22 Outcomes for alternatives B1, B2 and B3

Criteria		Ratings of alternatives in relation to corresponding criteria		
	Weights	B1	B2	B3
Performance (P1)	0.43	0.73	0.19	0.08
Camera quality (P2)	0.34	0.10	0.23	0.67
Battery life (P3)	0.18	0.09	0.64	0.27
Price (P4)	0.06	0.27	0.28	0.45
Total		1.19	1.34	1.47

From the above, we can see that B3 has the highest rating. Therefore, it can be concluded that the mobile B3 will be the most suitable option for Raj in consideration with four criteria and preferences as per him.

3.2 *The F-TOPSIS Method*

The Hierarchical F-TOPSIS is a comprehensive approach in the domain of fuzzy MCDM methods. It is tailor-made especially to deal with the uncertainty and complexity that the decisions in the actual world inhere, with greater ease and precision. By reducing the amount of numerical calculations involved in determining the order of ranking of alternatives, Hierarchical F-TOPSIS enhances the practical applicability of fuzzy MCDM methods to real-world problems, which was the key motivation behind the development of the former. Not only does it have a systematic incorporation of a hierarchical structure, it also offers efficiency for qualitative assessments, like BPO solutions. Its structured approach and ability to streamline the decision-making process make it a valuable tool in various domains, from business to engineering and beyond. Now, in the methodology given below, we discuss a vertex method to further enhance the TOPSIS procedure in a fuzzy environment, offering a simpler yet effective approach for handling uncertainty in decision-making [2].

Step 1: Decide upon the ones making the decision and recognise the criteria of evaluation.

Step 2: In order to evaluate the weight of the given conditions, select the suitable linguistic (qualitative) variables and determine the ratings given to alternatives in comparison with the respective criteria.

Step 3: Combine the weightage of the given criteria to get the consolidated fuzzy weightage for each condition. Pool the opinions of the ones making the decision to attain aggregated ratings under each criteria.

Step 4: Create a fuzzified decision matrix and then create another normalised decision matrix containing fuzzy values.

Step 5: Formulate the Adjusted standard (normal) fuzzy decision matrix.

Step 6: Identify Ideal-positive fuzzy solution ($FPIS, A^*$) and Ideal-negative fuzzy solution ($FNIS, A^-$):

$$A^* = \left(v_1^*, v_2^*, \ldots, v_n^*\right)$$
$$A^- = \left(v_1^-, v_2^-, \ldots, v_n^-\right)$$

where $v_j^* = (1, 1, 1)$ and, $v_j^- = (0, 0, 0)$, j = 1, 2, 3, 4,… n.

Step 7: Determine the Euclidean distance of every option from both the positive–negative ideal solution.

$$d_i^* = \sum_{j=1}^{n} d\left(v_{ij}, v_j^*\right), i = 1, 2, 3, \ldots, m$$
$$d_i^- = \sum_{j=1}^{n} d\left(v_{ij}, v_j^-\right), i = 1, 2, 3, \ldots, m$$

Step 8: Estimate the coefficient of closeness (determine the ranking of the given choices) of each option using the formula:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, i = 1, 2, 3, ..., m$$

Step 9: As per the coefficient of closeness, determine the order of ranking by arranging the alternatives in descending order. The alternative who has the highest coefficient of closeness should be identified as the perfect choice.

Solving by Fuzzy TOPSIS Method:

Problem 3 Assume that an IT company wants to recruit a Software System Engineer. There are three candidates A1, A2, and A3 who have been selected for the final round of the recruitment process. A committee of three people (M1, M2 and M3) is formulated to successfully execute the final round of selection and decide upon the most deserving candidate for the given designation. The four criteria of benefit considered by them are:

- Emotional Resilience (E1)
- Communication Skill (E2)
- Personality and Self Esteem (E3)
- Relevant Experience (E4) (Fig. 3)

.

Consider Tables 23 and 24 as references that will help us in determining the linguistic & fuzzy weights of each criteria whereas Tables 26 and 27 are the remaining part of the question.

Problem Solving Using TOPSIS Method:

Step 1: Refer the linguistic weighting and rating variables used by the decision makers presented in Tables 23 and 24.

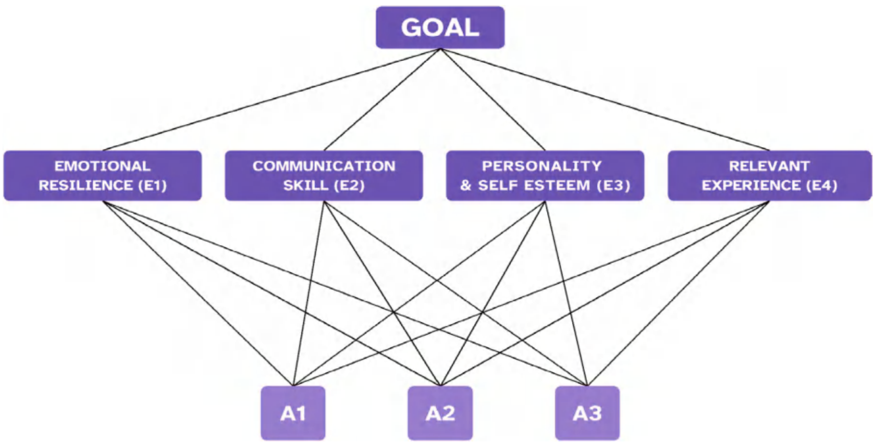

Fig. 3 Hierarchical representation of Problem 3

Table 23 Linguistic terms representing the relative importance of each criterion

Negligible (N)	(0.0, 0.0, 0.1)
Low (L)	(0.0, 0.1, 0.3)
Mild (MD)	(0.1, 0.3, 0.5)
Moderate (M)	(0.3, 0.5, 0.7)
Strong (S)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Intense (I)	(0.9, 1.0, 1.0)

Table 24 Terminologies describing the scoring levels (in crisp form)

Very Poor	Poor	Fair	Good	Very good
VP	P	F	G	VG
(0, 0, 1)	(0, 1, 3)	(3, 5, 7)	(5, 7, 9)	(9, 10, 10)

Step 2: Create a decision matrix and derive the fuzzy weight of each criteria by considering evaluations based on the given linguistic terms (ref. Tables 25 and 26) into fuzzy triangular numbers.

Step 3: Formation of the fuzzy normalised decision matrix by applying linear transformation. Here, criteria E1 has to be reduced whereas E2, E3 and E4 needs maximisation. So, we have normalised the matrix accordingly (Table 28).

Step 4: Now, let's create a fuzzy weighted normalised decision matrix (Table 29).

Step 5: Determination of $(FPIS, A^*)$ and $(FNIS, A^-)$.

We know that,

$$A^* = [(1, 1, 1), (1, 1, 1), (1, 1, 1), (1, 1, 1)]$$
$$A^- = [(0, 0, 0), (0, 0, 0), (0, 0, 0), (0, 0, 0)]$$

Step 6: Calculating the Euclidean distance of every candidate from the 'Fuzzy-Ideal Positive and Ideal Negative Solutions' respectively (Table 30).

Step 7: Calculating the closeness coefficient of A1, A2 and A3.

Table 25 Importance weight of the benefit criteria

	M1	M2	M3
E1	H	I	S
E2	I	I	L
E3	M	I	H
E4	I	MD	L

Table 26 Evaluations of the candidates by recruitment team across the standard criteria

Criteria of benefit	Candidates	Selection committee		
		M1	M2	M3
E1	A1	F	G	F
	A2	G	VG	F
	A3	VG	F	F
E2	A1	G	F	VG
	A2	G	G	G
	A3	F	G	VG
E3	A1	VG	VG	G
	A2	F	G	G
	A3	VG	G	G
E4	A1	F	F	F
	A2	G	VG	G
	A3	F	F	VG

Table 27 Determination of fuzzy weights of criteria individually

	E1	E2	E3	E4
A1	(3.7, 5.7, 7.7)	(5.7, 7.3, 8.7)	(7.7, 9, 9.7)	(3, 5, 7)
A2	(5.7, 7.3, 8.7)	(5, 7, 9)	(4.3, 6.3, 8.3)	(6.3, 8, 9.3)
A3	(5, 6.7, 8)	(5.7, 7.3, 8.7)	(6.3, 8, 9.3)	(5, 6.7, 8)
Weight	(0.87, 0.87, 0.97)	(0.6, 0.7, 0.77)	(0.63, 0.8, 0.9)	(0.33, 0.47, 0.6)

Table 28 Fuzzy decision matrix after normalisation

	E1	E2	E3	E4
A1	(0.43, 0.66, 0.89)	(0.57, 0.73, 0.87)	(0.77, 0.9, 0.97)	(0.3, 0.5, 0.7)
A2	(0.66, 0.84, 1)	(0.5, 0.7, 0.9)	(0.43, 0.63, 0.83)	(0.63, 0.8, 0.93)
A3	(0.57, 0.77, 0.92)	(0.57, 0.73, 0.87)	(0.63, 0.8, 0.93)	(0.5, 0.67, 0.8)

Table 29 Fuzzy weighted normalised decision matrix

	E1	E2	E3	E4
A1	(0.37, 0.57, 0.86)	(0.34, 0.51, 0.67)	(0.49, 0.72, 0.87)	(0.1, 0.24, 0.42)
A2	(0.57, 0.73, 0.97)	(0.3, 0.49, 0.69)	(0.27, 0.5, 0.75)	(0.2, 0.38, 0.56)
A3	(0.5, 0.67, 0.9)	(0.34, 0.51, 0.67)	(0.34, 0.64, 0.84)	(0.17, 0.31, 0.48)

Table 30 Euclidean distance of A1, A2 and A3 from FPIS and FNIS

	$A*$	A^{-}
A1	3.57	3.62
A2	3.44	3.88
A3	3.46	3.81

$$CC_1 = 0.46,$$
$$CC_2 = 0.53,$$
$$CC_3 = 0.52.$$

By observing the closeness coefficients, we can conclude the order of ranking of the three candidates as:

$$\text{A2} > \text{A3} > \text{A1}.$$

And, from the above ranking order, it is evident that A2 is the best candidate for selection.

4 Conclusion

Fuzzy logic serves as a more accurate and dependable tool in decision making problems involving multiple criteria (MCDM). The applications of MCDM are extensive, spanning fields and domains given that the nature of decision-making is pervasive. Fuzzy MCDM finds application in diverse areas including project selection, hospital performance evaluation, agriculture strategy formulation, software development, and global logistics management, highlighting its broad scope. To tackle such challenges, Fuzzy Analytic Hierarchy Process (F-AHP) and Fuzzy Technique for Order Preference by Similarity to Ideal Solution (F-TOPSIS) emerge as highly regarded methods, with research usage rates of 9.53 and 7.31% [4], respectively. In conclusion, the integration of fuzzy logic in MCDM not only enhances precision and applicability across various domains but also provides clarity and reduces ambiguity in complex decision-making scenarios. Thus, it stands as a vital tool for addressing multifaceted challenges in today's dynamic world.

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A Case Study on Pharmaceutical Supplier Selection by Using the Integrated Fuzzy AHP MABAC Method

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Abstract In the pharmaceutical sector, choosing suppliers is essential to guarantee the effectiveness, dependability, and quality of the supply chain. Pharmaceutical firms face a vital problem in strategically selecting their suppliers due to strict regulatory constraints, increasing demands for innovation, and an emphasis on cost-effectiveness. Securing a strong supply chain, reducing risks, and maintaining the pharmaceutical industry's dedication to providing safe and effective drugs all depend on efficient supplier selection procedures. This changing environment emphasizes how important it is for the pharmaceutical industry to choose suppliers carefully. In this chapter, the fuzzy Analytical Hierarchy Process (AHP) is employed to establish the normalized weight attributes. Subsequently, the fuzzy Multi-Attributive Border Approximation Area Comparison (MABAC) approach is utilized for pharmaceutical supplier selection. The comprehensive supplier assessment involves evaluating six criteria, each with five alternatives. A normalization process is applied to derive the weighted normalized fuzzy decision matrix. Beneficial and non-beneficial criteria are computed for each alternative, leading to the determination of an overall performance index. Next, based on the final table's descending total performance index values, the alternatives are ranked from best to worst. According to the study's outcomes, when choosing a pharmaceutical supplier, the alternative with the greatest overall performance index is the best option.

Keywords Fuzzy set · MCDM · AHP · MABAC · Supply chain management · Suppliers · Pharmaceutical supplier selections

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1 Introduction

Pharmaceutical industry research, which spans a variety of fields and aims to advance drug discovery, development, and patient care, is a pillar of medical advancement. This important study explores cutting-edge technologies, novel chemicals, and creative therapies to address unmet medical needs. By navigating complex regulatory environments and streamlining manufacturing procedures, it protects the quality and safety of pharmaceutical products. Through rigorous trials, clinical research in this field assesses patient outcomes, safety profiles, and therapeutic efficacy. This study also examines healthcare economics, enabling equitable healthcare delivery and economic access to necessary medications. Despite challenges including rising costs and technological advancements, strong research remains essential to creativity, enhancing outcomes for patients, and shaping the direction of global health.

Making decisions in the complex and quickly changing pharmaceutical sector requires sophisticated techniques that take uncertainty and imprecision into consideration. To address the complex nature of evaluating many criteria in pharmaceutical industry research, the integrated fuzzy AHP MABAC emerges as a potent instrument. Researchers and decision-makers can successfully handle the subjective judgments, ambiguity, and incomplete information endemic to this field using this strategy, which blends the principles of AHP-MABAC with fuzzy logic.

The primary research questions (RQ) that served as the basis for this study are as follows:

- RQ1: What are the primary difficulties facing the pharmaceutical industry?
- RQ2: What connections exist between these difficulties?
- RQ3: Why do we choose to rank the Supplier using this specific integrated fuzzy AHP MABAC approach (MCDM)?
- RQ4: How could this research's contribution to the industry be used?

The subsequent research objectives (RO) will help this book chapter to address the aforementioned research questions.

- RO1: Identify the main challenges in securing a reliable supplier.
- RO2: Identify the connections between these challenges.
- RO3: Explain the advantages of utilizing this specific integrated fuzzy AHP-MABAC technique.
- RO4: Offer both industrial and academic implications and recommendations.

The pharmaceutical sector confronts many obstacles, such as strict laws, rising R&D expenses, and problems with intellectual property. Complications related to market access, delays in drug approval, and pricing pressures contribute to complexity, and ongoing adaptation is required due to technological upheavals. Issues with public trust and competition from developing markets influence the dynamics of the industry. Resilience is needed in light of supply chain vulnerabilities and global health concerns. Overcoming these obstacles requires a strategic mindset, flexibility, and a dedication to innovation to guarantee that healthcare technology keeps improving.

The pharmaceutical industry relies on a complex network of suppliers for raw materials and components. The integrated fuzzy AHP-MABAC facilitates the assessment of suppliers by considering factors like product quality, reliability, and delivery performance. Uncertainties related to supplier performance can be effectively modelled using fuzzy membership functions. Incorporating fuzzy logic into the fuzzy MCDM method for supplier selection is essential because it effectively handles the uncertainty and subjectivity prevalent in the pharmaceutical industry. Fuzzy logic quantifies ambiguous and imprecise data, leading to more accurate evaluations of criteria like reliability, quality, and service. It enhances decision-making by converting human judgments, typically expressed in linguistic terms, into fuzzy numbers. This results in more precise criteria weighting and robust evaluations, particularly in complex scenarios. By thoroughly considering both quantitative and qualitative factors, fuzzy logic ensures more reliable and comprehensive supplier selection decisions.

In this book chapter, we employ the integrated fuzzy AHP-MABAC method to address the pharmaceutical supplier selection problem. The integrated fuzzy AHP-MABAC method offers a thorough solution for selecting suppliers in the pharmaceutical sector, adeptly addressing the inherent uncertainties and complexities in decision-making. Combining fuzzy logic with the AHP helps manage subjective judgments and ambiguous data, resulting in a more refined evaluation of criteria importance. The F-MABAC method enhances this process by assessing alternatives against the defined criteria and calculating distances from the border approximation area to identify the most appropriate supplier. This integrated approach enables decision-makers to systematically prioritize and assess potential suppliers, balancing qualitative and quantitative factors, ultimately facilitating more dependable and well-informed supplier selection decisions.

To achieve the main goal, the remaining portions of this chapter is structured as follows: Sect. 2 provides a literature review, while Sect. 3 explores the intricacies of the integrated fuzzy MCDM method. Section 4 presents the case study and discusses the findings. Section 5 examines the implications of the study. Section 6 expresses the Research limitations and finally, Sect. 7 summarizes the conclusions of this chapter.

2 Literature Review

The pharmaceutical sector exhibits ongoing difficulties such as complicated intellectual property, increasingly expensive research and development (R&D), and strict laws. The need for the sector to proactively manage the complexities of market access and pricing pressures is emphasized by academics. Discussions on adaptable methods are prompted by recurring topics such as drug approval delays, technology changes, and global health crises. Furthermore, researchers investigate the complex effects of public opinion, dynamics of trust, and increased competition from developing economies. Currently, a lot of researchers are working in that field to grow the pharmaceutical industry. The following are some of the most frequently cited approaches in the literature. Abdullah and Hasibuan use the application of pharmaceutical industry supplier selection [1]. Ayhan employed an F-AHP method to address supplier selection issues in a case study involving a gear motor company [2]. The fuzzy AHP approach is being used by Amallynd et al. in the clothing sector to pick suppliers [3]. AHP is used by Isa et al. to solve the supplier selection issue [4]. Sheykhzadeh et al. employed a combined decision-making framework for selecting suppliers [5]. Kahraman et al. employed interval-valued intuitionistic F-TOPSIS methodology in the domain of pharmaceutical third-party logistics supplier selection [6]. The Fuzzy AHP-Vikor approach is employed by Rani et al. in Pharmaceutical Manufacturing Company supplier selection [7]. Sabbaghi et al. highlight the significance of project risk management in the pharmaceutical industry, specifically in the context of drug production, during the supplier selection process [8]. Sahoo and Debnath utilized the F-MABAC technique to determine the optimal location for implementing regenerative practices in tourism [9]. In a review paper, Sahoo et al. address supplier selection in the age of Industry 4.0 by applying the MCDM approach [10]. In the process of choosing green suppliers, Wang et al. apply the MABAC technique [11]. The MABAC approach is used by Wei et al. to choose suppliers for products intended for medicinal consumption [12]. For the auto-making industry's supplier selection, Mishra et al. employ the MABAC approach [13]. Sahoo and Debnath employed a hybrid decision model to select an optimal location for hydroelectric power plant [14].

While earlier research emphasized the use of intricate mathematical methods for selecting the most suitable supplier in the pharmaceutical industry, there is an ongoing need to employ a straightforward and comprehensible MCDM technique to address challenges in system selection.

Two prevalent and effective combined strategies for this purpose are the fuzzy AHP and fuzzy MABAC. These strategies create a decision-making framework that aids in assessing and choosing alternatives based on various factors. They play a crucial role in making optimal choices from a diverse array of options, taking into account various factors that may align with different objectives. The process involves normalizing the criteria, assigning weights, and calculating the performance ratings for each alternative. By considering these aspects, the technique aims to determine the best option. Ultimately, the alternative with the highest score is deemed the optimal choice. This simplified yet effective approach provides decision-makers with a practical means of navigating the complexities associated with system selection in the pharmaceutical industry.

3 Methodology

Supplier selection is a crucial element of supply chain management, involving the process of selecting suppliers who ensure the movement of goods and services from raw material providers to final consumers. This chapter aims to introduce a novel strategy that improves supplier selection in a multi-supplier setting by utilizing the integrated fuzzy AHP MABAC Method. It also seeks to illustrate the reliability and significance of the criteria by managing uncertain information throughout the decision-making process.

Here first we use the F-AHP method to get the weight attribute, then we employ the F-MABAC technique to identify the best supplier for the pharmaceutical sector.

The decision-maker currently compares the available options or requirements by utilizing the linguistic phrases outlined in the provided Tables 1 and 2.

Fuzzy AHP Method:

One of the most effective methods for multi-criteria decision-making is the "Analytical Hierarchy Process (AHP)". This method aids decision-makers in navigating complex, multi-objective problems by systematically evaluating and selecting appropriate solutions. AHP operates on the principle of measurement through pairwise comparisons, providing a structured approach to assigning values to criteria and alternatives.

The F-AHP is an upgraded version of the AHP that arises in the growth of decision-making methodologies. F-AHP, in contrast to its predecessor, takes into

Table 1 List of abbreviations

Abbreviation	Description
MCDM	"Multi-Criteria Decision-Making"
AHP	"Analytical Hierarchy Process"
MABAC	"Multi-Attributive Border Approximation area Comparison"
LVs	"Linguistic variables"
TFNs	"Triangular Fuzzy Numbers"
BAA	"Border Approximation Area"
F-AHP	"Fuzzy Analytical Hierarchy Process"
F-MABAC	"Fuzzy Multi-Attributive Border Approximation area Comparison"

Table 2 Linguistic variables and the corresponding triangular fuzzy numbers

LVs	TFNs
Equal importance	(1, 1, 1)
Very less importance	(1, 3, 4)
Less importance	(2, 3, 4)
Moderately less importance	(4, 5, 6)
Moderate	(5, 6, 7)
Moderately strong importance	(6, 7, 7)
Strong importance	(7, 8, 9)
Absolutely important	(8, 8, 9)

account the inherent ambiguity and uncertainty that frequently surround decision-making processes. F-AHP transcends the strict confines of conventional decision-making techniques by introducing the idea of fuzziness and enabling the expression of personal evaluations using fuzzy numbers.

By utilizing fuzzy integers and fuzzy arithmetic processes, F-AHP provides a more accurate depiction of decision-makers' opinions. This realism, rooted in the acknowledgment of imprecision, establishes a resilient framework for decision-making. The utilization of fuzzy AHP enables decision-makers to navigate the complexities of decision scenarios with a nuanced understanding of uncertainties, Ultimately enhancing the process of decision-making's efficiency [15].

The steps of F-AHP for finding weight is as follows:

Step 1: Development of a Pairwise comparison matrix utilizing fuzzy numbers.

The pairwise comparison matrix with fuzzy elements is illustrated below [16].

$$\mathrm{A} = \begin{bmatrix} 1 & a_{12} & \ldots & a_{1n} \\ \ldots & \cdots & \ldots & \ldots \\ \vdots & \ddots & \vdots & \ldots \\ \ldots & \cdots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & 1 \end{bmatrix}$$

Here each a_{ij} is expressed as a triangular fuzzy number that is $a_{ij} = (a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u})$

Step 2:

Next, for each criterion, the geometric mean of the fuzzy comparison values is determined.

By using the following formula [2, 2]

$$r_i = \sqrt[n]{\prod_{j=1}^{n} a_{ij}} = \left(\sqrt[n]{\prod_{j=1}^{n} a_{ij}^{l}}, \sqrt[n]{\prod_{j=1}^{n} a_{ij}^{m}}, \sqrt[n]{\prod_{j=1}^{n} a_{ij}^{u}} \right), \quad \mathrm{i} = 1, 2, \ldots, \mathrm{n} \tag{1}$$

Step 3:

Next, another table is created and contains the geometric means of the fuzzy comparison values for each criterion.

Then we do each column sum. Write each column sum and its inverse on the same table. After that, we write the inverse value of each column sum in increasing order.

Step 4:

This phase involves multiplying each r_i by the reverse vector to determine the weight of the criterion, w_i.

which is expressed in increasing order.

The formula is written below

$$W_i = r_i * (r_1 + r_2 + \ldots + r_n)^{-1} = (lw_i, mw_i, uw_i) \quad (2)$$

Step 5:

As we are dealing with uncertain or imprecise triangular numbers, it is necessary to remove the fuzziness using the "centre-of-area method" introduced by Chou and Chang [17].

This involves applying the below Equation to achieve de-fuzzification.

$$M_i = \frac{(lw_i + mw_i + uw_i)}{3} \quad (3)$$

Step 6:

In the above step, we get the crisp number. But it needs to be normalized by using the below Eq. [2]

$$N_i = \frac{M_i}{\sum_{i=1}^{n} M_i} \quad (4)$$

Here we get the required normalized weight attribute by using the fuzzy AHP method.

Next, we use the fuzzy MABAC technique to get the best supplier.

Fuzzy MABAC Method:

The MABAC approach gains a more sophisticated comprehension of criteria and their relative significance by integrating fuzzy logic. To ensure a thorough understanding of the approach, this section offers a detailed examination of the stages required in applying the MABAC method in a fuzzy environment.

A real-world scenario will be used to demonstrate the application of the MABAC approach, aiming to improve comprehension by guiding readers through each step of understanding and applying the concept. Several noteworthy studies in the realm of MABAC in uncertain environments include those conducted by Yu et al. [18], Wang et al. [19], Büyüközkan et al. [20], and Pamucar et al. [21]. These studies contribute valuable insights and perspectives to the understanding of MABAC under conditions of uncertainty.

The steps are given below.

Step 1:

A decision matrix is created below using triangular fuzzy numbers while considering the qualitative information [22, 22]:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & \dots & \dots & y_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{bmatrix}$$

where $y_{ij} = (y_{ij}^l, y_{ij}^m, y_{ij}^u)$ is a TFN.

Step 2: Normalized decision matrix [9]

The decision matrix in its normalized form is provided below.

$$N = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1n} \\ n_{21} & \dots & \dots & n_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ n_{m1} & n_{m2} & \dots & n_{mn} \end{bmatrix}$$

The value of n_{ij} we get by employing the following formula.
For benefit type criteria (A greater value of the criteria is preferred):

$$\text{Where } n_{ij} = \frac{(y_{(ij)} - y_i^-)}{(y_i^+ - y_i^-)} \tag{5}$$

Here $y_i^+ = \max_i\{y_{ij}^u\}; j = 1,2 \dots, n.$
$y_i^- = \min_i\{y_{ij}^l\}; j = 1,2 \dots, n.$
For non-benefit type criteria (a lower value of the criteria is preferable)

$$\text{Where } n_{ij} = \frac{(y_{(ij)} - y_i^+)}{(y_i^- - y_i^+)} \tag{6}$$

Here $y_i^+ = \max_i\{y_{ij}^u\}; j = 1,2 \dots, n.$
$y_i^- = \min_i\{y_{ij}^l\}; j = 1,2 \dots, n.$

Step 3: The weighted normalized decision matrix [9, 21]:

The components of the weighted normalized decision matrix are derived through the application of the following equation.

$$v_{ij} = N_j.\left(n_{ij} + 1\right); i = 1, \ldots, m, j = 1, \ldots, n. \tag{7}$$

where $N_1, \ldots .N_j$ is the weight vector obtained by using the weight that we get from Eq. (4) of the fuzzy AHP method

Using the above equation, the weight-normalized decision matrix V represent below [21]

$$\mathrm{V} = \begin{bmatrix} v_{11} & v_{12} & \ldots & v_{1n} \\ v_{21} & v_{22} & \ldots & v_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ v_{m1} & v_{m2} & \ldots & v_{mn} \end{bmatrix}$$
$$= \begin{bmatrix} N_1.(n_{11}+1) & N_2.(n_{12}+1) & \ldots & N_n.(n_{1n}+1) \\ N_1.(n_{21}+1) & N_2.(n_{22}+1) & \ldots & N_n.(n_{2n}+1) \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ N_1.(n_{m1}+1) & N_2.(n_{m2}+1) & \ldots & N_n.(n_{mn}+1) \end{bmatrix} \tag{8}$$

Here "v_{ij}" represent the corresponding elements of the weight-normalized decision matrix V.

Step 4: The Border Approximation Area Matrix

The BAA for each criterion is given as below [22].

$$\text{where } g_j = \sqrt[m]{\prod_{i=1}^{m} v_{ij}} = \left(\sqrt[m]{\prod_{i=1}^{m} v_{ij}^{l}}, \sqrt[m]{\prod_{i=1}^{m} v_{ij}^{m}}, \sqrt[m]{\prod_{i=1}^{m} v_{ij}^{u}}\right) \quad j = 1, .., n \tag{9}$$

In this context, V represents the elements of the weighted matrix, with each element denoted as v_{ij}, and m signifies the no. of alternatives.

A BAA matrix G is created, once the value g_j for each criterion has been determined, structured as an n × 1 matrix. (Here n represents the total no. of criteria used for selecting alternatives from the options provided).

The BAA matrix is as below [21]

$$G = \begin{array}{c} C_1 \; C_2 \; \ldots \; C_n \\ \left[g_1 \; g_2 \; \ldots \; g_n \right] \end{array} \tag{10}$$

Step 5: Next, calculate the distance of each alternative from the BAA for the elements in the matrix (Q) [9, 22]

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \cdots & \ldots & \ldots & \ldots \\ q_{m1} & q_{m2} & \ldots & q_{mn} \end{bmatrix}$$

Here Q = V–G.

$$\text{Where } \mathrm{q}_{ij} = \begin{cases} d\left(v_{ij,}g_j\right) & \text{for } v_{ij} > g_j \\ o & \text{for } v_{ij} = g_j \\ -d\left(v_{ij,}g_j\right) & \text{for } v_{ij} < g_j \end{cases} \tag{11}$$

And $d\left(v_{ij,}g_j\right) = \sqrt[2]{\frac{1}{3}\left[\left(v_{ij}^l - g_j^l\right)^2 + (v_{ij}^m - g_j^m)^2 + (v_{ij}^u - g_j^u)^2\right]}$

Here

$$v_{ij} = \left(v_{ij}^l, v_{ij}^m, v_{ij}^u\right) \, and \; g_j = \left(g_j^l, g_j^m, g_j^u\right)$$

And if $\frac{(v_{ij}^l + 4v_{ij}^m + v_{ij}^u)}{6} > \frac{(g_j^l + 4g_j^m + g_j^u)}{6}$ then $v_{ij} > g_j$.

Alternative A_i may be associated with the BAA. The approximation region is divided into three sections:

- "Border approximation area (represented by G)"
- "Lower approximation area (represented by G^-)"
- "Upper approximation area (represented by G^+)"

$$A_i \in \begin{cases} G^+ \; if \; q_{ij} > 0 \\ G \; \; if \; q_{ij} = 0 \\ G^- \; if \; q_{ij} < 0 \end{cases} \tag{12}$$

It clearly expresses that "if the value of $q_{ij} > 0$, $then A_i \in G^+$ and the alternative A_i is near or equal to the ideal alternative". "If the value $q_{ij} < 0$, then, $A_i \in G^-$ and the alternative A_i is near or equal to the anti-ideal alternative". "If $q_{ij} = 0$ then A_i belongs to G and it lies in the border approximation area" [21].

Step 6: Ranking of the alternatives.

The final crisp values of the alternatives are obtained by using the below formula [21].

$$S_i = \sum_{j=1}^{n} q_{ij}, j = 1,2, .., n, i = 1,2, .., m \tag{13}$$

where n denotes the no. of criteria and m denotes the no. of alternatives.

4 Case Study and Result Discussion

In this book chapter, we take the case study of pharmaceutical supplier selection for this we take 5 alternatives M_1, M_2, M_3, M_4 *and* M_5 and 6 criteria that are given below.

1. **Assurance of Supply (C_1)** [1]:

In the pharmaceutical sector, Assurance of Supply (AoS) guarantees consistent product supply. It entails minimizing risks, constructing a robust supply chain, and adhering to legal requirements. Collaboration and communication with stakeholders are critical, and technological integration, inventory management, and strategic sourcing all improve efficiency. Because the industry is worldwide in scope, geopolitical considerations are crucial. Ensuring a dependable and robust pharmaceutical supply chain necessitates ongoing enhancement via consistent evaluations and feedback systems.

2. **Quality and Regulatory Compliance (C_2)** [1]:

In the pharmaceutical sector, ensuring quality and regulatory compliance is crucial to guaranteeing the efficacy and safety of medications. It is imperative to implement strict quality control procedures, comply with regulatory criteria, and follow Good Manufacturing Practices (GMP). While regulatory compliance guarantees respect to national and international standards, quality assurance manages product consistency through methodical methods. Sustaining public health, satisfying patient expectations, and gaining regulatory clearances all depend on these two factors. Upholding strict standards and promoting confidence in the pharmaceutical industry need constant observation, record-keeping, and coordination with regulatory agencies.

3. **Technical Capability (C_3)** [10]:

In the pharmaceutical sector, technical capability compliance is the conformance of manufacturing facilities and procedures to regulatory criteria. To guarantee product quality, safety, and effectiveness, it includes putting cutting-edge tools, techniques, and technologies into practice. Adhering to verified procedures, upholding a strong quality management system, and following Good Manufacturing Practice (GMP) rules are all part of compliance. This includes adhering to strict documentation procedures, automating processes to increase accuracy, and using sophisticated instrumentation for testing and monitoring. Pharmaceutical businesses must adhere to technical capabilities compliance in order to continuously manufacture high-quality pharmaceuticals, reduce risks, and fulfill regulatory obligations, protecting public health and upholding industry confidence.

4. **Communication (C_4)** [5]:

Patient safety, regulatory compliance, and the successful development of new drugs all depend on effective communication in the pharmaceutical sector. Critical information interchange is facilitated by open and honest communication amongst stakeholders, including manufacturers, researchers, and regulatory agencies. To fulfill legal obligations, address safety concerns, and maintain strict quality control procedures, this is essential. To manage clinical trial data, disclose adverse events, and preserve public confidence, prompt and accurate communication is crucial. Moreover, collaboration, the advancement of the pharmaceutical industry's goal to improve global health, and educating patients and healthcare professionals about the advantages and disadvantages of pharmaceutical goods all depend heavily on efficient communication.

5. **Environment (C_5)** [23]:

Environmental factors are essential to maintaining sustainable and ethical practices in the pharmaceutical sector. Strict rules must be followed during the manufacturing process to reduce the negative effects on the environment. Businesses work to lower emissions, waste production, and energy use by supporting environmentally friendly products and programs. In order to meet environmental sustainability targets, waste management, efficient water use, and sustainable raw material sourcing are given top priority. Furthermore, compliance with international standards and Good Manufacturing Practices (GMP) guarantees product quality while reducing environmental impact. Pharmaceutical businesses are investing more in environmentally friendly technology research and development as the sector develops in an effort to lessen their ecological impact and support international environmental conservation efforts.

6. **Personnel Capabilities (C_6)** [24]:

In the pharmaceutical sector, human resources are essential to success and compliance. The staff needs to be highly qualified in a variety of fields, including chemistry, pharmacology, and regulatory affairs. Teams working on research and development projects need to be knowledgeable about clinical trials and medication discovery, and manufacturing staff members need to follow tight quality control guidelines. Experts in regulatory affairs handle intricate compliance mandates. Furthermore, prompt product development depends on having excellent project management abilities. Cooperation and efficient communication between departments is essential to tackling issues like strict laws and developing technology. Continual training and flexibility are crucial for keeping staff members up to date with developments and supporting the innovation and steady expansion of the business (Fig. 1).

A pairwise comparison decision matrix is shown below based on the decision maker's perspective (Table 3).

After that we calculate the geometric mean of fuzzy comparison values of criteria by using Eq. (1).

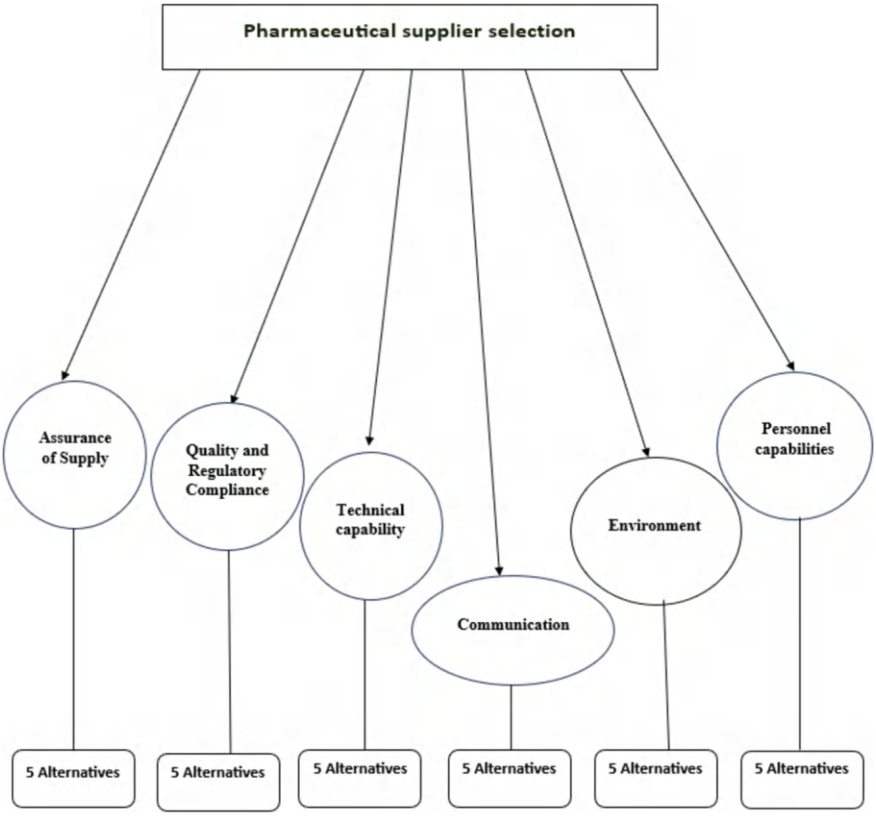


Fig. 1 Display the diagram of pharmaceutical supplier selection, the decision-maker choose six criteria and each criteria has five alternatives

Table 3 Criteria comparison table

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1, 1, 1)	(6, 7, 7)	(4, 5, 6)	(2, 3, 4)	(5, 6, 7)	(6, 7, 7)
C_2	(1/7, 1/7, 1/6)	(1, 1, 1)	(7, 8, 9)	(5, 6, 7)	(4, 5, 6)	(2, 3, 4)
C_3	(1/6, 1/5, 1/4)	(1/9, 1/8, 1/7)	(1, 1, 1)	(5, 6, 7)	(6, 7, 7)	(1/6, 1/5, 1/4)
C_4	(1/4, 1/3, 1/2)	(1/7, 1/6, 1/5)	(1/7, 1/6, 1/5)	(1, 1, 1)	(5, 6, 7)	(1/7, 1/6, 1/5)
C_5	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1/7, 1/7, 1/6)	(1/7, 1/6, 1/5)	(1, 1, 1)	(6, 7, 7)
C_6	(1/7, 1/7, 1/6)	(1/4, 1/3, 1/2)	(4, 5, 6)	(5, 6, 7)	(1/7, 1/7, 1/6)	(1, 1, 1)

$$r_i = \sqrt[n]{\prod_{j=1}^{n} d_{ij}} = \left(\sqrt[n]{\prod_{j=1}^{n} d_{ij}^{l}}, \sqrt[n]{\prod_{j=1}^{n} d_{ij}^{m}}, \sqrt[n]{\prod_{j=1}^{n} d_{ij}^{n}} \right)$$

For $i = 1$,

Table 4 Geometric means of fuzzy comparison values

Criteria	r_i		
C_1	3.36	4.05	4.493
C_2	1.849	2.165	2.513
C_3	0.673	0.771	0.871
C_4	0.392	0.458	0.551
C_5	0.273	0.421	0.476
C_6	0.684	0.767	0.914
Total	7.23	8.632	9.818
Reverse	0.138	0.116	0.102
Ascending order	0.102	0.116	0.138

Table 5 Relative fuzzy weight of each criterion

Criteria	W_i		
C_1	0.3427	0.47	0.62
C_2	0.1886	0.251	0.347
C_3	0.0686	0.089	0.12
C_4	0.04	0.053	0.076
C_5	0.0279	0.049	0.066
C_6	0.0697	0.089	0.126

$$r_1 = \left(\sqrt[6]{(1*6*4*2*5*6}, \sqrt[6]{1*7*5*3*6*7}, \sqrt[6]{1*7*6*4*7*7}\right)$$

$$= (3.3604215{,}4.049546886{,}4.49349463).$$

Similarly, we can calculate for the other values of r_i (Table 4).

Next, we proceed to calculate the relative weight of each criterion by applying **step 5** of the fuzzy AHP method. To achieve this, we multiply each r_i by the inverse of Ascending order, that is represent in Table 5.

In the standardized method, we determine the relative, non-fuzzy weight of each criterion (M_i) by calculating the average of fuzzy numbers associated with each criterion. Subsequently, we compute and present the normalized weights (N_i) of each criterion in Table 6 using the non-fuzzy M_i values (Fig. 2).

Next, we use the F-MABAC method, for the supplier selection process.

In constructing Table 7 for the fuzzy decision matrix, we utilize the linguistic terms accompanied by "Triangular fuzzy numbers" as despite in Table 2

Next, we normalized the fuzzy decision matrix of Table 7 by using the formula of step 2 of the fuzzy MABAC method and represented it in Table 8. In this problem, we treat all six criteria as beneficial and apply the normalization formula accordingly.

We calculate the elements of the weighted matrix V (see Table 9) by multiplying the criteria's weight coefficients (found in Table 6) with the elements of the normalized matrix (presented in Table 8). The determination of each element v_i in

Table 6 Averaged and normalized relative weights of criteria

Criteria	M_i	N_i
C_1	0.478	0.463
C_2	0.262	0.254
C_3	0.093	0.09
C_4	0.056	0.055
C_5	0.047	0.046
C_6	0.095	0.092

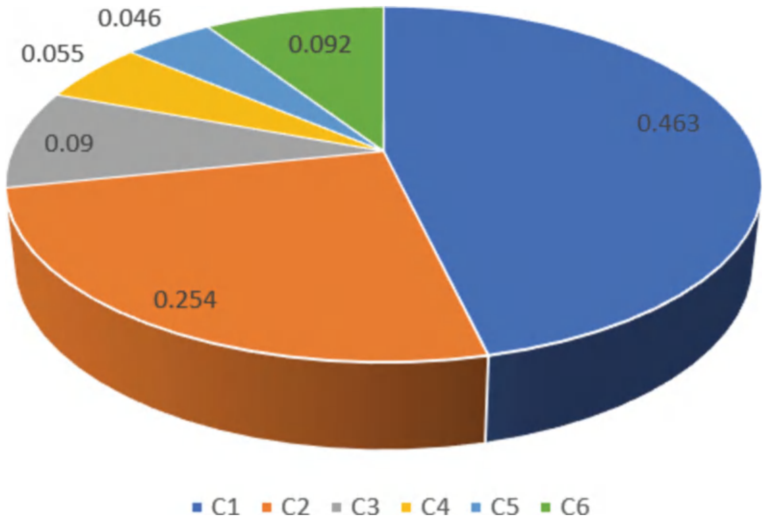


Fig. 2 Normalized criteria weights

Table 7 Fuzzy decision matrix for pharmaceutical supplier selection

Alt	C_1	C_2	C_3	C_4	C_5	C_6
M_1	(6, 7, 7)	(5, 6, 7)	(4, 5, 6)	(7, 8, 9)	(5, 6, 7)	(6, 7, 7)
M_2	(5, 6, 7)	(7, 8, 9)	(6, 7, 7)	(5, 6, 7)	(4, 5, 6)	(7, 8, 9)
M_3	(7, 8, 9)	(6, 7, 7)	(5, 6, 7)	(4, 5, 6)	(6, 7, 7)	(4, 5, 6)
M_4	(4, 5, 6)	(4, 5, 6)	(6, 7, 7)	(6, 7, 7)	(5, 6, 7)	(5, 6, 7)
M_5	(5, 6, 7)	(6, 7, 7)	(5, 6, 7)	(5, 6, 7)	(4, 5, 6)	(5, 6, 7)

the weighted matrix, V is accomplished by applying step 3 of the fuzzy MABAC method [21].

Using Table 9 and step 4 of the fuzzy MABAC method, we construct the BAA matrix (G), which is displayed in Table 10.

Table 8 Normalized matrix

Alt	C_1	C_2	C_3	C_4	C_5	C_6
M_1	0.4,0.6,0.6	0.2,0.4,0.6	0,0.33,0.66	0.6,0.8,1	0.33,0.66,1	0.4,0.6,0.6
M_2	0.2,0.4,0.6	0.6,0.8,1	0.66,1,1	0.2,0.4,0.6	0,0.33,0.66	0.6,0.8,1
M_3	0.6,0.8,1	0.4,0.6,0.6	0.33,0.66,1	0,0.2,0.4	0.66,1,1	0,0.2,0.4
M_4	0,0.2,0.4	0,0.2,0.4	0.66,1,1	0.4,0.6,0.6	0.33,0.66,1	0.2,0.4,0.6
M_5	0.2,0.4,0.6	0.4,0.6,0.6	0.33,0.66,1	0.2,0.4,0.6	0,0.33,0.66	0.2,0.4,0.6

Table 9 Weight normalized matrix (V)

Alt	C_1	C_2	C_3	C_4	C_5	C_6
M_1	0.648, 0.741, 0.741	0.304, 0.355, 0.406	0.09, 0.119, 0.149	0.088, 0.099, 0.11	0.061, 0.076, 0.092	0.128, 0.147, 0.147
M_2	0.556, 0.648, 0.740	0.406, 0.457, 0.508	0.149, 0.18, 0.18	0.066, 0.077, 0.088	0.046, 0.061, 0.076	0.147, 0.165, 0.184
M_3	0.741, 0.833, 0.926	0.355, 0.406, 0.406	0.120, 0.149, 0.18	0.055, 0.066, 0.077	0.076, 0.092, 0.092	0.092, 0.110, 0.129
M_4	0.463, 0.556, 0.648	0.254, 0.304, 0.355	0.149, 0.18, 0.18	0.077, 0.088, 0.088	0.061, 0.076, 0.092	0.110, 0.129, 0.147
M_5	0.556, 0.648, 0.741	0.355, 0.406, 0.406	0.119, 0.149, 0.18	0.066, 0.077, 0.088	0.046, 0.061, 0.076	0.110, 0.129, 0.147

With the help of Table 10 and Eq. 9 from the fuzzy MABAC method, we get the required Q matrix which is displayed in Table 11.

The elements in each row of the matrix (Q) are combined to determine the values of the criteria function for the alternatives. Table 12 illustrates the values of the criteria functions and the ranks of the alternatives.

Table 10 BAA matrix (G)

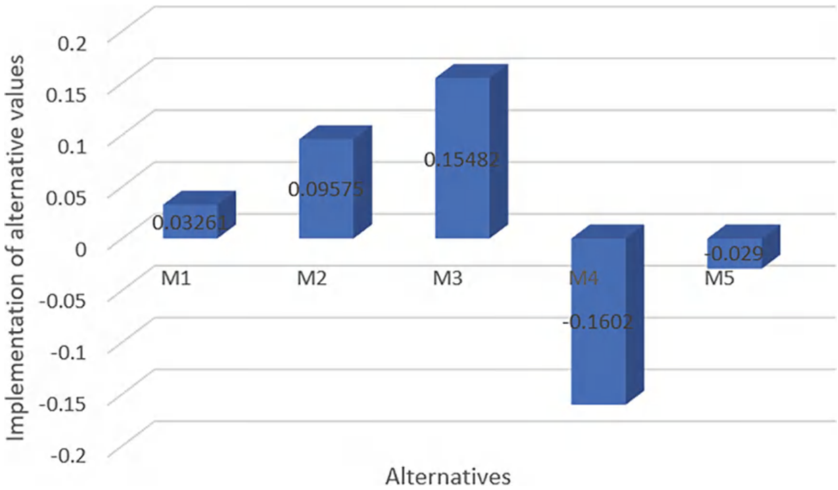
	C_1	C_2	C_3	C_4	C_5	C_6
g_i	0.585, 0.679, 0.754	0.33, 0.38, 0.414	0.12, 0.154, 0.173	0.0695, 0.081, 0.09	0.057, 0.071, 0.09	0.12, 0.13, 0.15

Table 11 (Q-matrix) [Distance from the BAA]

Alt	C1	C2	C3	C4	C5	C6
M_1	0.051	−0.022	−0.031	0.01901	0.0054	0.0102
M_2	−0.026	0.0815	0.0214	−0.003	−0.00995	0.03180
M_3	0.16	0.0201	−0.005	−0.0139	0.01682	−0.0232
M_4	−0.117	−0.071	0.0214	0.0061	0.00543	−0.0051
M_5	−0.026	0.0201	−0.005	−0.003	−0.00996	−0.0051

Table 12 Ranking of alternatives

Alt	Crisp value (S_i)	Rank
M_1	0.03261	3
M_2	0.09575	2
M_3	0.15482	1
M_4	−0.1602	5
M_5	−0.029	4



As depicted in the above figure and Table 12, the analysis indicates that the 3rd supplier stands out as the most favorable choice for the pharmaceutical sector. This determination is derived from the descending values of the overall performance indicator, employed to rank the options from the most favorable to the least. Conversely, based on Table 12, the 4th supplier is identified as the worst preferable option for pharmaceutical supplier selection.

5 Implications of This Study

(a) **Industrial Implication**

Here we express some implications by studying and optimizing the supplier selection process:

1. **Cost Reduction**

Finding vendors who provide competitive prices without sacrificing quality can be accomplished by reviewing the supplier selection process. Organizations may benefit from lower procurement costs and higher profit margins as a result.

2. **Quality Improvement**

Selecting suppliers with a track record of supplying high-quality materials and products can be accomplished by carefully examining providers' capabilities and performance measures. Customer happiness and end product quality both increase as a result.

3. **Risk Mitigation**

Supply chain disruptions can be reduced by assessing possible suppliers based on aspects including their financial standing, geography, and geopolitical risks. In industries where supply continuity is critical, this is significant.

4. **Innovation and Technology**

By learning about the supplier selection process, businesses can locate suppliers who offer cutting-edge technologies and practises. Working with such providers can lead to continued development and the adoption of cutting-edge technologies.

5. **Time-to-Market**

By expediting the procurement process, effective supplier selection can reduce lead times and assist firms in launching products more quickly. This is essential in industries where consumer preferences are constantly shifting.

6. **Relationship Building**

A well-organized supplier selection procedure makes it easier for businesses and their suppliers to communicate and work together. Stronger connections with suppliers can result in better terms, first access to resources, and information sharing.

7. **Customization and Flexibility**

Companies can choose partners who can deliver personalized solutions and quickly adjust to changing production needs by understanding suppliers' skills.

8. **Competitive Advantage**

By assuring a dependable and high-quality supply chain, an optimized supplier selection process may give businesses a competitive edge, improving customer loyalty and growing market share.

9. **Global Sourcing**

Industries that rely on global sourcing can find trustworthy suppliers from around the world by analysing the supplier selection process and taking into account elements like logistics, legislation, and cultural differences.

10. **Operational Efficiency**

The administrative burden is lessened, unnecessary work is minimized, and resource allocation is optimized through a streamlined supplier selection procedure.

In conclusion, researching the optimum supplier selection procedure has broad industrial ramifications. An optimized supplier selection process helps to overall business success in an increasingly competitive and complicated global market, from cost savings and quality improvements to risk mitigation and innovation.

(b) **Academic Implications**

Studying the best supplier choice process has significant academic implications, as it contributes to various fields of study and advances our understanding of supply chain management, decision-making, and business operations. Here are some academic implications of studying this process:

1. **Supply Chain Management Research**:

It ensures the smooth flow of goods, services, and information, which is better understood through research on supplier selection. This study can help with the creation of fresh supply chain models, plans, and best practices.

2. **Operations Management**:

A crucial component of operations management is supplier selection. Academic study in this field may result in the creation of frameworks, algorithms, and optimization models that assist businesses in selecting suppliers that will increase productivity and efficiency.

3. **Decision Science**:

Making decisions while facing risk and uncertainty is a factor in supplier selection research. Understanding decision-making processes, heuristics, and biases through academic study in this field can provide insights into how people and organizations make complicated judgments.

4. **Quantitative Analysis**:

Quantitative analysis, which might include mathematical modelling, statistical methods, and data analysis, is frequently required while analysing supplier selection. In the classroom, this helps students strengthen their analytical techniques.

5. **Innovation and Technology Management**:

An academic investigation of supplier selection can offer light on how businesses discover and work with suppliers to generate innovation because suppliers frequently play a role in bringing innovation and new technologies to an organization.

6. **Environmental and Social Responsibility Studies**:

To better understand sustainable business practices, the academic study might look at how organizations incorporate environmental and social responsibility criteria into their supplier selection process.

In conclusion, research on the optimal supplier selection method has broad academic ramifications that cut across many disciplines. It fosters a deeper understanding of how organizations manage their supply chains and make crucial business decisions through advancing theoretical knowledge, real-world applications, and a variety of academic subjects.

6 Research Limitations

1. **Data Availability and Quality**:

Accessing accurate and comprehensive information on suppliers' capabilities, performances, and compliance can be challenging. Some vendors can refuse to share specific private information, which could reduce the scope of the investigation.

2. **Confidentiality and Non-Disclosure Agreements**:

Pharmaceutical companies routinely sign strict confidentiality agreements with their suppliers, which may restrict the amount and type of data that can be acquired and analysed.

3. **Limited Collaboration with Suppliers**:

Some company's reluctance to fully collaborate with researchers may limit the depth of insights that might be gained.

4. **Subjectivity in Supplier Evaluation**:

Choosing a supplier may involve irrational factors that are challenging to quantify or analyse, like relationships, trust, and cultural compatibility.

5. **Long-Term Performance Prediction**:

It might be challenging to forecast a supplier's long-term performance and dependability based on historical data and current capabilities given how quickly market and industry conditions can change over time.

7 Conclusion

In this chapter, M_3 provider emerges as the ideal choice for the pharmaceutical sector. To arrive at this decision, the alternatives are ranked from best to worst using the overall performance indicator's reducing values. The chapter employs an integrated fuzzy AHP-MABAC technique to find credible suppliers for the pharmaceutical business. The F-AHP approach is used in this chapter to establish the weight of the criteria, and the F-MABAC approach is used to identify the most suitable alternative for the pharmaceutical sector. This method's adaptability makes it useful in a variety of sociocultural contexts outside of the healthcare industry. It's a useful tool for solving problems with ranking and choice. The chapter illustrates how decision-makers in the Pharmaceutical industry can utilize F-AHP and F-MABAC techniques to select suppliers adapted to the particular needs of their business by highlighting its applicability outside of the pharmaceutical sector. The integrated Fuzzy AHP-MABAC method extends beyond pharmaceutical supplier selection, finding applications across diverse fields to determine optimal choices from multiple options. It excels in managing intricate decision-making scenarios characterized by subjective assessments and uncertain information. For example, it proves valuable in manufacturing for supplier assessment, in project management for task prioritization, and in environmental management for assessing sustainability initiatives. By integrating qualitative and quantitative factors seamlessly, this method guarantees sound and dependable decision-making outcomes across various sectors. Similarly, suppliers for the pharmaceutical industry can be chosen by employing various other MCDM methods.

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A Novel Approach for Multi-cluster-Based River Flood Early Warning System Using Fuzzy-Logic-Based Learning and Rule Optimization

S M Nazmuz Sakib

Abstract River flooding is a significant natural hazard that can cause substantial damage to society. This paper presents a novel approach for a multi-cluster-based river flood early warning system (EWS) using fuzzy-logic-based learning (FLBL) and fuzzy rule optimization. The proposed system consists of cluster-based IoT nodes that collect sensor data from water level and rainfall sensors, and a server-side component that handles missing data imputation, FLBL for generating fuzzy rules, and an inference engine for predicting incoming flood events. The FLBL technique automatically learns and generates optimized fuzzy rules from the dataset, eliminating the need for manual trial-and-error processes. The system was showcased in three districts in Semarang City, Indonesia, and the results demonstrated that the proposed method outperformed baseline methods using decision trees and neural networks, achieving an average accuracy of 97.87% in predicting flood events. The proposed approach addresses the challenges of multi-cluster-based EWS and provides a reliable solution for early flood event prediction.

Keywords River flood early warning system · Fuzzy-logic-based learning · Fuzzy rule optimization · Missing data imputation · Multi-cluster IoT architecture

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1 Introduction

River flooding is considered one type of flood that can cause significant damage to society. This typical flood occurs when the water level exceeds the river's capacity, so the water overflows into the surrounding occupied area. Some government agencies have developed a preventive flood-related disaster plan to mitigate the harmful effects of river floods, such as river maintenance and an early warning system (EWS) [1]. With the advancement of IoT technologies, providing an EWS solution has become a preference for improving people's situational awareness of incoming flood events [2, 3].

In establishing a reliable IoT-based EWS architecture, critical requirements should be fulfilled, including a set of reliable sensory tools, good connectivity, reliable electric power sources, reasoning engines to infer the data from the sensors to generate early warning alarms, and apps to convey critical flood-related situations to related parties [4–6]. The architecture of an IoT-based EWS becomes more challenging when dealing with broader areas (e.g., inter-city) having many rivers traversing them. In such a case, the rain in a particular area might cause river flooding in other areas even though no rain is detected. Moreover, there are many possibilities of impacted areas depending on the water levels of the traversing rivers and the land contour. Consequently, the performance of EWS's reasoning engine is at stake to infer incoming flood events, given the complexity of natural flood-related factors from areas of concern and data transmission problems coming from the IoT devices [7–10]. Therefore, a reliable reasoning engine for EWS is highly required to deliver flood-event alarms in critical times [11, 12] and to impute missing data caused by connectivity problems.

In developing the reasoning engine for river flooding prediction, previous studies relied on three main approaches: rule-based reasoning (e.g., [3]), machine learning (e.g., decision tree [4], neural network [13, 14]), and fuzzy-based approach [15–18]. Some studies combine machine learning techniques with fuzzy theory, such as neuro-fuzzy [11, 19]. Rule-based reasoning may have higher reliability than machine learning techniques, particularly when data is insufficient to train as a learning model [15]. However, manually developing rules for complex situations is also a difficult process, which can be mitigated by machine learning techniques when good quality recorded data is available. On the other hand, [15] viewed the fuzzy-based technique as promising to improve real-time river flood prediction. Even in their early experiments, the fuzzy-based technique better predicted flood events than the machine learning technique. However, trial and error cannot be avoided in complex situations in determining fuzzy rules and the degree of membership values [17]. Furthermore, some of the above studies also implemented missing data handlers, such as [13], which used the mean value of previously recorded data. Other techniques of missing data imputation are the rolling statistical technique, linear models, and regression imputation [20].

Even though the performance of methods proposed in previous studies had been evaluated in terms of accuracy, there are some hurdles, particularly in setting them up in an EWS for a wide area involving multiple clusters of observation points. Their performance in such settings has not been extensively examined yet. Moreover, this paper views optimizing fuzzy rule construction as an option to eliminate the trial-and-error process when determining rules and degree of membership. This option can be achieved by implementing the automatic learning capability from a dataset to generate fuzzy rules without losing the uncertainty model provided by the fuzzy and applying a rule optimization strategy. To fill these gaps, this study aims to develop a novel approach for multiple-clusters-based EWS to infer incoming flood events based on fuzzy-logic-based learning (FLBL) and fuzzy rule optimization strategy, including missing value handler.

The rolling statistical technique is selected for its reliability and easy implementation to impute missing data compared to similar methods, such as linear models with autoregressive moving averages suitable for time series data. Autoregressive and moving-average models frequently express this association in error terms [21]. Furthermore, the fuzzy rule model is generated by the FLBL. Even though the FLBL has been introduced in different domains [22], none have been implemented in the hydrological field, especially in predicting incoming flood events. Moreover, the other FLBLs from previous studies were implemented in the Takagi–Sugeno [23] and Mamdani models [22]. Our paper implements FLBL in a different setting based on fuzzy logic that uses logical mapping from input to output to generate fuzzy rules. The rules optimization strategy is also applied in our FLBL by finding similar rules using the Szymkiewicz–Simpson coefficient and reducing them using the maximum function on their degree of membership values.

To sum up, the main contributions of this paper are as follows:

(1) A LoRa-based EWS architecture for flood-related data transmissions that supports multi-clusters-based observation points.
(2) The FLBL technique with optimization strategy is used to produce the fuzzy rules model to enhance incoming flood event alert predictions, including the simple moving average technique to impute the missing data.
(3) The demonstration of the proposed approach to prove the performance of the missing data handler and the fuzzy rules model.

The proposed method was showcased in three districts in Semarang City: Tugu, Ngaliyan, and West Semarang Districts. The flood event in those districts is affected by three traversing rivers: Plumbon, Bringin, and Silandak. The water level and rainfall rate sensors are two primary sensing tools used in this research. The experiments showed that the proposed method outperformed the baseline method in terms of accuracy in predicting flood events in selected areas. The average accuracy of our proposed method was 97.87%, while the accuracy of baseline methods using the decision tree algorithm and neural network was 96.83 and 73.07%, respectively.

The rest of this paper is structured as follows. Section 2 presents related works in flood-event EWS. Section 3 explains the proposed method, followed by the experiments and results in Sect. 4. Finally, conclusions are drawn in Sect. 5.

2 Related Works

IoT-based flood detection for early warning has attracted researchers around the globe, and various methods of inference engines have been proposed. Using ultrasonic sensors to measure the river's water level, [3] used a rule-based method to produce early warning information to the control room and open the dam gate. Even though the developed rules were reliable for the designated case, the proposed flood detection in this study was configured for a single spot where the dam gate is located, and there was no centralized mechanism to infer incoming flood events from multiple spots or clusters.

When multiple sensors collect flood-related data at an observation point, researchers prefer to use a machine-learning technique to develop a classifier to trigger flood-event alerts. For example, [4] proposed an IoT architecture to send data obtained from temperature, humidity, and water level sensors to the cloud over a Wi-Fi module embedded in the microcontroller. The decision tree model installed in the cloud was used to infer incoming floods and achieved 99.6% accuracy during the laboratory experimental setup. Furthermore, other researchers used neural networks to infer flood status. The two-class neural network was proposed by [14] and applied in the cloud to support the inference module that obtains data over the Wi-Fi module from the miniaturized water level sensor. During the laboratory experimental test, the accuracy achieved 98.9%. A more sophisticated machine-learning approach came from [13] that used the deep convolutional neural network to generate a learning model from IoT-based big data. The flood-related data consists of four variables: water flow, water level, rainfall rate, and humidity. The classifier's output indicates whether there is a chance of an incoming flood event. The learning model was trained using the National Weather Service, and as a result, this method achieved 93.23% accuracy in flood prediction. Even though the accuracy of machine learning methods was well-evaluated, their performance in tackling multi-cluster problems should be further examined.

Another widely used method to predict incoming floods is based on fuzzy models, such as Takagi–Sugeno and Mamdani. The capability of the Takagi–Sugeno model has been examined by [15] and compared to a machine learning technique called an artificial neural network (ANN) using rainfall rate data of a river in India. The results showed that the Takagi–Sugeno model outperformed the ANN. In [16], data anomaly detection was added before processing the Fuzzy inference system. Their data anomaly detection consists of three algorithms: Median-Interquartile Range, Multi-Layer Perceptron, and Recurrent Neural Network. Unlike the others that rely on sensory tools to infer incoming floods, [16] used remote sensing climate data to generate early warnings of flood events. Two other researchers who showed the capability of the fuzzy-based approach to alert incoming floods are [17, 18]. Researcher [18] compared fuzzy with impact-based forecasting by developing a relation matrix between the level of risks and their impacts. However, trial-and-error in determining fuzzy sets and degree of membership must be conducted carefully to avoid performance degradation of the fuzzy inference system [17].

Even though previously mentioned fuzzy-based techniques to generate flood-related early warnings were well-proven already, some issues still need to be solved, particularly in optimizing the construction of fuzzy rules. Some researchers [10] suggested combining fuzzy and machine-learning approaches to transform complex fuzzy inference rules or functions into a machine-learning model. However, when the fuzzy rules have been manually set, the impact of the machine-learning model to replace the fuzzy rules is relatively trivial as those rules are already reliable and can be used to infer the target state. The FLBL that can automatically learn and generate fuzzy rules becomes an option. This technique has been used in other domains, such as [22]. However, [23] viewed that the fuzzy rules produced by FLBL should be optimized. In this regard, they proposed fuzzy set fusions. As it can increase complexities, a new strategy is needed to reduce fuzzy rules.

3 The Proposed River Flood EWS Using FLBL

This section explains the proposed river flood EWS exploiting FLBL. As illustrated in Fig. 1, there are two main blocks in the architecture of the proposed systems: the cluster-based IoT nodes and the server cloud. There are five processing phases in this proposal: (1) The development board installed at a cluster collects the sensor reading values periodically based on the pre-defined settings; (2) The collected data is sent to the cloud through LoRa platform; (3) At the server, a missing data handler will be activated if there is no data received in a determined time interval; (4) The received data will be evaluated using the fuzzy rule model generated by the proposed FLBL; (5) The flood prediction results will be forwarded to monitoring and EWS. The following subsections explain the details of the proposed system.

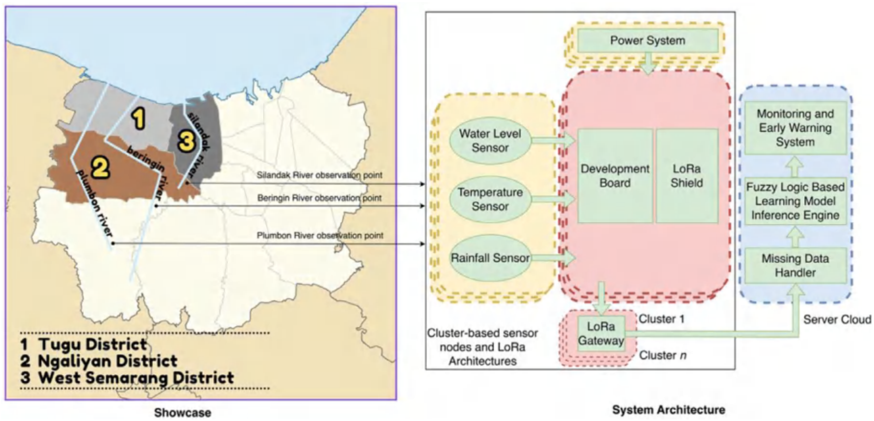


Fig. 1 The proposed system architecture for flood river early warning

3.1 Cluster-Based Sensor Nodes

This paper assumes that every observation point is a cluster, and every cluster is commonly associated with a specific observed region. However, an observation point may be close to another, particularly in river branches, and each branch has a potential flood impact on different areas. At an observation point, an IoT node is installed. This node comprises seven components: (1) development board; (2) water level sensor; (3) temperature sensor; (4) rainfall sensor; (5) LoRa shield; (6) LoRa gateway; (7) power system. The water level and rainfall rate sensors are the primary sources for calculating the chance of incoming river flood events in the proposed system. Each IoT node also has a temperature sensor, mainly used to confirm whether the rainfall rate sensors work well. Thus, this addition makes the proposed system more robust.

3.2 Server-Side Processing

This sub-section highlights two main parts of the server: the missing data handler and the FLBL inference engine. The output of FLBL is a fuzzy rule model. In generating the fuzzy rule model, FLBL has four steps, as illustrated in Fig. 2: (1) input data preparation, (2) associating data to the corresponding fuzzy set fuzzification process, (3) the learning process to generate fuzzy rules, and (4) rule optimization. The first step processes the input data from sensors and changes the data into a fuzzy membership belonging to the corresponding fuzzy set. The linguistic term resulted from the fuzzification process was then mapped to produce several groups of rules. After that, the optimization process is performed to produce the final fuzzy rule model. This model infers incoming flood events in the EWS using real-time data. The detailed explanations are provided below.

3.2.1 Missing Value Handler

The IoT-based architecture is vulnerable to data transmission failure, and therefore, our proposed approach implements a handler to impute sensor reading missing values. In this regard, our proposed system used a rolling statistical technique called simple moving average to impute missing values. Suppose that d is the data generated by a sensor at time t. The task of the simple moving average is aggregating previous captured values to replace the missing value at time $t(M_t)$ using following equation:

$$M_t = \frac{(d_{t-1} + d_{t-2} + d_{t-3} + \cdots + d_{t-n})}{n} \tag{1}$$

where n is the determined number of captured values from previous reading data.

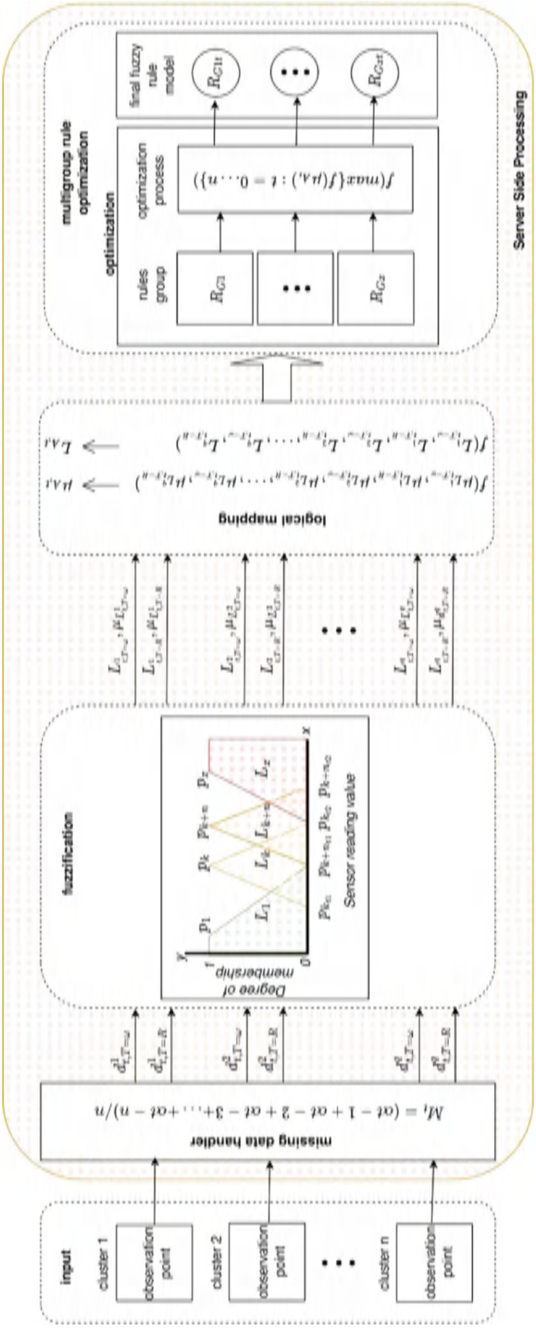


Fig. 2 Fuzzy-logic-based learning

3.3 Generating Fuzzy Rules Model

A. Input Data Preparation

As illustrated in Fig. 2, the learning process uses the data collected from sensors installed in the observation point cluster. Each sensor controller at the observation points periodically sends a tuple of data S_t^q, such that:

$$S_t^q = \left\{d_{t,T=\omega}^q, d_{t,T=R}^q\right\} \quad (2)$$

where d is the data generated by sensor T from the cluster number q at time t. There are two possible values of T based on the proposed EWS architecture: the water level sensor (denoted by ω) and the rainfall rate sensor (denoted by R). The temperature sensor is excluded from the dataset when generating the fuzzy rule model.

From the data collection, the universe of discourse U_{dT} is determined for each sensor type, such that:

$$U_{d_T} = [\max(d_T), \min(d_T)] \quad (3)$$

where d_T is a set of data collected from a type of sensor at all clusters. Therefore, there will be $U_{d_{T=\omega}}$ and $U_{d_{T=R}}$. Furthermore, the dataset used for the further process (denoted by D) consists of the unification of S_t^q collected during a specific period such that:

$$D = \left\{\left(d_{1,T=\omega}^1, d_{1,T=R}^1\right), \left(d_{1,T=\omega}^2, d_{1,T=R}^2\right), \ldots, \left(d_{t+n,T=\omega}^q, d_{t+n,T=R}^q\right)\right\} \quad (4)$$

where n is the end period of collected data. The number of clusters and sensors in D is correlated with the number of attributes in the dataset. For example, if the number of clusters is 3 and there are 2 sensors in each cluster, then D will have six attributes.

B. Fuzzification Process

In this step, $\forall d_j \in D$ will be fuzzified to associate them with the corresponding fuzzy set. Let $L = \{L_1, L_2, L_3, \ldots, L_x\}$ denotes a set of linguistic terms (e.g., low, normal, high, extreme) for fuzzy sets used to describe the reading value from a sensor, where x indicates the number of linguistic terms of the fuzzy sets. For each sensor, the proposed FLBL will determine the fuzzy sets based on two parameters, namely x and I denoting the desired interval value for each fuzzy set. Furthermore, the fuzzy sets indicated by the lowest and the highest x will have a trapezoid membership function. The intermediate fuzzy set(s) uses the triangular membership function.

Some critical points need to be determined by FLBL, following the number of linguistic terms used for each sensor (x). Those critical points (illustrated in Fig. 3) are:

(1) p_1 denoting the endpoint of the trapezoid membership function belongs to L_1 having $\mu_{L_1} = 1$.

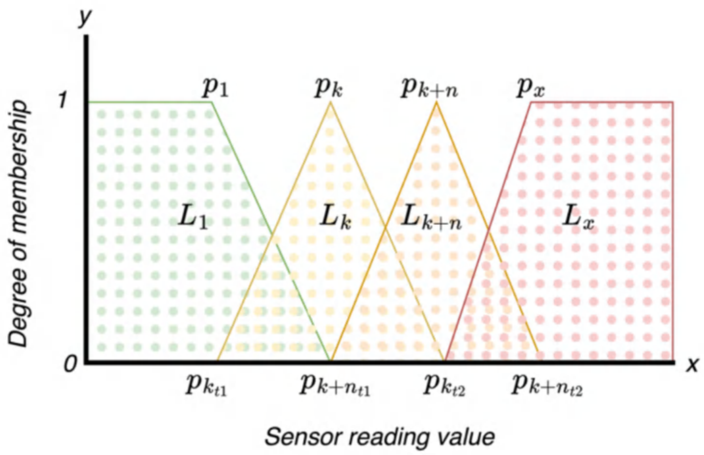


Fig. 3 Critical points of fuzzy sets

(2) p_x denoting the endpoint of the trapezoid membership function belongs to L_x having $\mu_{L_x} = 1$.
(3) p_k denoting the endpoint of the trapezoid membership function belongs to L_k having μ_{L_k}, where k represents the infinite integer number of fuzzy sets having a triangular membership function located between L_1 and L_k, and $1 < k < x$.
(4) $p_{k_{t1}}$ and $p_{k_{t2}}$ denoting the left and right threshold in the x-axis belongs to L_k, respectively, having $\mu_{L_k} = 0$.

Now suppose that $v_{\max}$ and $v_{\min}$ are the minimum and maximum values in U_s, respectively, and $v \in s$. The points of p_1 and p_x are determined based on the value of $v_{\max}$, $v_{\min}$, and I as formulated below:

$$p_x = v_{\max} - v_{\max}(modI) \tag{5}$$

$$p_1 = \frac{p_x}{x} \tag{6}$$

Furthermore, p_k is calculated as follows:

$$p_k = p_1 \times k \tag{7}$$

It should be noted that $\min(k) = 2$ and $\max(k) = x - 1$. Recalling that d_j represents the sensor reading value in the x-axis, the membership values of d_j in the fuzzy set having linguistic term $L_x\left(\mu_{d_j}^{L_x}\right)$ can be defines as follows:

$$\mu_{d_j}^{L_x} = \begin{cases} 1 & d_j \geq p_x \\ 0 & d_j \leq p_{k=x-1} \\ \frac{d_j - p_{x-1}}{p_x - p_{x-1}} & p_{x-1} \leq d_j \leq p_x \end{cases} \tag{8}$$

Similarly, the membership values of d_j in the fuzzy set having linguistic terms $L_1\left(\mu_{d_j}^{L_x}\right)$ can be defined as follows:

$$\mu_{d_j}^{L_1} = \begin{cases} 1 & d_j \leq p_1 \\ 0 & d_j \geq p_{k=2} \\ \frac{p_{x-1} - d_j}{p_{x-1} - p_1} & p_1 \leq d_j \leq p_{k=2} \end{cases} \tag{9}$$

Next, $p_{k_{t1}}$ and $p_{k_{t2}}$ can be determined as follows:

$$p_{k_{t1}} = p_{k-1} \tag{10}$$

$$p_{k_{t2}} = p_{k+1} \tag{11}$$

Thus, the membership values of d_j in the fuzzy set having linguistic terms $L_k\left(\mu_{d_j}^{L_k}\right)$ can be formulated as follow:

$$\mu_{d_j}^{L_k} = \begin{cases} 1 & d_j = p_{k+1_{t1}} \text{ where } k < x-1 \\ 1 & d_j = p_x \text{ where } k+1 = x \\ 0 & d_j \leq p_{k_{t1}} \text{ OR } d_j \geq p_{k_{t2}} \\ \frac{d_j - p_{k_{t1}}}{p_{k+1_{t1}} - p_{k_{t1}}} & p_{k_{t1}} \leq d_j \leq p_{k+1_{t1}} \text{ where } k < x-1 \\ \frac{d_j - p_{k_{t1}}}{p_x - p_{k_{t1}}} & p_{k_{t1}} \leq d_j \leq p_x \text{ where } k+1 = x \\ \frac{p_{k_{t2}} - d_j}{p_{k_{t2}} - p_{k+1_{t1}}} & p_{k+1_{t1}} \leq d_j \leq p_{k_{t2}} \text{ where } k < x-1 \\ \frac{p_x - d_j}{p_x - p_{k_{t2}}} & p_{k_{t2}} \leq d_j \leq p_x \text{ where } k+1 = x \end{cases} \tag{12}$$

At the end of this stage, it can be assumed that $\forall d_{t,T=\omega}^q \in D$ and $\forall d_{t,T=R}^q \in D$, a linguistic term has been set, namely $L_{t,T=\omega}^q$ and $L_{t,T=R}^q$, respectively, including their degree of membership on the corresponding fuzzy set $\mu_{L_{t,T=\omega}^q}$ and $\mu_{L_{t,T=R}^q}$, respectively.

For further processing, the dataset D, that was previously defined in Eq. 3, will be replaced with the fuzzified dataset $\left(D'\right)$. It should be noted that besides representing data collection time, t can also be viewed as the row number in the dataset. Now, it can be assumed that D_t' is one row of the fuzzified dataset, and it contains two tuples information from each cluster at time t, such that

$$D_t' = \left\{\left(\left(L_{t,T=\omega}^1, \mu_{t,T=\omega}^1\right)\left(L_{t,T=R}^1, \mu_{t,T=R}^1\right)\right), \ldots, \left(\left(L_{t,T=\omega}^q, \mu_{t,T=\omega}^q\right)\left(L_{t,T=R}^q, \mu_{t,T=R}^q\right)\right)\right\} \tag{13}$$

The information in D' will be used to feed the FLBL learning process.

C. Fuzzy-Logic-Based Learning Process

The FLBL learning process is initiated after the input dataset fuzzified into its linguistic term. The target of the learning process is to optimize fuzzy rules from the given the dataset. There are two steps in the learning process: (1) producing rules using a logical mapping function and (2) performing fuzzy rules optimization. While producing rules, there are two terms: the antecedent and the consequent, denoted by ∂ and τ, respectively. The former is the assigned degree of membership of from input sensor values. In other words, ∂ at time τ is constructed as follows:

$$\partial_t = \left\{ \mu_{L^1_{t,T=\omega}}, \mu_{L^1_{t,T=R}}, \mu_{L^2_{t,T=\omega}}, \mu_{L^2_{t,T=R}}, \ldots, \mu_{L^q_{t,T=\omega}}, \mu_{L^q_{t,T=R}} \right\} \tag{14}$$

Furthermore, τ_t is the output variable that can be the assigned label (A_t) for each row in dataset D'. The fuzzification process is also applied for A_t resulting in its linguistic term denoted by L_{A_t} and its degree of membership in the corresponding fuzzy set $\left(\mu_{A_t}\right)$. The fuzzy rule generation is structured based on the relation of ∂_t and τ_t, and is determined as follows:

$$f\left(\mu_{L^1_{t,T=\omega}}, \mu_{L^1_{t,T=R}}, \mu_{L^2_{t,T=\omega}}, \mu_{L^2_{t,T=R}}, \ldots, \mu_{L^q_{t,T=\omega}}, \mu_{L^q_{t,T=R}} \right) \rightarrow \mu_{A_t} \tag{15}$$

Based on the logical mapping function above, the linguistic terms of the antecedent can be mapped to the consequent for each row in the dataset, such that:

$$f\left(L^1_{t,T=\omega}, L^1_{t,T=R}, L^2_{t.T=\omega}, L^2_{t,T=R}, \ldots, L^q_{t,T=\omega}, L^q_{t,T=R}\right) \rightarrow L_{A_t} \tag{16}$$

Now, the fuzzy rules for the specific time t can be generated forming AND functions among antecedent's linguistic terms with L_{A_t} as the returning value.

The second process in the FLBL learning is performing fuzzy rules optimization. This process is conducted through several steps. The first step is the unification three sets μ_A, L_A, and D'. The sets μ_A and L_A are sets of consequent's linguistic terms and its degree of membership generated by Eqs. 15 and 16, respectively. The matching process uses index matching driven by t. Before this step is executed, the pre-requisite must be met which can be defined as follows:

$$\left|L_A\right| = \left|D'\right| = \left|\mu_A\right| \tag{17}$$

Which means that the size of both sets must be equal. Once the pre-requisite is fulfilled, the unification step is initiated to produce a set of Fuzzy rules (FR), such that:

$$FR = \left\{\left(D', L_A, \mu_A\right)\right\} \tag{18}$$

The size of $FR(|FR|)$ is considered the total of initial fuzzy rules that will be optimized by reducing them. The optimization strategy contains two main procedures: rules grouping and rules fusion. The grouping procedure follows a similarity check called Szymkiewicz–Simpson coefficient (SSC) that can be determined as follows:

$$SSC = \frac{|R_G \cap FR_t|}{\min(|R_G|, |FR_t|)} \tag{19}$$

where t indicates row number. Firstly, the grouping procedure will insert the initial row ($FR_{t=0}$) to a rule group set (R_G) where G indicates the group number. Hence, $R_{G=0}$ will have the same rule value compared to $FR_{t=0}$. In the second loop, the grouping procedure will compare the FR value for the next t (e.g. $t = 1$) to each existing R_G. When the SSC value is 1, no R_G will be formed, and the rule represented in e.g. $FR_{t=1}$ will be added as a new element to the corresponding R_G. Otherwise, new R_G (e.g. $R_{G=1}$) will be created. Thus, the total rule group can be determined by $G + 1$.

After all, rows have been examined for the grouping process, it can be assumed that each R_G contains a set of the same rules, but they have different membership degrees in the consequent part. In the proposed approach, the rule having the highest membership degree will be selected from each R_G. The rule optimization is regulated by the following operation:

$$R_{G_t} = f\left(\max\left\{f\left(\mu_{A_t}\right) : t = 0...n\right\}\right) \tag{20}$$

where t is the row index and R_{G_t} is the selected rule in a certain R_G having index t and highest membership degree. Thus, the optimized fuzzy rule model (M) can be described as follows:

$$M = \left\{R_{G=0_t}, R_{G=1_t}, ...R_{G=k_t}\right\} \tag{21}$$

where k is the rule group number.

4 Experiment and Result

4.1 The Swowcase Area

Table 1 displays the characteristics of three rivers in Semarang. Silandak is the shortest river, while the other two are similar in length. There is a significant difference in the water debit of the three rivers, with Bringing having the highest at 381.74 m^3/s, followed by Plumbon with 244 m^3/s, and the lowest being Silandak with 130.32 m^3/s. Table 2 presents the observed water travel times for the three rivers and regions with flooding risk. Each river has three observation points, each with a corresponding

Table 1 River profile in the Tugu area

River name	Length (km)	Water debit (m^3/s)
Plumbon River	19.5	244
Bringin River	21.6	381.74
Silandak River	10.88	130.32

Table 2 Flood affected area

Observation point	Affected area	Travelling time
Plumbon	Ngaliyan district	30 min
Bringin	Ngaliyan and Tugu district	120 min
Silandak	Ngaliyan and West Semarang district	30 inutes

affected area. All three observation points potentially affected the Ngaliyan district. Two observation points, Plumbon and Silandak, have the same travel time to the affected area in 30 min, and Bringin has the longest travel time of 120 min. When a higher risk water level is detected, certain regions could be potentially impacted by river flooding, particularly the districts near the observation points. For example, if the water level at the Plumbon Dam observation point rises to a high level, it could adversely affect the Ngaliyan district.

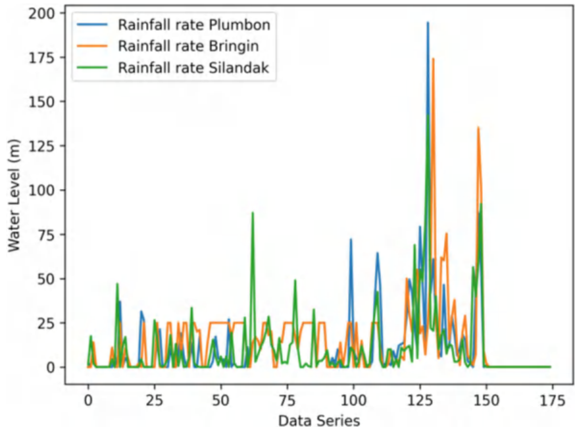
4.2 *The Dataset*

The dataset used to train the fuzzy rules model was collected from the Pemali Juana Office, the government's technical operation unit managing the Central Java River in Indonesia during 2020–2021. The dataset has three main attributes for each observation point: water level, rainfall rates, and temperature. However, the fuzzy rules generation only considers water level and rainfall rate data. The visualization of rainfall rates and water level data in the dataset can be seen in Figs. 4 and 5, respectively.

4.3 *The Data Imputation Performance*

This section illustrates how the missing data from sensors was handled in the proposed system as written in Eq. 1. Table 3 shows a replaced value assuming that at $t = 0$, the data from sensors was missing. By examining the data from $t - 5$ to $t - 1$, the predicted value from the water level sensor at $t = 0$ was 0.82. Meanwhile, the predicted value for the rainfall rate sensor was 6.7. Those values are used to replace the missing data.

Fig. 4 Rainfall rates at three observation points: Plumbon, Bringin, and Silandak Dams in 2020–2021



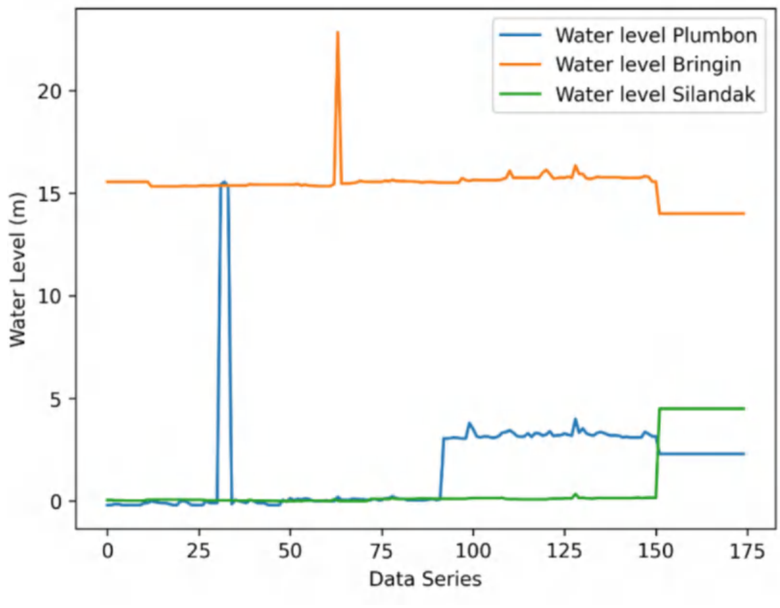


Fig. 5 Water level at three observation points: Plumbon, Bringin, and Silandak Dams in 2020–2021

Table 3 Imputing missing data

Sensors	$t-5$	$t-4$	$t-3$	$t-2$	$t-1$	$t-0$
Water level	0.1	0.07	0.12	0.1	0.2	0.82*
Rainfall rate	3.3	17.2	5.1	3.4	4.1	6.7*

Note * = a new value replacing missing data

4.4 Generating Fuzzy Membership and Fuzzy Rules

Before generating Fuzzy membership, both Fuzzy sets and their linguistic terms were defined for the main attributes (water level and rainfall rate) used to predict flood events. Additionally, this research set a fuzzy set for the incoming flood-event alert level. As described in Table 4, rainfall rates and water level had similar linguistic terms for their fuzzy sets:'Low','Medium', and'High'. However, the rainfall rates had another fuzzy set called'Extreme'. Furthermore, the alert levels have three linguistic terms for their fuzzy set:'Alert','Normal', and'Danger'. The alert level is the flood-event label assigned for combinations of the main attributes based on historical data.

Next was the learning process to generate fuzzy rules. It should be noted that from the three attributes in Table 4, the rainfall rates and water level are the antecedent, and the alert level is the consequent. For the starter, the proposed approach calculated the universe of discourse of each attribute by using Eq. 3. After that, critical points were calculated using Eqs. 5–7 based on the number of fuzzy sets determined for each attribute in the dataset. As the rainfall rate had four fuzzy sets (see Table 4), there were four critical points as references for two trapezoidal and two triangular membership functions (see Fig. 6a). Differently, the water and alert levels had only three fuzzy sets, so only three critical points were set for two trapezoidal membership functions and one triangular membership function (see Fig. 6b, c).

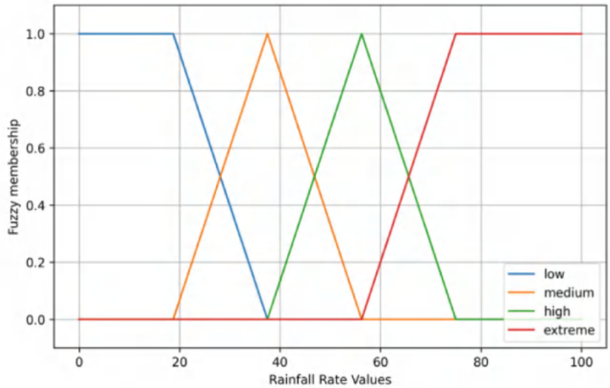
Once the critical points were set for each attribute, their linguistic terms can be determined, including their degree of membership by using Eqs. 8–13. For example, Table 5 shows an example from rainfall rates data mapped to their corresponding fuzzy set's linguistic terms. Similarly, Table 6 provides the linguistic terms for water level data and its degree of membership.

Regarding the alert levels, as previously mentioned in Eqs. 15–16, they were mapped into three linguistic terms, and their degree membership can be seen in Table 7. The alert level 0 has a full (1) degree membership value in the'Normal' fuzzy set. Furthermore, alert levels 1 and 2 had half (0.5) and total (1) degree membership values in the'Alert' and'Danger' fuzzy sets, respectively.

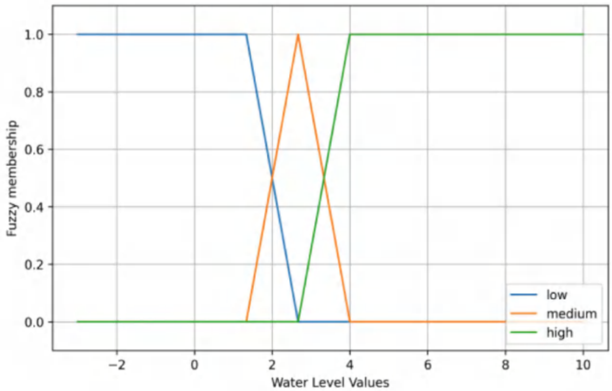
After all the sensor values were mapped into linguistic terms related to corresponding fuzzy sets, the proposed method summarized the fuzzy rules extracted from the relation between the antecedent and the consequent. The example of fuzzy rules generated by the proposed method can be seen in Table 8. The fuzzy rules were region-based, meaning that West Semarang, Tugu, and Ngaliyan districts might have different alert levels even though they had the same antecedent.

Table 4 List of linguistic term for each sensor

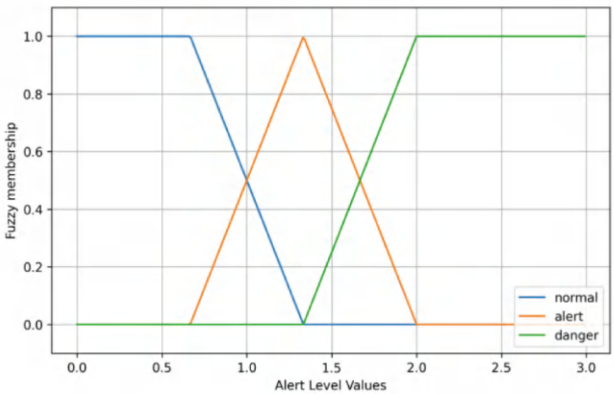
Attributes	Linguistic terms and symbols
Rainfall rates	Low (L)–Medium (M)–High (H)–Extreme (E)
Water level	Low (L)–Medium (M)–High (H)
Alert level	Normal (N)–Alert (A)–Danger (D)



(a) Fuzzy Membership of Rainfall Rate Values



(b) Fuzzy Membership of Water Level Values



(c) Fuzzy Membership of Alert Levels

Fig. 6 The fuzzy membership values for main attributes

Table 5 Example data mapping rainfall rate values to their corresponding fuzzy sets

Sensors values	Low	Medium	High	Extreme	Linguistic terms	Membership degree
2.3	1	0	0	0	Low	1
4.3	1	0	0	0	Low	1
19.1	0.98	0.02	0	0	Low	0.98
37	0.03	0.97	0	0	Medium	0.97
31.4	0.33	0.67	0	0	Medium	0.67
46	0	0.55	0.45	0	Medium	0.55
56.2	0	0.01	0.99	0	High	0.99
49.5	0	0.36	0.64	0	High	0.64
64.4	0	0	0.57	0.43	High	0.57
87	0	0	0	1	Extreme	1
72.1	0	0	0.15	0.85	Extreme	0.85
79.3	0	0	0	1	Extreme	1

Table 6 Example data mapping water level values to their corresponding fuzzy sets

Sensor values	Low	Medium	High	Linguistic term	Membership degree
-0.2	1	0	0	Low	1
0.23	1	0	0	Low	1
0.1	1	0	0	Low	1
2.3	0.28	0.72	0	Medium	0.72
3.17	0	0.62	0.38	Medium	0.62
3.24	0	0.57	0.43	Medium	0.57
3.36	0	0.48	0.52	High	0.52
3.45	0	0.41	0.59	High	0.59
4	0	0	1	High	1

Table 7 Mapping alert level values to their corresponding fuzzy sets

Alert level	Normal	Alert	Danger	Linguistic term	Membership degree
0	1	0	0	Normal	1
1	0.5	0.5	0	Alert	0.5
2	0	0	1	Danger	1

4.5 Results and Discussion

This section provides a comparison result between the baseline and the proposed method. There are two baseline methods by [4, 13] that used decision tree and neural network algorithms, respectively, to trigger the EWS alert. The decision three method

Table 8 Example of generated fuzzy rules for flood-event alert in west Semarang district

RP	RB	RS	WP	WB	WS	Alert level
E	E	H	H	H	L	D
E	L	H	M	H	L	A
H	E	E	M	H	L	D
M	L	M	M	H	L	N
M	L	H	M	H	L	A
L	L	H	L	H	L	A

Notes RP, *RB*, *RS* Rainfall rate for Plumbon, Bringin, and Silandak, respectively
WP, *WB*, *WS* Water level for Plumbon, Bringin, and Silandak, respectively
Other symbols see Table 4

was selected because it generates a set of rules, and the performance is comparable to the fuzzy rules from the proposed method. Meanwhile, the neural network approach was selected as it is well-proven in a previous study.

The results can be seen in Table 9, which show five trials conducted using the proposed method and two baseline methods, as mentioned. From the results, the proposed method in most districts in all five trials achieved the highest accuracy except in the second trial in the Tugu district and the fourth trial in the Ngaliyan district. The accuracy of the proposed method was 96% in those trials, while the baseline method using the decision tree showed 98% accuracy in comparison. However, the other trials on three districts showed that the proposed method is more accurate.

In the districts of Tugu, Ngaliyan, and Western Semarang, the suggested method's average accuracy ratings were 99.6, 97.2, and 96.8%, respectively. The decision tree achieved an average accuracy of 97.6, 95.2, and 95.2% in compared to the baseline approaches. The average accuracy of the neural network approach was then

Table 9 Prediction performance

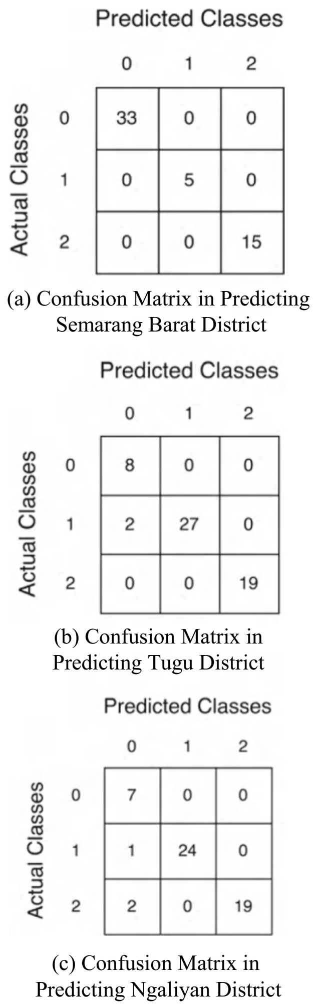
trial no	West Semarang			Tugu			Ngaliyan		
	A	B	C	A	B	C	A	B	C
1	0.98	0.98	0.92	0.98	0.96	0.64	0.98	0.92	0.64
2	1	0.98	0.94	0.96	0.98	0.64	0.94	0.94	0.64
3	1	0.98	0.88	1	0.96	0.61	0.98	0.96	0.64
4	1	0.98	0.97	0.96	0.92	0.63	0.96	0.98	0.64
5	1	0.96	0.91	0.96	0.94	0.62	0.98	0.96	0.64
Score	0.996	0.976	0.924	0.972	0.952	0.628	0.968	0.952	0.64
#Rules	29	4	N/A	29	4	N/A	29	8	N/A

Notes
A Proposed method
B Baseline method [4]
C Baseline method [13]
N/A Not Available

92.4, 62.8, and 64% for each of the three selected districts. The proposed approach demonstrated the highest average accuracy. Furthermore, the confusion matrix of the proposed method can be seen in Fig. 7. Based on these experiments, it can be concluded that the proposed method outperformed the baseline methods in terms of its accuracy, so it can reduce false alarms to notify flood events in the EWS. Regarding the rule optimization process, the proposed method had 29 final fuzzy rules to achieve in all districts, while the baseline method using decision tree had 4 final rules for West Semarang district and Tugu district, but had 8 final rules for Ngaliyan district. The rules were not available in baseline method using Neural Network.

Comparing the self-determined fuzzy rules approach with once generated by the proposed FLBL method, the former resulted in 1728 fuzzy rules, while the latter

Fig. 7 Confusion matrix based on trial no. 2



(a) Confusion Matrix in Predicting Semarang Barat District

(b) Confusion Matrix in Predicting Tugu District

(c) Confusion Matrix in Predicting Ngaliyan District only produced 29 fuzzy rules. Considering this rule gap, the proposed approach has benefits and drawbacks. When most of the incoming input data has already been characterized in the training dataset, the proposed method can optimize fuzzy rules. In the meantime, like the other machine learning approaches, the performance of the proposed approach will be poor when the training dataset does not reflect the daily incoming data.

5 Conclusion

This paper presents an IoT-based river flood EWS solution, including the hardware architecture, communication protocol, and a novel FLBL method to predict incoming floods. The proposed FLBL method involves two other techniques: a simple moving average to impute missing data and a join function between Szymkiewicz–Simpson coefficient similarity and max functions as the fuzzy rules optimization strategy. The proposed FLBL generates a fuzzy rules model from a given dataset. The solution was evaluated at three observation points: Plumbon, Silandak, and Bringin River, traversing three districts in Semarang City: Ngaliyan, Tugu, and West Semarang.

The results from the showcase indicated that the proposed method was more accurate in predicting incoming floods compared to the baseline method used in this study. However, the proposed FLBL still had some weaknesses. The method was designed to have two trapezoidal fuzzy membership functions with triangular membership functions in between. With such a design, it could not have other types of membership functions. Future direction can be directed to enhance the fuzzy set creation process.

Conflicts of Interest The author declare no conflict of interest.

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Evaluating Gamification Tools for Operating Management in Industrial Engineering: A Dual-Model Approach Using Fuzzy AHP and MACBETH

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Abstract Gamification is an innovative educational strategy that, if well executed, has the capacity to be highly impactful. The utilization of gamification has significantly increased since 2002. With so many possibilities, picking the right one can be challenging, especially when faced with the inherent uncertainty in making decisions in the real world. In order to determine the best gamification tool for the "Operating Management" module of the Master of Science in Industrial Engineering program, this research builds dual models. One model uses fuzzy analytical hierarchy process (AHP). A different model that measures attractiveness using a categorical based evaluation techniques (MACBETH) method is joint with fuzzy AHP. The instructor of

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the course served as the decision center. To our knowledge, no previous research has used fuzzy logic to determine which gamification tool would be most useful for a certain course. The study's findings indicate that this Masters course would benefit most from using Socrative as a gamification tool. However, when it comes to using models in degree courses, Quizizz is the superior alternative, especially when taught earlier in the degree program.

Keywords Analytic hierarchy process · Teaching · Gamification · Operations management · Application · Fuzzy analytical hierarchy process

1 Introduction

New educational trends advocate for more active and participatory pedagogies, as opposed to the lecture-based, traditionally passive form of instruction [1]. When it comes to this, gamification is a tool that applies game mechanics and strategies to non-gaming situations, either in education or otherwise. In contrast, gamification can be seen as an improvement method that incorporates elements of games in order to enhance user activities [2]. The goal of gamification of learning is to create learning environments that are more like video games to engage students and encourage them to study. There are numerous advantages to incorporating gamification into teaching [3]. Enhancing the active engagement and motivation of the participants; integrating theoretical concepts with practical application to facilitate learning assimilation; provide prompt feedback, so encouraging students to persist in their efforts even after making mistakes; Providing ongoing mental stimulation through constant interaction with a computer; transforming the learning of complex concepts and materials into an enjoyable experience; utilizing a robust multidisciplinary approach; enhancing group dynamics; encouraging healthy competition; fostering creativity. This course aims to help students develop digital skills, improve their problem-solving abilities, visualize simulations, interact with one another, open up new communication channels, offer part-time learning options that are compatible with other methods of instruction, incorporate a strong interdisciplinary approach, and enhance their ability to search for and select information. By testing students' knowledge across several domains as they study, rather than merely at the end of the course, gamification offers a fresh perspective on student evaluation [4]. Additionally, the ability to effectively use digital resources has grown in significance in recent years. Furthermore, numerous studies have examined the effects of eSports and gamification on health and fitness. The findings indicate that the implementation of gamification can result in weight reduction and enhanced physical activity. Additionally, it encourages greater exercise participation among children and adolescents, as well as higher energy expenditure compared to conventional video games or a sedentary lifestyle. In addition, it supports the use of wearables and mobile devices to increase physical activity in outdoor settings. A number of sources have reported success stories involving gamification in the classroom. For instance, authors [5] reviewed 24 gamification research and found favorable outcomes for student learning. In contrast, authors [6] examined 93 research that used Kahoot and found that gamification can improve learning outcomes, teacher and student attitudes, classroom dynamics, and anxiety levels. Notable favorable experiences, particularly for university courses, include the following: 97.30% of students reported an improved understanding of the ideas, while 95.95% found it easier to concentrate compared to typical theoretical/practical lectures. Additionally, 83.78% preferred group revision over individual revision. Researchers [7] found that implementing gamification-based teaching methods enhances students' attitudes and academic performance in the classroom. Expanding the use of gamification beyond the conventional lecture style resulted in higher levels of learning (62.50%), motivation (91.67%), collaboration (70.83%), curiosity (75.00%), and an overall positive evaluation of classes including novel technology (83.33%). Authors [8] found that students hold a more favorable perception of the class, exhibit increased engagement during class discussions, and have a higher likelihood of successfully completing the course. Researchers reveal that 88% of respondents were inspired by the concept, while an impressive 96.2% of students found it to be useful. In addition to being a great way to test students' knowledge, understanding, and progress, revision games are entertaining ways to review course material and have a strong correlation with students' final grades. The authors of various studies [9, 10] have shown improvements in participants' grades, especially in the context of higher education. While gamification platforms like Kahoot and Socrative did boost student engagement and enthusiasm, researchers discovered that it had no effect on students' performance in the class or their participation in ongoing evaluations. As a result of increased interest in business gamification, as predicted by authors [11] who said that "By 2015, gamification will have revolutionized corporate operations for 40% of the world's top 1000 companies," 55% of Spain's Top Employers have implemented gamification-related methodologies [12]. Data from this set has found applications in the following domains: advertising, marketing, health, education, enterprise resource planning, science, communication and activity inside organizations, public services, commerce, exercise, and health [13]. University students will encounter these approaches in their future careers, thus it is a huge step forward for them to be familiar with them. Not only can gamification be applied in the aforementioned business settings, but it can also be used in other contexts. For instance, a gamified educational platform is introduced to help the long-term unemployed develop their socio-economic entrepreneurship skills. The training program includes lessons and real-life business tasks like creating a business plan, as well as individual and group activities to test students' knowledge and skills. A marking system is also used to show how each student and group are doing. More and more new applications are being developed with the intention of gamification is noted, due to the growing interest in the concept [14]. Educational activities utilizing Quizizz were shown to be less beneficial than using Kahoot! in terms of academic accomplishment, according to several academics that have compared the applications statistically. Using the terms "multi-criteria gamification" and "educational game multi-criteria", researchers searched Science Direct, Hindawi, Emerald, Scopus and Proquest. However, the only prior work that fulfilled these criteria [15]. This work used the AHP to create a model for expanding gamification into the business world, but it focused on seller characteristics (supplier credibility, product competitiveness, contract terms, etc.) rather than gamification software specifically for teaching purposes. Thus far, no model has been discovered to aid university or secondary school educators in selecting the most appropriate gamification application for a certain subject. One model employs fuzzy AHP, while the another integrates fuzzy AHP and MACBETH system, which measures attractiveness using a categorical-based evaluation technique. Selecting the most appropriate gamification tool for a certain course has never been done before in published works utilizing fuzzy logic. Fuzzy logic is a useful tool for dealing with the doubts, uncertainties, uncertainties, and indecision that are common in real-life decision-making situations. Fuzzy AHP was chosen above other fuzzy strategies because it has been proven useful in real-world problems in previous studies [16, 17]. Fuzzy AHP combined with the MACBETH technique has evolved as a methodology to handle unclear, imprecise, inadequate, or non-existent values. The capacity to construct quantitative measuring scales from qualitative evaluations was a major factor in MACBETH's selection over other multi criteria techniques [18]. This is the first published work to merge the two multi criteria methods. With the use of MACBETH and the best–worst technique, to determine which of Iran's nineteen international airports would be the greatest fit for a hub airport. In the meanwhile, combines MACBETH, the Choquet integral, and the analytic network approach to assess ERP alternatives. Solar, wind, hydropower, and geothermal power are among the renewable energy sources that the authors [19] investigate while comparing the MACBETH and fuzzy AHP models. Similar to this, authors [20] assess the AHP, Delphi, and MACBETH models for analyzing the credit risk of mortgage loans by modifying the trade-offs. According to the findings, AHP and MACBETH are practically identical with respect to accuracy, and they both outperform Delphi in terms of behavior. Authors [21] extending MACBETH for fuzzy environments using the approach, this study uses a combination of methodologies that equally benefits from ambiguity and imprecise information, making it easier to apply. The purpose of the application was to assess potential solutions for tyre waste in reverse logistics. Kahoot, Quizizz, and Socrative are the gamification apps that have been evaluated in the models. This is due to the fact that these three apps include the two most popular gamification tools at the moment, Kahoot and Socrative, and because their free versions do not limit the number of users or questions [22]. The first step is to explain how to make a fuzzy analytic hierarchy. Subsequently, the structure, criteria, decision matrix, and intermittent values of the fuzzy analytic hierarchy approach are elaborated upon. The model derived from the integrated fuzzy analytical hierarchy process with MACBETH method is shown in below. The two models culminate in a sensitivity analysis, findings, and proposals for further development.

2 Fuzzy AHP Methodology

Well-known multi-criteria techniques like AHP have their foundational parts. Human judgments are often based on language, which can lead to inaccurate patterns when dealing with complex problems [23]. Uncertainty resulting from human judgements' imprecision or vagueness was the primary concern of first application of fuzzy set theory. The imprecision of language is compensated for by using fuzzy numbers, namely trapezoidal, interval, and type-2 fuzzy numbers. However, triangular numbers garner the most citations in the literature due to their computational efficiency and intuitiveness making them the preferred representation. Uncertain judgments are defined using a triangle fuzzy number M˜ = (l, m, u). In this case, the minimum value is l, the mode is m, and the maximum is u. Conversely, we obtain a precise integer for all values of l, m, and u. Triangular fuzzy numbers are defined as continuous membership functions that link real numbers on the interval [0, 1]. Equation (1) shows that the membership function $\mu(x \mid \tilde{M})$ of $\tilde{M}$. The degree to which x is a member of the fuzzy set A increases as the distance between $\mu(x \mid \tilde{M})$ and 1 decrease [24].

$$\mu(x \mid \tilde{M}) = \begin{cases} (x-l)/(m-l) & l \le x \le m \\ (u-x)/(u-m) & m \le x \le u \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

A large number of studies have used fuzzy AHP in conjunction with other tools, like in our own study. This is supported by a literature review conducted by authors [25], which examines 190 papers published between 2004 and 2016. The review reveals that the government, manufacturing, and industry sectors are the most common users of fuzzy AHP. On the other hand, the MACBETH method is never used in its entirety. Several fuzzy AHP techniques have been presented to determine the comparative relevance of the criterion. Geometric mean, fuzzy preference programming, fuzzy minimum square priority, synthetic extended analysis, dual-stage logarithm goal software design, and logarithmic least squares are all tools in this set. The extent analysis method is commonly employed; however, its findings are subject to debate. This inquiry should utilize the geometric mean methodology, since it ensures a singular solution and is simpler to apply and comprehend compared to alternative approaches. To implement Buckley's geometric mean methodology, one must follow these steps:

- The model's results might be validated with the help of a decision-maker who offer pertinent data, views, and insights.
- An organizational structure is built from the problem's pertinent criteria and sub-criteria after their identification and characterization. Rank order is as follows: problem objective at the top, criteria and sub-criteria at the bottom, and options at the very bottom.
- Decision difficulties in the actual world sometimes involve uncertainties, such as uncertainty, ambiguity, hesitating, or confusing situations. Instead of using these

components, the original Saaty scale used integers from 1 to 9, or their inverses, to represent each judgment. The literature suggests many fuzzy scales. Based on its closer relationship to Saaty's initial scale proposal in the crisp AHP, the scale provided by Lamata was used for this investigation.

- Fuzzy judgments should be used by the decision maker or group while comparing options, criteria/sub-criteria, and the levels of each scale. Along the same hierarchical level, fuzzy values, $\tilde{M}_{ij}$, reveal how the decision maker rates element i in comparison to element $a$using the fuzzy scale from Table 1. $\tilde{A}$ is the fuzzy pairwise comparison matrix that includes these values.

$$\tilde{A} = \begin{pmatrix} \tilde{1} & \tilde{M}_{12} & \cdots & \tilde{M}_{1n} \\ \tilde{M}_{21} & \tilde{1} & \cdots & \tilde{M}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{M}_{n1} & \tilde{M}_{n2} & \cdots & \tilde{1} \end{pmatrix} A = \begin{pmatrix} \tilde{1} & \tilde{M}_{12} & \cdots & \tilde{M}_{1n} \\ 1/\tilde{M}_{12} & \tilde{1} & \cdots & \tilde{M}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{M}_{1n} & 1/\tilde{M}_{2n} & \cdots & \tilde{1} \end{pmatrix} \tag{2}$$

- Equations (3) and (4) are utilized to construct fuzzy weights for each criterion/sub criterion, in accordance with the geometric mean technique [26]:

$$\tilde{w}_i = \left(\alpha_{ij}, \beta_{ij}, \gamma_{ij}\right) \otimes (\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3 \oplus \cdots\cdots \oplus \tilde{r}_n)^{-1} \forall \mathrm{i} \tag{3}$$

$$r_i = \left(\tilde{M}_{i1} \otimes \tilde{M}_{i2} \otimes \cdots \otimes \tilde{M}_{in}\right)^{\frac{1}{n}} \tag{4}$$

- In order to determine the optimal non-fuzzy performance value (BNP), the defuzzification method must be performed. Theβ-cut, center of area (COA), and Mean of maximum (MOM), are three different approaches that can be employed for this purpose. In this study, we used the COA, or centroid technique because it is easy to implement, does not require any assessor preferences to be considered, and yields reliable results. The result of Eq. (4) should be used in Eq. (5):

$$w_i = (l_i + m_i + u_i)/3, i = 1{,}2, 3 \ldots, n \tag{5}$$

Table 1 Triangular fuzzy number operations

Actions	Outcome
Addition	$\tilde{M}_1 \oplus \tilde{M}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$
Subtraction	$\tilde{M}_1 \ominus \tilde{M}_2 = (l_1 - u_2, m_1 - m_2, u_1 - l_2)$
Multiplication	$\tilde{M}_1 \otimes \tilde{M}_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2)$
Division	$\tilde{M}_1 \oslash \tilde{M}_2 = (l_1/u_2, m_1/m_2, u_1/l_2)$
Inverse	$\tilde{M}_1^{-1} \approx (1/u_1, 1/m_1, 1/l_1)$ for $l, m, u > 0$
Scalar multiplication	$k \otimes \tilde{M}_1 = (kl_1, km_1, ku_1), k > 0, k \in R$
	$\mathrm{k} \otimes \tilde{M}_1 = (ku_1, km_1, kl_1), k < 0, k \in R$

- Equation (6) can be used to normalize, *wi*.

$$z_i = \frac{w_i}{\sum_i^n w_i}, i = 1{,}2, 3 \ldots, n \tag{6}$$

- Compare the reliability of the decisions.

  Fuzzy evaluation matrix $\tilde{R} = \left[\tilde{r}_{ij}\right]$
  A crisp matrix

$$R = \left(\beta_{ij}\right)$$

  If $R \& \tilde{R}$ is consistent.

To measure the reliability of each paired matrix's judgments, Saaty devised the consistency ratio (CR) (see Eq. (7)) [27, 28].

$$CR = \frac{CI}{RCI'} \tag{7}$$

The consistency index CI is found by solving Eq. (8), where $\lambda_{\max}$ is the maximum Eigenvalue and n denotes dimension of matrix. The simulation involves employing random matrices of the same dimension as the one being assessed to determine the random consistency index (RCI).

$$CI = \frac{(\lambda_{\max} - n)}{(n - 1)} \tag{8}$$

To ensure responsiveness, the CR < 0.05 (3 $\times$ 3 matrix), CR = 0.08 (4 $\times$ 4 matrix), or CR = 0.1 (more than 4 $\times$ 4). The decision of the pair relative matrix must be re-obtained if the CR is more than 0.1.

3 An AHP Model Utilizing Fuzzy Logic for Decision Making in the Context of Gamification

Hierarchical problem structure is presented in this section. Following is an explanation of the steps involved in weight assignment, including the calculations related to the geometric mean technique. At long last, the method of giving each descriptor's scale levels weights is detailed.

3.1 Organizing

The study's criteria are novel and tailored to the Master of Science in Industrial Engineering program's Business Management course; nevertheless, they are flexible enough to be modified for use in other programs. Every criterion or sub-criterion that will be evaluated must have a descriptive term. The alternatives' effects on each criterion can be objectively described using a descriptor, which is a ranked collection of potential efficiency or function levels in a criterion. Researchers won't have to judge the criteria subjectively, and researchers won't lose any information about the criterion, owing to the descriptor definition, which will make the final evaluation process for each criterion much easier. These descriptions could be either direct or indirect. Scale levels in direct descriptors represent effects directly, but in indirect descriptors they suggest causes. In cases where there are information gaps, multiple interconnected aspects, or the criteria is individual or imperceptible, no direct or indirect descriptors can be created; in such cases, a built descriptor is employed. Constructed descriptors can have quantitative, qualitative, or mixed scale levels, and they are made by describing the potential outcomes of cases with several aspects. All of the descriptive terms used in this study are made up. The following decision criteria were developed after reviewing the gamification research and taking into account the outcomes of one's own classroom experience with gamification and the alternatives evaluated: The app's adaptability is tested by seeing if they can accommodate various question formats (multiple choice, short answer, etc.), the ability to grade every questionnaire separately, the maximum number of replies per questionnaire, and the inclusion of multimedia elements (photos, videos, etc.) in questions. The descriptor's scale levels are as follows: L11. The level of flexibility in question format is extremely high; it supports all conceivable question types, allows an unlimited number of answers, incorporates images and video into questions, allows for unlimited character limits for additional explanations for every question, and can be download as a document. (Good). L12. Many options are available for how to word the questions. Researchers can use it to create surveys with any kind of question researchers can imagine, including multiple-choice, short, single-and true/false, multimedia (pictures and videos), character limits for explanations, and the ability to save the results as a file. L13. There is a moderate amount of flexibility in the questions. It supports every conceivable question type (true/false, brief, single, multiple choice), but only up to four answers per question; questions can incorporate media like images and videos; questions cannot have additional explanations; and the inquiry form cannot be saved as a document. L14. Formulating the questions allows for very little flexibility. Only up to four answers are allowed per question; questions cannot contain media like images or videos; questions cannot have further explanations; and the questionnaire cannot be saved as a file. It supports all possible question categories, including true/false, brief, single, and multiple choices. L15. Changing the language of the questions is not given much flexibility. It restricts the following features: the use of media files in questions; a maximum of four answers; the ability to attach supplementary explanations to questions; the ability to save the questionnaire as a file; and the inclusion of all question types (including true/false, short, and single, multiple choices, and others). Learning rhythm (LRH). A person's learning pace can be categorized into three levels: rapid, medium, and slow. It is critical that the app lets researchers manage the pace of the activities and, by extension, the learning. Consequently, this criterion relies on the teacher's capacity to manage the pace in one of three ways: by setting a time restriction, by granting students unlimited time, or by enabling them to take the test as many times as necessary. This is a hierarchical list of the descriptor scale levels, ordered by function: L21. Possibility to be managed by the instructor, who can choose to establish a time restriction or allow students to work at their own pace, without restriction on number of attempts or waiting for others to finish (Good). L22. Possibility to be managed by the instructor, who can choose to establish a time limit or let students to work at their own pace, answering the questions independently and without interruption. L23. Each question needs to have a time limit of fifteen minutes or less. L24. Each question needs to have a time limit of 120 s or less. The criteria used to evaluate Assessment of the Questionnaire (AQU) is the variable time it takes to answer each question and the degree of flexibility in evaluating them. Levels of scale for this description are as follows: L31. It is possible to assign separate scores to each question and to utilize students' individual time in the event of a tie (Good). L32. It takes the amount of time it takes to answer each question and the percentage of correct answers to determine the final score. L33. The number of correct responses is used to evaluate each question. It is neutral. L34. The time it takes to answer each question is used to determine its score. L35. No points are deducted from the questions. The capacity to gather data and display player performance in-game is Obtained Results and Report (ORR) evaluated. Considering the option to conceal names is worth noted, as there were students who wished their results to remain confidential. In order of how appealing they are, the following are the scale levels: L41. Access the participants' results question by question in an Excel file. Examining the outcomes visually on the platform. While playing, researchers have the option to conceal the identities of players on the scoreboard. Those who took part can see their final tally in real time (Good). L42. Access the participants' results question by question in an Excel file. Examining the outcomes visually on the platform. No one can hide their names from the score list while the game is in progress. In real time, researchers may see the contestants' final score lists. L43. Access the participants' results question by question in an Excel file. The platform does not allow visual scanning of the results. No one can hide their names from the score list while the game is in progress. In real time, researchers may see the contestants' final score lists. On the neutral spectrum. L44. Get the Excel file that contains all of the participants' results, not just a breakdown by question. The platform does not allow visual scanning of the results. No one can hide their names from the score list while the game is in progress. Participants' final score lists are not displayed in real-time. L45. Neither downloading nor converting to Excel is possible for an Excel file that contains the outcomes of participants' questions individually. It is not feasible to visually scan the results on the platform. No player's identity can be concealed from view on the scoreboard while the game is in progress. There is no real-time display of the contestants' final score lists.Weighed against one another is a weedy method of JITT (Just in Time Teaching) that relies on students having completed prior work (the former uses open-ended questions to gauge students' understanding of the material, while the latter employs closed-ended questions to assess students' mastery of the material). The descriptor utilizes the following scale levels, arranged in descending order of function: L51. JITT is simple to use (Good). L52. Using JITT is possible, but it takes more work to create the questions. On the neutral spectrum. L53. The information gathered is limited while using JITT since the questionnaire cannot include questions that require short answers. L54. JITT is inapplicable since, by default, it does not have the capability to collect short replies or to grade the responses to the surveys.

Gamification features that have an effect on the student and inspire them to learn (EGS). Capacity to assess the app's evolution to add game-like gamification elements. The following are the levels of the criterion-assessing descriptors: L61. Every student can create their own avatar or utilize one of the many accessible on the app. They can also send and receive positive comments (memes), attach photographs, embed movies from YouTube, and even add music to their inquiries. Here is the entertaining final tally (Good). L62. Using the app, students can access the platform's avatars, add photographs, and embed videos from YouTube; however, the app does not provide students with the option to add music to questions or uplifting comments through memes. An entertaining rundown of the final scores is displayed. On the neutral spectrum. L63. While the software does allow users to upload photos and embed movies from YouTube, it does not currently support avatars. Neither the option to add music to questions nor the ability to send encouraging words (memes) are available. Here is the entertaining final tally. L64. It is a serious application that doesn't have any enjoyable features.

Quality of the question library (QQL). This criterion is evaluated based on the quantity of publicly available surveys, how easy it is to share, copy, or modify them, and the quality of the platform for receiving and sharing information. The following are the scale levels: L71. More than two million questionnaires that are available to the public are housed in its library. The surveys are editable, duplicable, and shareable. An active online forum is available for the purpose of sharing knowledge and experiences (optimum). L72. It offers a collection of up to half a million freely accessible surveys. Researchers can copy and share the surveys, but researchers can't change the questions. So far, no major application-related forums have been located. L73. Neither a library nor the materials accumulated by other users are at disposal. No one may copy, paste, or make changes to a questionnaire. So far, no major application-related forums have been located.

Ease of use in class (EUC). Various devices and the necessity of supplementary equipment are used to assess the class's adaptability. The following are the levels on the descriptor's scale, from most attractive to least: L81. Works with desktop computers, laptops, and tablets in the classroom. Excellent; doesn't necessitate a projector. L82. Useful for classroom use with mobile or laptop devices. Can be viewed without a projector (without bias). L83. Useful for classroom use with mobile devices (not all game types are compatible with laptops). No projector is necessary. L84. Playable on mobile devices (not on laptops, though, in certain game modes) in the classroom. Needs a projector (for certain game styles). L85. Use it exclusively in class with mobile device. Must have a projector on hand. With an average of about 25 students in a Business Management class, the decisional makers did not think crew rivalry was a significant factor. However, students are encouraged to discuss questions with their classmates during the test, which serves more as a learning opportunity than an evaluation tool. Additionally, this model does not take cost into account; the only choices evaluated were those that permitted the use of a free version of the application, since these were the only ones relevant to the course. The model's hierarchical structure is depicted in Fig. 1.

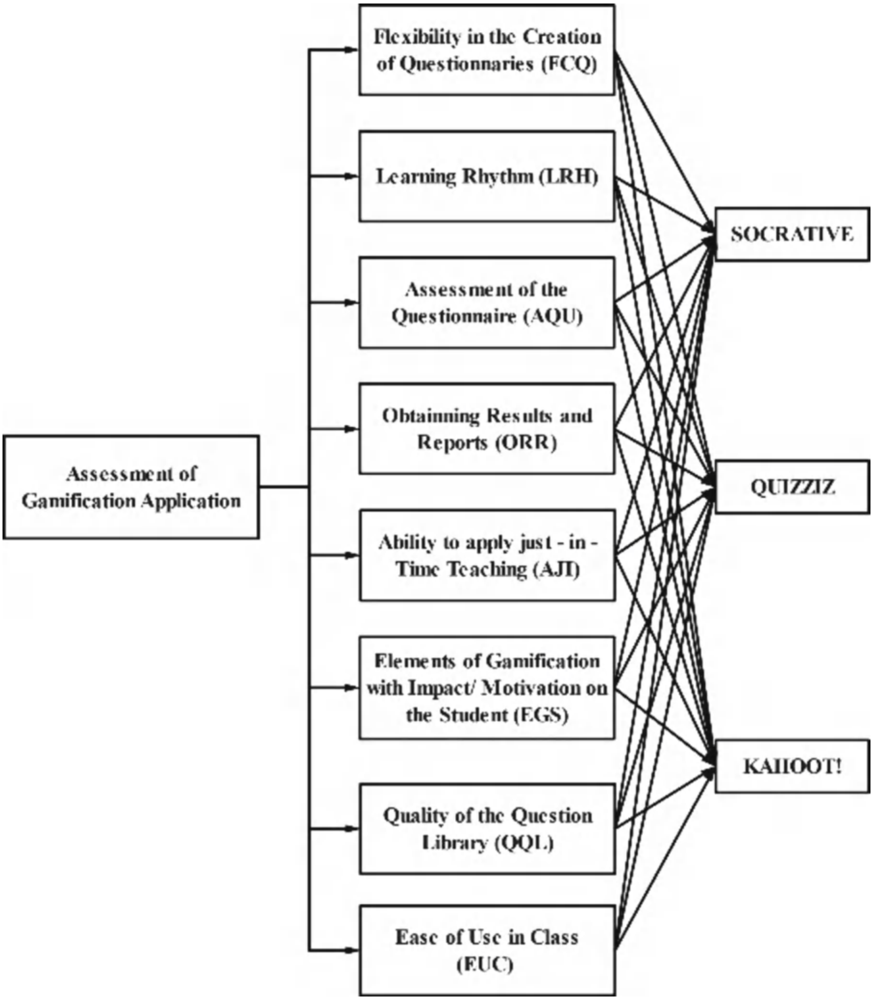


Fig. 1 Hierarchy structure

3.2 Weighting

Fuzzy judgments for matrix $\tilde{A}$ of Eq. (3) are made by the course instructor who also serves as the decision maker. The fuzzy scale that is displayed in Table 1 was utilized for the evaluations.

$$\tilde{r}_{FCQ} = [(1, 1, 1) \otimes (1, 2, 3) \otimes (1, 2, 3) \otimes (1, 2, 3) \otimes (2, 3, 4) \otimes (3, 4, 5) \otimes (3, 4, 5) \otimes (4, 5, 6)]^{\frac{1}{8}} = (1.707, 2.573, 3.359)$$

$$\vec{P}_{LRH} = [(0.428, 0.623, 1) \otimes (1, 1, 1) \otimes (1, 1, 1) \otimes (1, 1, 1) \otimes (1, 2, 3) \otimes (2, 3, 4) \otimes (2, 3, 4) \otimes (3, 4, 5)]^{\frac{1}{8}} = (1.192, 1.571, 1.987)$$

$$\tilde{r}_{AJI} = (0.611, 0.921, 1.439)$$

$$\tilde{r}_{EGS} = (0.429, 0.562, 0.775)$$

$$\tilde{r}_{\mathrm{ORR}} = (1.192, 1.571, 1.987)$$

$$\tilde{r}_{QQL} = (0.429, 0.562, 0.775)$$

$$\tilde{r}_{EUC} = (0.282, 0.361, 0.516)$$

$$\tilde{r}_{AQU} = (1.192, 1.571, 1.987)$$

The fuzzy weight of the criterion iw_i can be found by applying Eq. (4), which leads to:

$$\begin{aligned}
&(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3 \oplus \cdots \oplus \tilde{r}_n)^{-1} \\
&\quad = [(1.192,1.571,1.987) \oplus (1.707,2.573,3.359) \\
&\quad \oplus (1.192,1.571,1.987) \oplus (1.192,1.571,1.987) \\
&\quad \oplus (0.611,0.921,1.439) \oplus (0.429,0.562,0.775) \\
&\quad \oplus (0.429,0.562,0.775) \oplus (0.282,0.361,0.516)]^{-1} \\
&\quad = \left(\frac{1}{8.235}, \frac{1}{11.357}, \frac{1}{15.243}\right)^{-1} = \left(\frac{1}{15.243}, \frac{1}{11.357}, \frac{1}{8.235}\right)
\end{aligned}$$

$$w_{FCQ} = \left(\frac{1.710}{15.243}, \frac{2.577}{11.357}, \frac{3.363}{8.235}\right) = (0.112,0.227,0.408)$$

$$\tilde{w}_{\mathrm{LRH}} = \left(\frac{1.192}{15.243}, \frac{1.571}{11.357}, \frac{1.987}{8.235}\right) = (0.078{,}0.138{,}0.241)$$

$$\tilde{w}_{AQU} = (0.078{,}0.138{,}0.241)$$

$$\tilde{w}_{\mathrm{ORR}} = (0.078{,}0.138{,}0.241)$$

$$\tilde{w}_{AJI} = (0.040, 0.081, 0.175)$$

$$\tilde{w}_{EGS} = (0.028, 0.049, 0.094)$$

$$\tilde{w}_{\mathrm{QQL}} = (0.028, 0.049, 0.094)$$

$$\tilde{w}_{EUC} = (0.019, 0.032, 0.063)$$

The criteria' precise weights are subsequently derived using a centroid technique. By inserting the results into Eq. (5), we get:

$$w_{FCQ} = \frac{0.112 + 0.227 + 0.408}{3} = 0.249$$

$$w_{LRH} = \frac{0.078 + 0.138 + 0.241}{3} = 0.153$$

$$w_{AQU} = 0.153;\ w_{ORR} = 0.153;\ w_{AJI} = 0.099;\ w_{EGS} = 0.057;\ w_{QQL} = 0.057;\ w_{EUC} = 0.038$$

Following the normalization process, the weights of the crisps that are:

$$w_{FCQ} = 0.260;\ w_{LRH} = 0.159;\ w_{Orr} = 0.159;\ w_{AJI} = 0.103;\ w_{EGS} = 0.060;\ w_{QQL} = 0.060,\ w_{AQU} = 0.159,\ w_{EUC} = 0.039.$$

Using the values m_{ij} from the judgement matrix in Table 2, CR is calculated to ensure that the decision maker's judgements are consistent while generating the pairwise comparison matrix.

$$CI = \frac{(8.092 - 8)}{(8 - 1)} = 0.01314$$

$$CR = \frac{0.01314}{1.41} = 0.00932 < 0.1$$

Table 2 Specifications of fuzzy logic scale

Saaty's Scale	Description of fuzzy numbers	Numerical fuzzy	Reciprocation of numerical fuzzy
1	Equal importance	$\tilde{1} = (1, 1, 1)$	(1, 1, 1)
2	Assessment results ranging from moderately to evenly	$\tilde{2} = (1, 2, 3)$	(1/3, 1/2, 1)
3	Slightly more crucial	$\tilde{3} = (2, 3, 4)$	(1/4, 1/3, 1/2)
4	Evaluation scales ranging from highly to mildly	$\tilde{4} = (3, 4, 5)$	(1/5, 1/4, 1/3)
5	Critically more vital	$\tilde{5} = (4, 5, 6)$	(1/6, 1/5, 1/4)
6	Values ranging from extremely strongly to very strongly	$\tilde{6} = (5, 6, 7)$	(1/7, 1/6, 1/5)
7	Significantly more vital	$\tilde{7} = (6, 7, 8)$	(1/8, 1/7, 1/6)
8	Extremely high and very low judgment values	$\tilde{8} = (7, 8, 9)$	(1/9, 1/8, 1/7)
9	Much more crucial	$\tilde{9} = (8, 9, 9)$	(1/9, 1/9, 1/8)

According to the instructor's evaluations, researchers created a pairwise comparison matrix comparing the indicator scale stages and then used AHP. Table 3 displays the utility vectors (UV) that were generated from the valuations of every scale level for every indicator using the precise numbers from Table 1. All of the utility vector's components are associated with the descriptor levels that were defined in the structuring section. For a descriptor, 1 is the most useful level while 0 is the least useful level. All of the multicriteria model's pairwise comparison matrices have consistency ratios that are less than 10% or 0.1%.

Table 3 UV represent the scale stages of the criteria descriptors

Descriptor	UV	CR
AQU	(1, 0.476, 0.199, 0.063, 0)	0.0535
FCQ	(1, 0.557, 0.244, 0.083, 0)	0.0155
EGS	(1, 0.473, 0.212, 0)	0.0337
LRH	(1, 0.406, 0.122, 0)	0.0438
ORR	(1, 0.570, 0.223, 0.070, 0)	0.0416
EUC	(1, 0.563, 0.275, 0.099, 0)	0.0153
AJI	(1, 0.386, 0.108, 0)	0.0304
QQL	(1, 0.232, 0)	0.0355

4 The Fuzzy AHP, in Conjunction with the MACBETH Model Utilized for Decision Making in the Field of Gamification

The researchers developed the MACBETH, a comprehensive multicriteria approach that uses qualitative evaluations from individuals or groups to get a numerical value for the options. The theoretical foundations and practical applications can be notedby the authors [29, 30]. By outlining indicators linked to every criterion, assigning reference stages to the scale stages of every signifier, constructing value roles to ensure that the criteria can be compared on a normal scale, validating the values given to every substitute, and maintaining constancy in the decisions given the MACBETH provides a comprehensive methodology that aids in objective decision making. The M-MACBETH software, used for constructing MACBETH models.

4.1 Organizing

According to MACBETH each descriptor must be assigned two scale levels: neutral and good. If a decision maker finds a stage to be in the middle of the spectrum, neither too good nor too bad, then it is good and neutral Fig. 1. shows the hierarchical structure of the models.

4.2 Weighting

At this point, the MACBETH and fuzzy AHP method come together. The criteria generated from the geometric mean method are given weights to do this. Then, researchers got the value functions for all the criteria. To achieve this goal, a novel model semantic category to make assessments between each descriptor's scale levels: moderate, no, weak, very weak, very strong, extreme, strong, or combination of two or more subsequent classifications. Assume a positive category when it is impossible to precisely measure the variance in attractiveness between the stages of the scale. As a result of this MACBETH feature, the fuzzy logic shown in fuzzy AHP is strengthened, and the decision maker's doubt remains intact. For instance, the decision maker made the assessments depicted in Fig. 2 for the criterion of Flexibility in devising questionnaires. When comparing levels L11 and L15, the decision maker was unsure whether to use extreme or very strong language, thus a range of very strong- extreme was allocated, as shown in the Fig. 2. Findings from comparing the other levels to L15 reveal a similar pattern.

Using linear programming, M-MACBETH determines that 0 is the neutral stage and 100 is the good level in relation to the reference levels and judgements concerns. The criterion's value function that follows Fig. 3 displays the levels of flexibility when

	L11	L12	L13	L14	L15
L11	no	week	moderate	strong	vstrg-extr
L12		no	week	moderate	str-vstr
L13			no	week	mod-strg
L14				no	weak-mod
L15					no

Consistent judgements

Current Scale
100
50
0
-50
-100

extreme
v-strong
strong
moderate
week
very week
no

Fig. 2 Judgment matrix of criterion for MACBETH adaptability in the development of questions

it comes to creating questionnaires. Figure 3 shows that by repeating the process for all of the remaining criteria, we were able to produce value functions and judgement matrices that were consistent. The decision maker should check these value functions to make sure they accurately portray the relative importance of their judgments [31] (Fig. 4).

The last step was the input into fuzzy AHP criterion weightings from Sect. 3.2 into the M-MACBETH program. In order to validate the weightings that were introduced, automatically completing the judgment matrix among the criteria is the M-MACBETH software.

5 Results and Discussions

Figure 5 displays the model's results that were obtained using fuzzy AHP.

Using a simple additive aggregation process that moves from the bottom of the hierarchical or value tree to the top, MACBETH evaluates gamification apps. Equation (9) is used to determine V(A) the performance of an alternative A, when considering n decision criteria.

$$V(A) = \sum_{i=1}^{n} w_i v_i(\text{ impact of A on criterion } i)$$

$$\sum_{i=1}^{n} w_i = 1 \text{ and } w_i > 0 \text{ and } \begin{cases} v_i(\text{ most attractive impact level on } i) = 100 \\ v_i\ (\text{least attractive impact level on } i) = 0 \end{cases}$$

The value score of A in criteria i is v_i, where w_i is the weight of every criterion.

Figure 6 displays the total measurements, and the following shows the computation of the final valuations for every alternative.

$$V(S) = w_{FCQ} * v(FCQ(S)) + w_{LRH} * v(LRH(S)) + w_{AQU} * v(AQU(S))$$

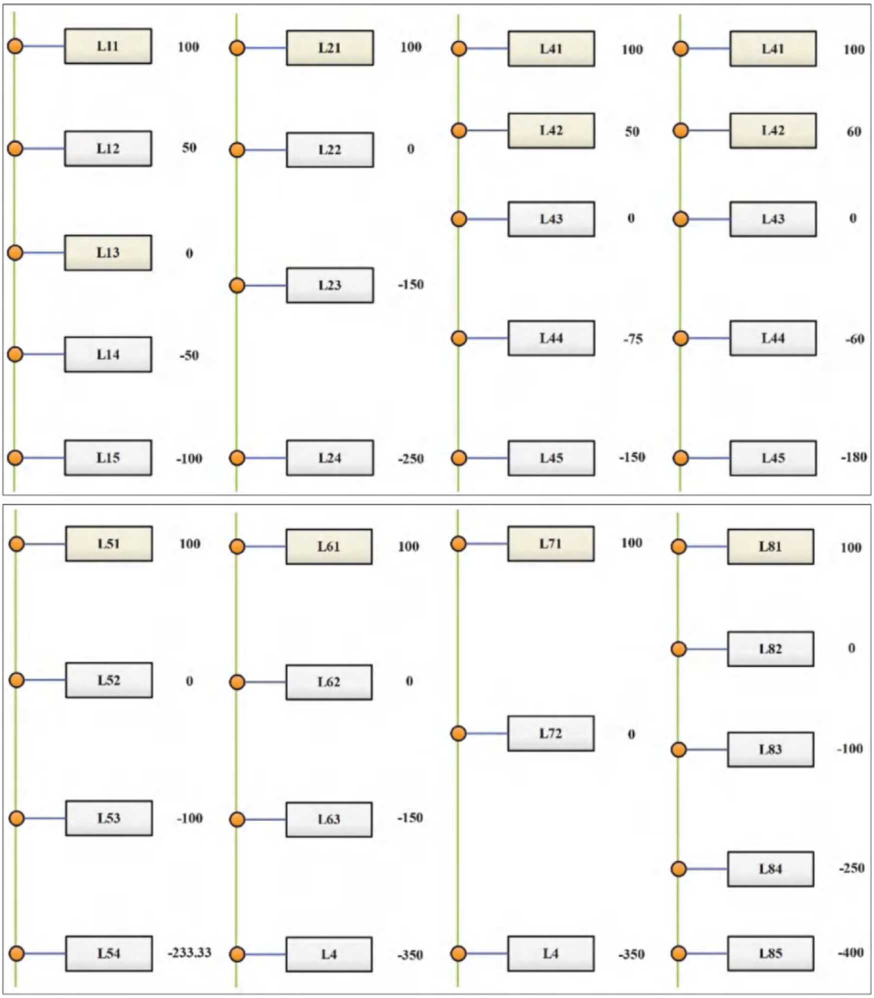


Fig. 3 Functions of criteria value

$$
\begin{aligned}
&+ w_{ORR} * v(\text{ ORR } (S)) + w_{AJI} * v(AJI(S)) + w_{EGS} \\
&* v(EGS(S)) + w_{QQL} * v(QQL(S)) + w_{EUC} * v(EUC(S)) \\
&= 0.260 * 100 + 0.159 * 100 + 0.159 * 0 + 0.103 * 100 + 0.060 * 100 \\
&+ 0.060 * (-150) + 0.159 * (-133.33) + 0.039 * 100 = 55.10
\end{aligned}
$$

$$
\begin{aligned}
V(Q) = {} & 0.260 * 50 + 0.159 * (-150) + 0.159 * 50 + 0.103 * 60 \\
& + 0.060 * (-233.33) + 0.060 * 100 + 0.159 * 0 + 0.039 * 100 = -7.44
\end{aligned}
$$

	[FCQ]	[QRR]	[LRH]	[AQU]	[AJI]	[EGS]	[QQL]	[EUC]	[all low]	Current Scale
[FCQ]	no	Positive	Positive	Positive	Positive	Positive	Positive	Positive	Positive	26.1
[QRR]		no	no	no	Positive	Positive	Positive	Positive	Positive	15.9
[LRH]		no	no	no	Positive	Positive	Positive	Positive	Positive	15.9
[AQU]		no	no	no	Positive	Positive	Positive	Positive	Positive	15.9
[AJI]					no	Positive	Positive	Positive	Positive	10.3
[EGS]						no	no	Positive	Positive	6.0
[QQL]						no	no	Positive	Positive	6.0
[EUC]								no	Positive	3.9
[all low]									no	0.0

extreme
v-strong
strong
moderate
week
very week
no

Fig. 4 The MACBETH judgment matrix is completed to determine the weightings for the criteria obtained from the fuzzy AHP

$$V(K) = (0.260 * 0 + 0.159 * (-250) + 0.159 * (-75) + 0.103 * 0 \\ + 0.060 * (-100) + 0.060 * 0 + 0.159 * 100 + 0.039 * (-100) = -59.88$$

Socrative, Quizizz, and Kahoot are the selected gamification applications, and the results for fuzzy analytic hierarchy process (per unit) as well as combining fuzzy analytic hierarchy process with the MACBETH (as a %) are comparable. In light of the findings from both models, we sought the perspective of the course instructor. As an alternative, they planned to implement was this one, and it was the most appropriate application for the kids and the program's unique qualities. Hence, the models' outputs are confirmed. The decision maker provided an explanation for why the criteria were given low weights. Elements of gamification (fun) with an effect on and motivation from the student population; this was a master's course, after all, and the students were well-prepared to enter the workforce with their prior degrees. If the evaluations were for a degree program, though, the decision-maker reasoned, this criterion would have been weightier, especially given the early stages of gamification's implementation. Regarding the standard, there are no ready questionnaires that serve to evaluate the information, either partially or whole, and the decision maker accorded the library of questions very little weight due to the specificity of the subject matter. But much as the fun criterion, degree program material is simpler, and researchers might find the ideas on some surveys made by other people. The first year of the Master of Science in Industrial Engineering program places a heavy emphasis on this criterion, while later year's shows a decrease in its weight. A sensitivity analysis confirms the stability of the models. In order to assess the alternatives, researchers maintained the relative weights of the criteria and made consistent adjustments to their weighting. The sensitivity analysis on the model that was constructed using fuzzy AHP. Putting more emphasis on the factors the ability to easily modify existing surveys, collect data, and compile reports there

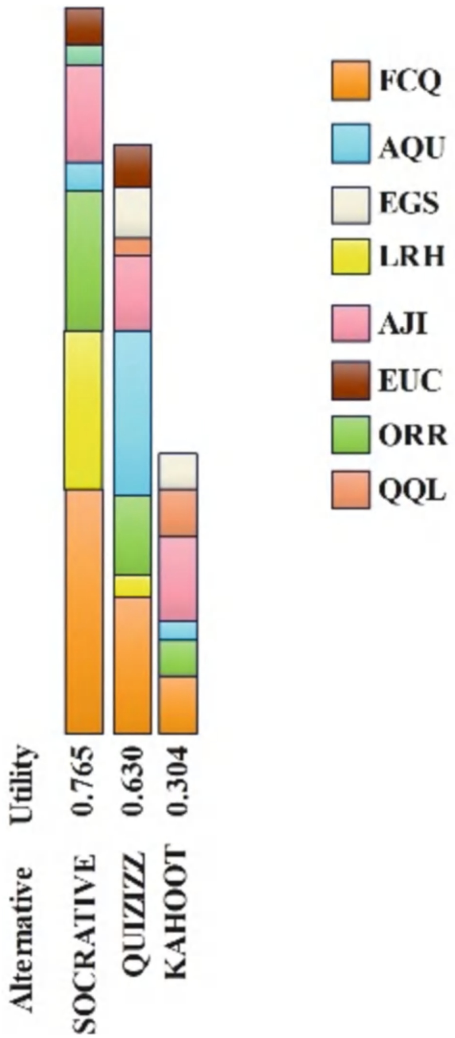


Fig. 5 Categorization of replacements with fuzzy analytic hierarchy process

is no difference in the categorization of the alternatives when comparing the ability to implement just-in-time instruction to 100% or reducing to 0%, and Socrative is always chosen. The alternative classification remains unchanged when the learning rhythm criterion's weight is increased to 100%, but it shifts in favour of Quizizz when the weight is decreased to 0.5%. It is not reasonable to give the criterion a weight of 0.5% since it would effectively disappear in practice. Quizizz takes over as the go-to option when the questionnaire's criterion valuation weights 29% (an increase of 82.39%). But even with this criterion's weight reduced to zero, Socrative remains the best option. Elements of gamification, which include enjoyment have a comparable effect on students' motivation and engagement. Quizizz is selected

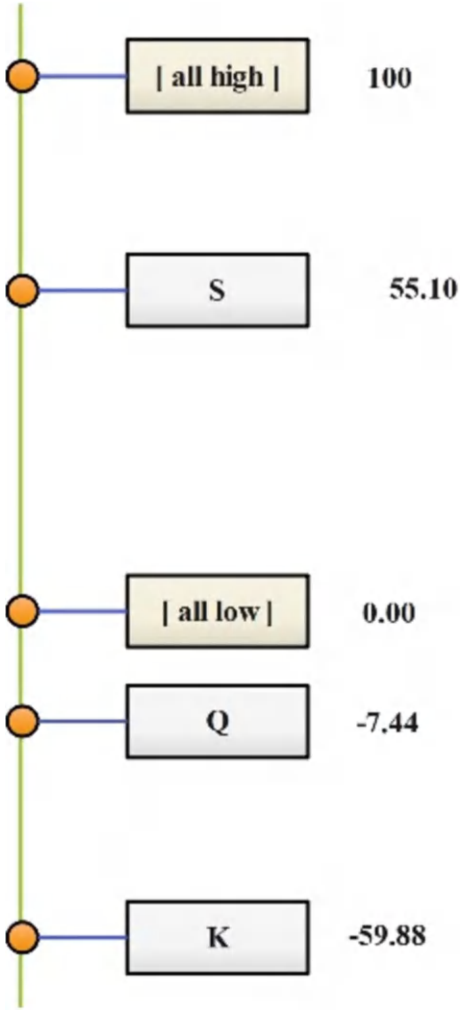


Fig. 6 Categorization of replacements combining the MACBETH and fuzzy analytic hierarchy process

when the weighting hits 20% (rise of 233.33% relative to the assigned weighting), while Socrative is selected if the weighting of this criterion is dropped. That this criterion would be more essential at lower stages lends credence to the decisions made by the decision centre, which likely would have put Quizizz ahead of Socrative. Kahoot would be selected as the alternative when the Quality of the question library weighting is raised to 36% (a 500% rise), whereas Socrative remains the same when the weighting is dropped from the actually allocated stage of 6%. Even with a 3.9% reduction, Quizizz would still be preferred above Socrative if the "Ease of use in class" criterion was increased to 92%. When considering the other criteria' relative importance, it is clear that the necessary weighting increases are excessive in all of these instances; furthermore, reducing the weightings of these criteria by any amount has no effect on the alternatives' categorization. This research concludes that the model is stable.

Changes to the criterion weightings in the model that combines fuzzy analytic hierarchy process and MACBETH are shown in Figs. 7, 8, 9, 10, 11 and 12 as variations in the alternative classifications. The decision maker's proposed weightings for a criterion, expressed as a percentage. As the criterion's weight changes, the y-axis displays the options' valuation. On the x-axis, researchers can see the range of possible criterion weightings, from 0 to 100%. Figure 7, 8, 9, 10, 11 and 12 indicates that depending on the weightings given to the criteria, the categorization of options remains unchanged for the following: Flexibility in the construction of questionnaires; Obtaining reports and outputs; and Learning rhythm. Both Socrative and Quizizz are considered unviable since their valuations differ when the criterion of Ease of use in class is given a 100% weight (all other criteria would be given a 0% weight). It would take a weighting of 62.7%, or 293.71%, for the choices to be inverted and Quizizz to take first position; at the moment, the questionnaire's criterion valuation is 16.23% weighted. To illustrate, if the alternatives were to be reclassified, for "Ability to utilize just-in-time teaching" it would rise by 245.63%, for "QQL" it would rise by 500% and the weighting for "EGS" would increase by 321.35%. The model is determined to be resilient because none of these increases are reasonable; minor changes in the criterion weightings do not affect the alternatives ultimate categorization.

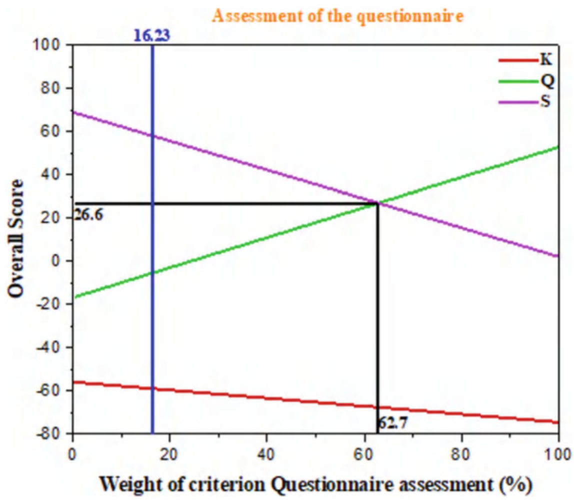


Fig. 7 Sensitivity analysis chart for assessment of questionnaire

Fig. 8 Sensitivity analysis chart for obtaining results and reports

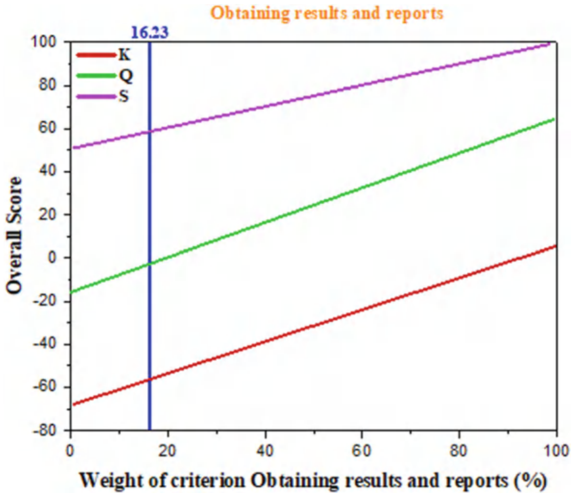
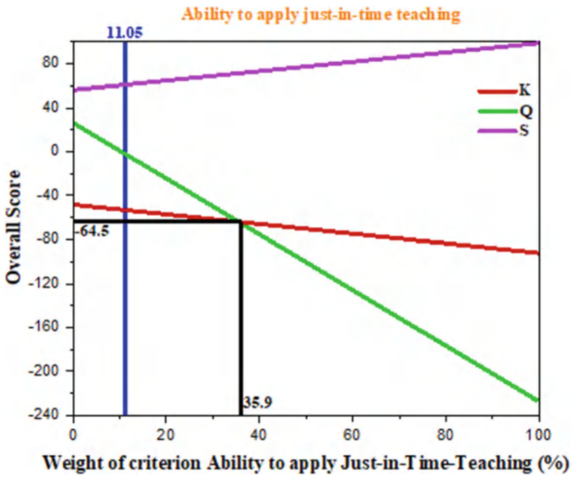


Fig. 9 Sensitivity analysis chart for ability to apply just-in-time teaching



6 Conclusions

The utilization of gamification in educational settings has been driven by the pursuit of more participatory approaches to teaching and learning. The advantages are considerable and compared with various real-world experiments. Many gamification applications have been developed as a result of this success. There are thousands of applications available, but the teacher must select one. This decision-making process could significantly impact students' outcomes; yet, no models to guide this process were found in the conducted literature study. Fuzzy AHP and its combination with the MACBETH approach were thus established as two techniques in this study to

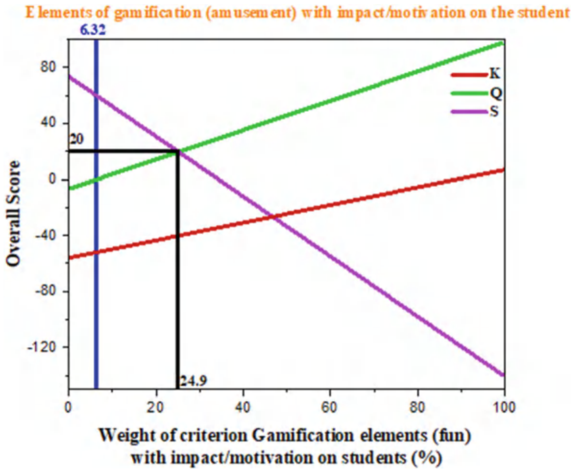


Fig. 10 Sensitivity analysis chart for elements of gamification (amusement) with impact/motivation on the student

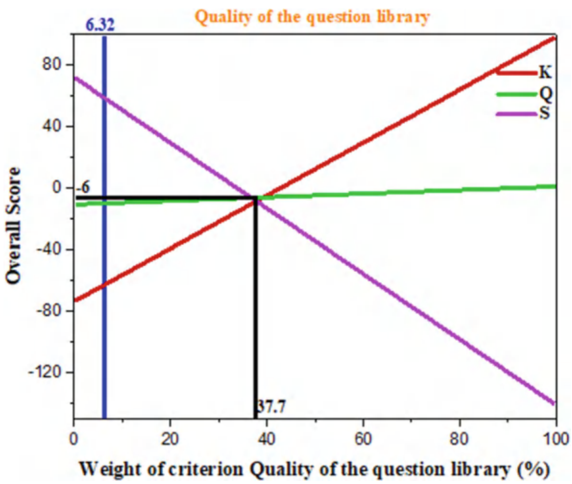


Fig. 11 Sensitivity analysis chart for quality of the question library

account for uncertainty resulting from human judgements being imprecise or nebulous in real-world circumstances. To our knowledge, this is the initial published work to merge fuzzy AHP with the MACBETH method. The weightings utilized to select a gamification application for a Master's program course were computed using the geometric mean methodology proposed by authors. To get the whole alternative classification for the two methods mentioned, we employ the crisp weights from the fuzzy AHP methods. Comparing the two sets of data, it's clear that Socrative, not Quizizz or Kahoot is the best gamification app for the class under review. But it's easy to see that Quizizz would be the obvious pick if the class were required early on in a degree program. This is because it is more crucial to incorporate aspects directly connected to games when working at early levels, when the incentive required is thought to

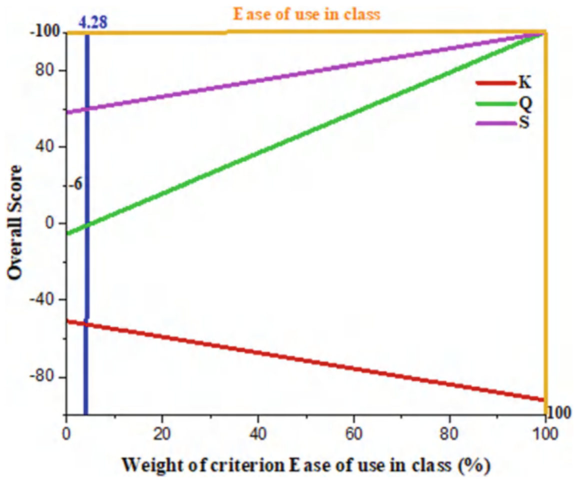


Fig. 12 Analysis of sensitivity for ease of use in class

be stronger. This course is appropriate for college students since it allows them to get feedback on their progress toward learning goals without giving any weight to the gamification aspects in terms of actual learning. Following the steps outlined in this study, the methodology, criteria/descriptors, and weightings can be applied to different courses or programs, or customized to meet the requirements of individual courses. Utilizing the group decision-making methods founded on card-arranging suggested in researchers to narrow the opening set of replacements to a possible one that specialists can comfortably analyse is one example of how future work aims to incorporate new gamification applications as replacements in this program. Further courses in undergraduate and graduate degree programs will also make use of both approaches, particularly in situations when multiple instructors are responsible for a given class and a decision-making collection needs to be assembled to ascertain qualitative evaluations of the relative importance of various factors.

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Exploring the Efficacy of Machine Learning Algorithms Across Diverse Feature Selection Strategies in Rice Classification Tasks

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Abstract Rice, a staple crop of paramount global importance, is instrumental in ensuring food security. As the world's population continues to grow, the demand for increased rice production becomes increasingly vital. Accurate and efficient classification of rice types is indispensable in agriculture. Differentiating various rice varieties is crucial for effective crop management, quality control, and addressing consumer preferences. Historically, the classification process has relied on manual assessments, making it labor-intensive and susceptible to human error. In recent years, the integration of machine learning techniques, particularly ensemble methods like Decision Trees and Random Forest, has revolutionized rice type prediction. This

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contribution of this work is primary focus is on developing a model capable of automatically categorizing rice grains into their respective varieties based on meticulously extracted morphological features. Additionally, feature selection techniques are applied to enhance the model's efficiency, mitigate over fitting, and improve prediction accuracy. This research is of paramount significance. Automating rice type classification not only reduces human error but also delivers swift and consistent results, benefiting both farmers and consumers. This advancement holds the potential to enhance crop management efficiency, optimize resource allocation, and elevate product quality control.

Keywords Rice type classification · Ensemble machine learning · Morphological features · Feature selection techniques · Crop

1 Introduction

Rice stands as a staple grain cherished worldwide for its widespread consumption. Particularly prominent in Southern Asia, both in production and consumption, rice boasts an annual global output of around 800 million tons, featuring numerous distinct varieties, each with its own defining traits. This study delves into the classification research of Osmancik and Cammeo rice varieties [1]. The cultivation and classification of rice, a globally vital staple crop, play a pivotal role in ensuring food security, particularly with the escalating demand driven by a growing world population. The accurate categorization of rice types holds immense significance in agriculture, influencing crop management, quality control, and meeting consumer preferences. Traditionally, manual assessments have been the primary method for classifying rice varieties but this approach is labor-intensive and susceptible to human error. In recent years, a transformative shift has occurred with machine learning techniques, specifically ensemble methods such as XGBoost, AdaBoost and Random Forests etc. that revolutionizing the prediction of rice types. This work focused primarily on the development of a model designed to automatically categorize rice grains into their respective varieties and classification is based on meticulously extracted morphological features, and the model's efficiency is further improved through the application of feature selection techniques. The primary objectives include mitigating over fitting and enhancing prediction accuracy. The paramount significance of this work lies in the automation of rice type classification, offering advantages such as the reduction of human error and the delivery of swift and consistent results. These advancements stand to benefit both farmers and consumers, with potential implications for enhancing crop management efficiency, optimizing resource allocation, and elevating product quality control. Ultimately, the adoption of ensemble machine learning models for rice type prediction holds promise in contributing to the sustainability and productivity of rice production. This, in turn, has the potential to make a meaningful impact on global food security, addressing the challenges posed by a dynamic and ever-changing world. The objective of the research work to is classifying the rice varieties based on the features using ML algorithms. The remaining paper was organized as Sect. 2 related work and Proposed work in Sect. 3, Sect. 4 Experimental setup and Results and conclusion in Sect. 5.

2 Related Work

In the realm of machine learning, the algorithms mentioned have been subjects of extensive research across various domains. XGBoost [2], a powerful gradient boosting algorithm, has garnered attention for its versatility and effectiveness. Researchers have focused on optimizing its hyper parameters, investigating its interpretability, and extending its applications, such as integrating it into ensemble methods and exploring its utility in time-series forecasting. AdaBoost [3], a popular boosting algorithm, has been widely studied both theoretically and practically. Researchers have delved into its convergence properties, examined its behavior with diverse weak learners, and adapted it for imbalanced datasets. K-Nearest Neighbors (KNN) has been a focal point in pattern recognition and clustering research [4, 5]. Studies have explored different distance metrics, efficient indexing structures, and extensions for semi-supervised and active learning scenarios. Singular Value Decomposition (SVD) [6, 7] and Principal Component Analysis (PCA) [8, 9], as dimensionality reduction techniques, have found applications in image compression, signal processing, and collaborative filtering. Recent work has aimed at improving their efficiency, scalability, and robustness to noise and outliers. Ensemble learning [10], combined with feature selection techniques, has been a topic of investigation to understand the impact of different feature selection methods on ensemble models' performance and stability. Ongoing research in these areas continues to refine and extend these algorithms, addressing challenges and exploring novel adaptations for emerging issues in machine learning and data analysis. Exploring these related works allows the research on ensemble methods for rice type classification to build upon established methodologies, draw lessons from successes and challenges in analogous applications, and contribute to the evolving landscape of machine learning applications in agriculture.

3 Proposed Work

The proposed methodology for classification of rice including series of steps and outlined in below sections

A. **Data Set**

A dataset consisting of 3810 images [11] of rice namely Cammeo and Osmancik, was collected and processed. From each image, 7 morphological features were extracted for analysis and classification purposes. These features provide valuable insights into the characteristics of each grain of rice, allowing for differentiation between the two species.

B. **Steps of Architecture**

The essential steps involved in the proposed model design are outlined as follows:

1. ***Data Pre-processing*** The data preprocessing phase involved several critical steps aimed at ensuring data quality and reliability. Initially, a comprehensive analysis of the data properties was conducted to understand its distribution and characteristics thoroughly. Following this, standard normalization techniques were implemented to standardize the scale of the features, facilitating consistent comparisons and analyses across variables. Furthermore, a rigorous approach was taken to detect and address outliers in the dataset. Outliers were identified using the Interquartile Range (IQR) method, a robust statistical technique capable of detecting data points significantly deviating from the central tendency. Subsequently, these outliers were replaced with values derived from the IQR, thereby maintaining data integrity while preserving the overall distribution. Through systematic data preprocessing, our aim was to establish a solid foundation of clean and standardized data. This approach enhances the reliability and validity of subsequent analyses and interpretations, ensuring robust findings
2. ***Features Extraction*** SVD [12], PCA [13], and LDA [14] techniques were applied to selecting a reduced set of features that retain the essential information necessary analysis tasks while eliminating redundant or less informative features (Fig. 1).
3. ***Model Development-*** KNN, XGBoost, AdaBoost, Logistic Regression, SVM, Decision Tree, Random Forest were trained with chosen features and evaluated performance.

4 Experimental Setup and Results

For evaluation of performance of the models, we have created application in Google Colab with python programming language along with GPU features. We collected data set from the kaggle and initially preprocessed data and verified null values. Then the distribution of instances of class attributes is verified and distribution class Osamncik is 2180 instance and Cammeo is 1630 shown in below Tables 1 and Fig. 2. Figure 3 depicted the quantile information and higher whisker, lower whisker values mentioned. For the classification of rice type, seven machine learning algorithms KNN, XGBoost, AdaBoost, LR, SVM, DT, RF were executed without features selection and evaluated results for each classifier with metrics accuracy in Fig. 4, evaluation metrics without feature selection in Table 2.

The provided results Table 3 show the performance for classifiers with feature selection techniques. From the results, we observe that AdaBoost has the highest

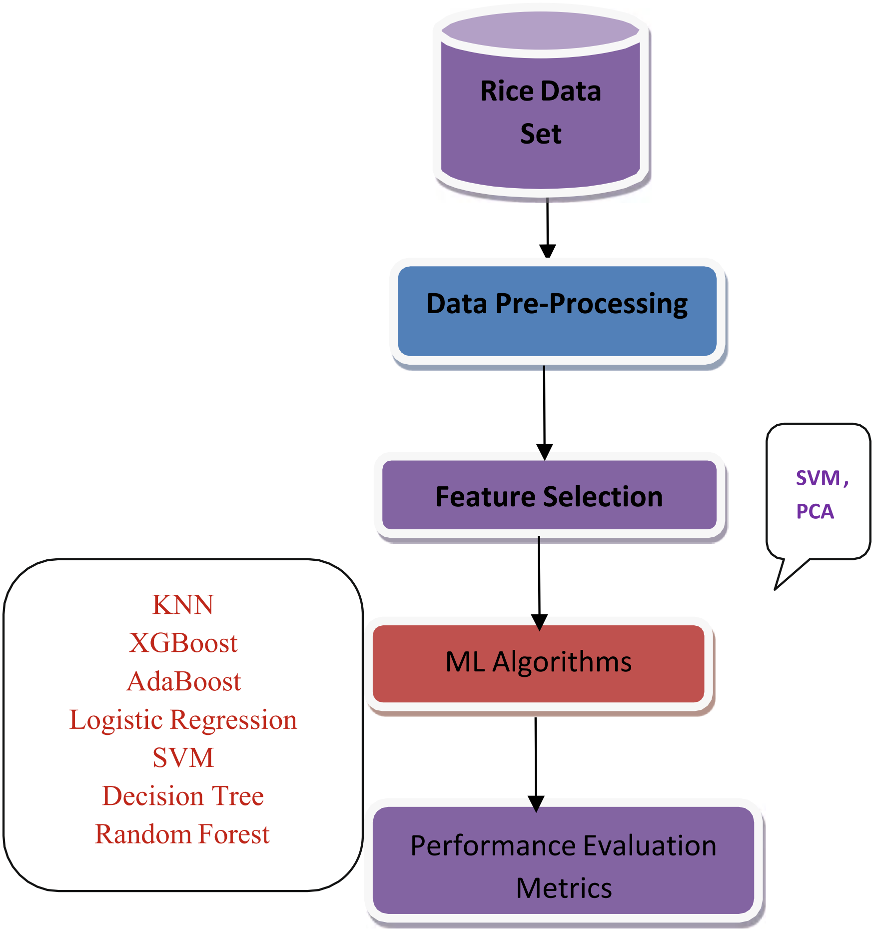

Fig. 1 Flow diagram of proposed work

Table 1 Count of target class

Class	Count of Instance
Osmancik(0)	2180
Cammeo(1)	1630

accuracy (92.78%), followed closely by XGBoost (91.86%) and Logistic Regression (92.52%). Random Forest also performs well with an accuracy of 92.26%. For both classes (Cammeo and Osmancik), AdaBoost achieves high precision and recall scores and consistently shows the highest F1-scores for both classes, indicating its robustness in capturing both precision and recall effectively. In conclusion, based on

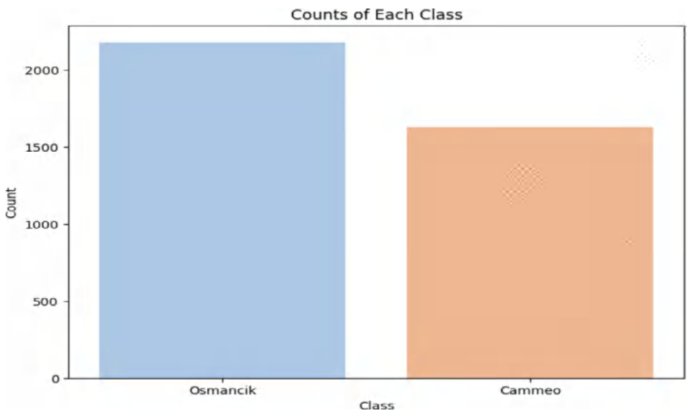

Fig. 2 Distribution of Instances count for class Osamncik and Cammeo

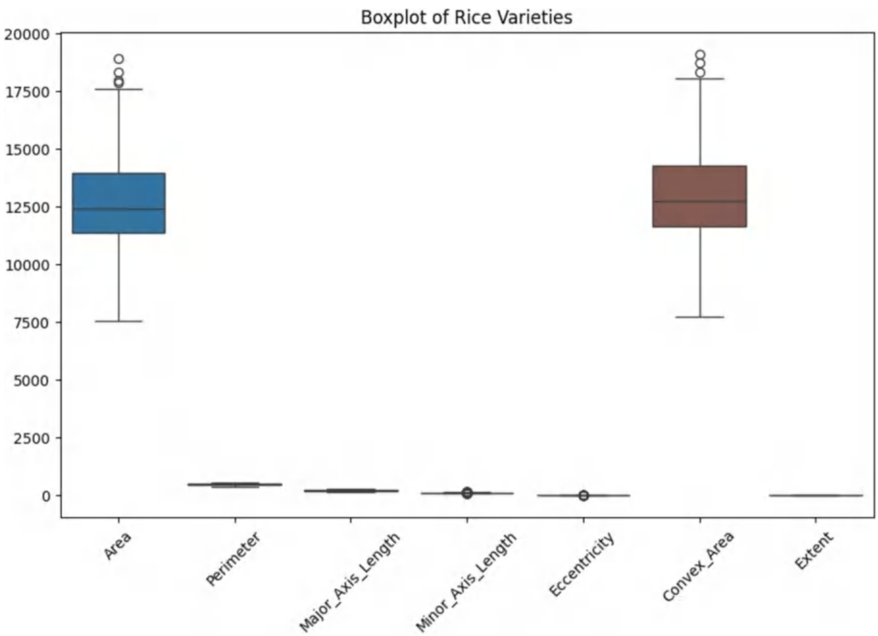

Fig. 3 Box plot

these results, AdaBoost demonstrates superior performance across multiple evaluation metrics, making it a promising choice for the classification task on the given dataset.

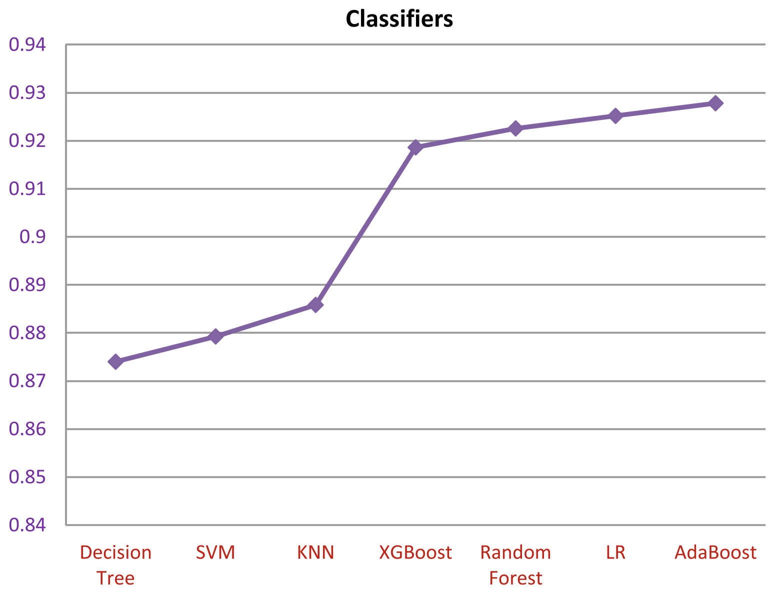

Fig. 4 Accuracy of classifiers

Table 2 Model evaluation metrics without feature selection

	Accuracy	Precision (0)	Precision (1)	Recall (0)	Recall (1)	F1-score (0)	F1-score (1)
KNN	0.885827	0.897281	0.87703	0.848571	0.917476	0.872247	0.896797
XGBoost	0.918635	0.909091	0.926829	0.914286	0.92233	0.911681	0.924574
AdaBoost	0.927822	0.913165	0.940741	0.931429	0.924757	0.922207	0.932681
Logistic regression	0.925197	0.924638	0.925659	0.911429	0.936893	0.917986	0.931242
SVM	0.879265	0.908228	0.858744	0.82	0.929612	0.861862	0.892774
Decision Tree	0.874016	0.877976	0.870892	0.842857	0.900485	0.860058	0.885442
Random Forest	0.922572	0.921739	0.923261	0.908571	0.934466	0.915108	0.92883

Table 3 Performance metrics of each classifier with feature selection

Model	Feature selection	Accuracy	Precision (0)	Precision (1)	Recall (0)	Recall (1)	F1-score (0)	F1-score (1)
KNN	SVD	0.860892388	0.87195122	0.852534562	0.817142857	0.898058252	0.843657817	0.874704492
	LDA	0.921259843	0.919075145	0.923076923	0.908571429	0.932038835	0.913793103	0.927536232
	PCA	0.860892388	0.87195122	0.852534562	0.817142857	0.898058252	0.843657817	0.874704492
XGBoost	SVD	0.86351706	0.87962963	0.851598174	0.814285714	0.905339806	0.845697329	0.877647059
	LDA	0.926509186	0.945454545	0.912037037	0.891428571	0.95631068	0.917647059	0.933649289
	PCA	0.860892388	0.883647799	0.844594595	0.802857143	0.910194175	0.841317365	0.876168224
AdaBoost	SVD	0.8832021	0.916932907	0.859688196	0.82	0.936893204	0.865761689	0.896631823
	LDA	0.92519685	0.945288754	0.909930716	0.888571429	0.95631068	0.916053019	0.932544379
	PCA	0.880577428	0.90851735	0.860674157	0.822857143	0.92961165	0.863568216	0.893815636
Logistic Regression	SVD	0.559055118	0.529166667	0.572796935	0.362857143	0.725728155	0.430508475	0.640256959
	LDA	0.927821522	0.930029155	0.92601432	0.911428571	0.941747573	0.920634921	0.933814681
	PCA	0.887139108	0.897590361	0.879069767	0.851428571	0.917475728	0.873900293	0.897862233
SVM	SVD	0.879265092	0.908227848	0.858744395	0.82	0.92961165	0.861861862	0.892773893
	LDA	0.927821522	0.930029155	0.92601432	0.911428571	0.941747573	0.920634921	0.933814681
	PCA	0.879265092	0.908227848	0.858744395	0.82	0.92961165	0.861861862	0.892773893
Decision Tree	SVD	0.835958005	0.824207493	0.845783133	0.817142857	0.851941748	0.820659971	0.84885127
	LDA	0.897637795	0.893063584	0.901442308	0.882857143	0.910194175	0.887931034	0.905797101
	PCA	0.811023622	0.787709497	0.831683168	0.805714286	0.815533981	0.796610169	0.823529412
Random Forest	SVD	0.87007874	0.895899054	0.851685393	0.811428571	0.919902913	0.851574213	0.884480747
	LDA	0.897637795	0.893063584	0.901442308	0.882857143	0.910194175	0.887931034	0.905797101
	PCA	0.864829396	0.88	0.853546911	0.817142857	0.905339806	0.847407407	0.878680801

5 Conclusion

In this study, we explored the efficacy of various machine learning algorithms combined with diverse feature selection strategies for the task of rice classification. The accurate and efficient classification of rice types is crucial for optimizing crop management, ensuring quality control, and meeting consumer preferences. By leveraging machine learning techniques, particularly ensemble methods such as Decision Trees and Random Forests, we aimed to automate the rice classification process, thereby reducing the labor-intensive and error-prone nature of manual assessments. Adaboost produced highest accuracy without feature selection. The produced findings underscore the importance of feature selection techniques in enhancing model performance. Techniques like SVD, LDA, and PCA were instrumental in improving prediction accuracy, mitigating overfitting, and boosting the overall efficiency of the classification models. Among the evaluated algorithms, models incorporating LDA for feature selection consistently demonstrated superior performance across multiple metrics, including accuracy, precision, recall, and F1-score. The AdaBoost model with LDA reached an accuracy of 0.9252, Logistic Regression and SVM models using LDA also showed high performance, both achieving an accuracy of 0.9278, with precision and F1-scores closely aligned, highlighting their robustness and reliability for this classification task. The automation of rice type classification through machine learning not only reduces human error but also provides rapid and consistent results. This advancement holds significant potential for enhancing crop management efficiency, optimizing resource allocation, and elevating product quality control.

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Industrial Intelligence: A Fuzzy Logic Approach

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Abstract This chapter, titled "Industrial Intelligence: A Fuzzy Logic Approach", delves into the transformative realm of Industrial Intelligence, presenting Fuzzy Logic as a powerful and adaptive methodology. In the face of increasing complexity and uncertainty in industrial settings, Fuzzy Logic emerges as a key enabler for intelligent decision-making. The abstract explores the symbiotic relationship between Industrial Intelligence and Fuzzy Logic, offering a comprehensive overview of foundational concepts, real-world applications, and the potential for shaping the future of smart and adaptive industrial systems. With a focus on enhancing automation, navigating uncertainty, and integrating with smart technologies, this chapter provides valuable insights into the dynamic landscape of Industry 4.0.

Keywords Industrial intelligence · Fuzzy logic · Decision-making · Automation · Adaptive systems · Uncertainty · Smart technologies · Engineering applications

1 Introduction to Industrial Intelligence

Industrial intelligence refers to the combination of advanced artificial intelligence (AI), data analytics, and machine learning (ML) in industrial processes to improve decision-making, optimize operations, and drive competence. By leveraging vast amounts of data generated by sensors, machines, and systems within an industrial setting, industrial intelligence enables predictive maintenance, real-time monitoring, and automated control of processes [1]. This convergence of technology not only reduces downtime and operational costs but also improves product quality and safety standards. Furthermore, industrial intelligence fosters innovation by providing actionable insights that facilitate continuous improvement and adaptive responses

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to changing market demands and operational challenges. As industries increasingly embrace digital transformation, industrial intelligence stands at the forefront, revolutionizing traditional manufacturing and production landscapes.

1.1 A Defining Industrial Intelligence and Its Role in Modern Engineering

Industrial intelligence involves the application of advanced technologies like data analytics, AI, and ML to industrial processes [2]. This approach enhances decision-making, optimizes performance, and improves efficiency by gathering and investigating data from various sources within an industrial environment. Figure 1 shows the key activities those include predictive maintenance, real-time monitoring, process automation, and quality control in modern industries. By leveraging real-time data, engineers can forecast equipment letdowns before they happen, decreasing downtime and maintenance costs, and extending machinery lifespan. Real-time monitoring allows for immediate adjustments, crucial in manufacturing for maintaining consistent product quality and minimizing waste. AI and machine learning algorithms detect patterns and anomalies that human operators might miss, leading to more accurate and timely interventions.

Moreover, industrial intelligence supports process automation, enabling AI-powered systems to perform complex tasks with greater precision and efficiency than manual methods. This not only increases output but also reduces human error and enhances workplace safety. Industrial intelligence fosters innovation by providing engineers with deep insights into operations and informing the development of new

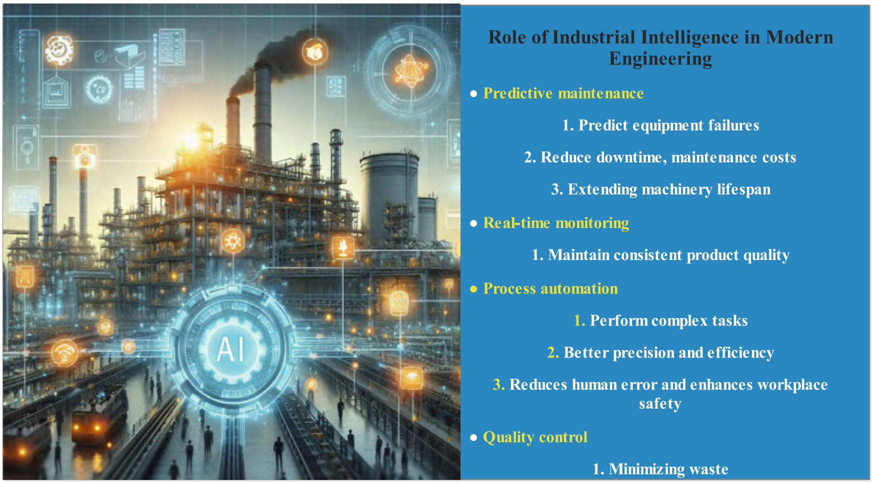

Fig. 1 Role of Industrial Intelligence in modern engineering

products and processes [3]. These insights drive continuous improvement and offer a competitive advantage. As industries become increasingly interconnected and reliant on digital technologies, industrial intelligence is essential for staying ahead of the curve and meeting evolving market demands.

1.2 Overview of Emerging Challenges in Decision-Making for Industries

In today's rapidly evolving industrial landscape, decision-making faces a host of emerging challenges that significantly impact operational efficiency, strategic planning, and overall competitiveness. One of the main challenges is the sheer volume and complexity of information generated by modern industrial systems. As industries adopt advanced technologies and IoT devices, they produce vast amounts of information that required to be processed, analyzed, as well as interpreted in real-time [4]. Managing this data deluge requires sophisticated data analytics tools and capabilities, which many organizations still struggle to implement effectively.

Another significant challenge is the integration of legacy systems with new technologies. Many industries operate with a mix of outdated and modern equipment, creating compatibility issues and hindering seamless data flow and communication. This integration challenge complicates the implementation of advanced decision-making tools and technologies, such as AI and machine learning, which rely on cohesive and comprehensive data sets [5].

Cybersecurity threats present another critical challenge in industrial decision-making. As industries become more interconnected, the risk of cyber-attacks increases, potentially compromising sensitive data and disrupting operations. Ensuring robust cybersecurity measures while maintaining operational efficiency is a delicate balance that decision-makers must achieve. Furthermore, the growing need for sustainability and regulatory compliance adds another layer of complexity. Industries are under increasing pressure to reduce their environmental impact and adhere to stringent regulations, which requires informed decision-making that aligns with both business goals and regulatory standards.

Lastly, workforce skills and cultural adaptation pose significant challenges. The adoption of advanced technologies requires a workforce skilled in new tools and methodologies, which often necessitates extensive training and development programs. Moreover, fostering a culture that embraces digital transformation and innovation is crucial for successful implementation. Resistance to change and lack of technical expertise can impede the effective utilization of industrial intelligence, ultimately affecting decision-making processes and outcomes. Addressing these emerging challenges is essential for industries to leverage the full potential of modern technologies and maintain a competitive edge.

2 Foundations of Fuzzy Logic

The foundations of fuzzy logic lie in its capability to handle uncertainty and imprecision by extending traditional binary logic to include partial truth values between 0 and 1. This framework is built on the concept of fuzzy sets, where elements have degrees of membership rather than strict inclusion or exclusion [6]. Fuzzy logic operations—such as union, intersection, and complement—are defined in terms of these membership functions, allowing for flexible manipulation of fuzzy sets. This approach is governed by principles that mirror classical set theory but accommodate the nuances of real-world scenarios. As an adaptive decision-making tool, fuzzy logic employs a set of "if–then" rules and an inference engine to model complex systems and reason under uncertainty [7]. By enabling human-like reasoning and handling vague or incomplete information, fuzzy logic proves invaluable in diverse applications, including control systems, pattern recognition, and artificial intelligence, where precision and adaptability are crucial. By accommodating the nuances of real-world scenarios, fuzzy logic provides innovative solutions to complex problems.

2.1 Basics of Fuzzy Sets, Operations, and Principles

Fuzzy logic extends traditional binary logic by introducing the concept of partial truth, where truth-values vary between 0 and 1. This approach is especially useful for dealing with uncertainties and imprecision in real-world scenarios. The fundamental element of fuzzy logic is the fuzzy set, which is characterized by a membership function that gives a degree of membership to each element in the set. Unlike classical sets, where elements either belong or do not belong to a set, fuzzy sets allow elements to have varying degrees of membership [8].

Operations on fuzzy sets comprise union, intersection, and complement, analogous to their classical set counterparts but defined in terms of membership functions. For instance, the union of two fuzzy sets A and B is determined by taking the maximum membership value of the two sets for each element, while the intersection is calculated by the minimum membership value. The complement of a fuzzy set is found by subtracting the membership value from 1. These operations are governed by principles such as the commutative, associative, and distributive laws, which help in manipulating and combining fuzzy sets effectively.

2.2 Understanding Fuzzy Logic as an Adaptive Decision-Making Tool

Fuzzy logic serves as an adaptive decision-making tool by providing a flexible and intuitive framework for modeling complex systems and reasoning under uncertainty.

Unlike traditional binary logic, which can be rigid and inadequate for capturing the nuances of real-world situations, fuzzy logic allows for more realistic and human-like reasoning [9]. This adaptability makes it particularly valuable in applications where precise information is unavailable or difficult to obtain.

In practical decision-making, fuzzy logic systems contain of a set of fuzzy rules and an inference engine. The rules are typically in the form of "if–then" statements that describe relationships between input variables and desired outputs. For example, in a temperature control system, a rule might be, "If the temperature is high, then reduce the heating." The inference engine evaluates these rules based on the current input values, using fuzzy set operations to combine the rules and derive a conclusion. The final output is then defuzzified to produce a specific action or decision. The whole steps of the fuzzy logic system's working are shown in Fig. 2.

The strength of fuzzy logic lies in its capability to handle vague, imprecise, or incomplete information, making it an ideal tool for adaptive decision-making in various fields such as control systems, pattern recognition, and artificial intelligence [10]. By mimicking human reasoning, fuzzy logic systems can adapt to varying conditions and make robust decisions in dynamic environments, enhancing their applicability in complex and uncertain scenarios.

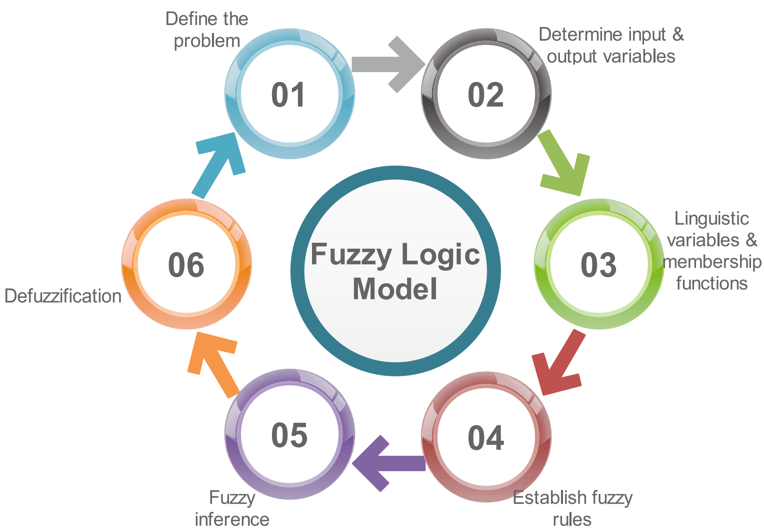

Fig. 2 Steps of working on the fuzzy logic system

3 Applications of Fuzzy Logic in Industrial Contexts

Fuzzy logic has proven to be a versatile and powerful tool in various industrial applications, providing effective solutions for complex and uncertain environments [11]. Its ability to handle imprecision and model human-like reasoning makes it particularly valuable in several key areas:

1. **Process Control**: Fuzzy logic is extensively used in process control systems to manage and optimize industrial processes. For example, in chemical plants, fuzzy logic controllers regulate temperature, pressure, and flow rates, ensuring that processes run smoothly despite variability in input materials and operating conditions [12]. By incorporating expert knowledge through fuzzy rules, these controllers can adapt to changing conditions more effectively than traditional control methods.
2. **Quality Control**: In manufacturing, fuzzy logic helps maintain high product quality by monitoring and adjusting production parameters. For instance, in the production of electronics, fuzzy logic systems can evaluate the quality of solder joints by assessing various attributes such as shape, size, and temperature profiles. This allows for real-time adjustments and reduces the likelihood of defects [13].
3. **Predictive Maintenance**: Fuzzy logic enhances predictive maintenance strategies by analyzing sensor data to predict machine failures before they happen. By considering multiple factors and their uncertainties, such as vibration levels, temperature changes, and operating hours, fuzzy systems can provide more accurate maintenance schedules [14]. This reduces downtime and extends the lifespan of machinery.
4. **Energy Management**: In industrial settings, energy management is crucial for cost savings and sustainability. Fuzzy logic systems optimize energy consumption by controlling HVAC systems, lighting, and machinery operation based on fuzzy rules that consider occupancy, time of day, and environmental conditions [15]. This results in significant energy savings and improved efficiency.
5. **Robotics and Automation**: Fuzzy logic is employed in robotics and automation for tasks that require adaptive and flexible control [16]. Robots equipped with fuzzy logic controllers can handle tasks such as assembly, welding, and material handling with greater precision and adaptability to varying conditions. This enhances productivity and reduces the need for human intervention.
6. **Supply Chain Management**: In supply chain management, fuzzy logic assists in decision-making processes related to inventory control, demand forecasting, and supplier selection [17]. By dealing with uncertainties in demand and supply, fuzzy logic models provide more reliable and efficient solutions, optimizing inventory levels and reducing costs.
7. **Environmental Monitoring**: Fuzzy logic systems are used to monitor and manage environmental parameters in industrial contexts, such as air quality, waste management, and pollution control. These systems can interpret sensor data to make real-time decisions, ensuring compliance with environmental regulations and reducing the ecological footprint of industrial activities [18].

In summary, fuzzy logic's ability to handle imprecise information and mimic human reasoning makes it a powerful tool in various industrial applications. From process control and quality assurance to predictive maintenance and energy management, fuzzy logic enhances efficiency, reduces costs, and improves decision-making in complex and dynamic industrial environments.

3.1 Fuzzy Logic Controllers for Industrial Processes

Fuzzy logic controllers (FLCs) are widely used in industrial processes to enhance control and optimization. Unlike traditional controllers that rely on precise mathematical models, FLCs leverage fuzzy logic to handle ambiguity and vagueness, making them ideal for complex and nonlinear systems. In industrial settings such as chemical plants, oil refineries, and manufacturing lines, FLCs are employed to regulate variables like temperature, pressure, flow rates, and speed [12]. For example, in a temperature control system, an FLC can maintain desired levels by adjusting heating or cooling based on fuzzy rules derived from expert knowledge. These rules allow the controller to respond to varying conditions more effectively than traditional methods, ensuring stable and efficient operation. Additionally, FLCs are beneficial in scenarios where precise models are difficult to obtain, offering robust performance with minimal tuning. By integrating FLCs, industries can achieve better process stability, reduce energy consumption, and improve overall product quality.

3.2 Fuzzy Decision Support Systems in Adaptive Work Environments

Fuzzy Decision Support Systems (FDSS) play a crucial role in adaptive work environments, where decision-making often involves ambiguity and incomplete information. FDSS utilize fuzzy logic to interpret and analyze complex data, providing actionable insights and recommendations. In adaptive work environments, such as dynamic manufacturing floors, logistics operations, and emergency response scenarios, FDSS help managers and operators make informed decisions under uncertainty. For instance, in a manufacturing setting, an FDSS can evaluate factors like machine performance, workforce availability, and supply chain status to optimize production schedules and resource allocation [19]. By incorporating fuzzy logic, these systems can process subjective inputs and qualitative data, such as expert opinions and risk assessments, alongside quantitative metrics. This holistic approach enables more flexible and adaptive decision-making, improving responsiveness and efficiency. FDSS enhances situational awareness, supports strategic planning, and facilitates continuous improvement, making them indispensable tools in modern industrial and operational contexts.

4 Enhancing Automation Through Fuzzy Logic

Enhancing automation through fuzzy logic represents a paradigm shift in industrial processes, where complex and uncertain environments demand adaptive and flexible control systems. Fuzzy logic offers a robust framework for modeling human-like reasoning and handling imprecise information, making it well-suited for automation tasks that involve variability and unpredictability. By integrating fuzzy logic into automation systems, industries can achieve greater efficiency, productivity, and reliability [20]. Fuzzy logic controllers adapt to changing conditions in real-time, adjusting parameters and making decisions based on fuzzy rules derived from expert knowledge. This adaptability enables automation systems to respond effectively to dynamic environments, optimizing performance and minimizing downtime. Moreover, fuzzy logic enhances the decision-making process by considering multiple factors and their uncertainties, leading to more informed and nuanced actions. Overall, by leveraging fuzzy logic, industries can enhance automation systems to meet the challenges of modern manufacturing, improve operational agility, and maintain a competitive edge in the market.

4.1 Integrating Fuzzy Logic in Automated Systems

Integrating fuzzy logic in automated systems involves embedding fuzzy logic controllers (FLCs) or fuzzy decision support systems (FDSS) to enhance control, decision-making, and adaptability. FLCs, in particular, play a pivotal role by enabling intelligent, context-aware control in dynamic environments. By incorporating fuzzy rules taken from expert knowledge and data-driven insights, FLCs can effectively manage uncertainties and imprecisions inherent in real-world systems. This integration allows automated systems to respond more robustly to changing conditions, optimize performance, and ensure safety. Moreover, fuzzy logic facilitates the incorporation of qualitative and subjective inputs alongside quantitative data, enabling a more holistic approach to automation [21]. In various industries, from manufacturing and robotics to energy management and transportation, integrating fuzzy logic in automated systems offers a powerful means of enhancing efficiency, reliability, and adaptability.

4.2 Case Studies Illustrating Improved Industrial Automation

Numerous case studies reveal the tangible benefits of integrating fuzzy logic in industrial automation:

1. **Temperature Control in Chemical Reactors**: Fuzzy logic controllers are employed to regulate temperature in chemical reactors, ensuring optimal reaction

conditions and product quality. By considering factors such as reactant concentrations, flow rates, and ambient conditions, fuzzy logic-based control systems can maintain stable and precise temperature control, minimizing energy consumption and enhancing process efficiency.

2. **Robotic Assembly Lines**: Fuzzy logic is utilized in robotic assembly lines to adaptively control robotic manipulators and optimize assembly processes. Fuzzy logic-based controllers adjust robot trajectories, gripper forces, and assembly sequences based on real-time sensor feedback and task requirements. This enables robots to handle variations in part sizes, shapes, and positions, improving assembly speed, accuracy, and flexibility.
3. **Energy Management Systems in Buildings**: Fuzzy logic-based energy management systems are deployed in commercial buildings to optimize Heating, Ventilation, and Air Conditioning (HVAC) operation and energy consumption. By considering aspects such as occupancy, outdoor temperature, and building thermal properties, these systems adjust HVAC settings dynamically to maintain comfort conditions while minimizing energy usage. This results in significant energy savings and improved occupant comfort.
4. **Traffic Signal Control Systems**: Fuzzy logic is applied in traffic signal control systems to improve traffic flow and decrease congestion [22]. By analyzing traffic density, vehicle speeds, and historical data, fuzzy logic-based controllers change signal timings in real-time to accommodate changing traffic patterns and prioritize traffic flow on congested routes. This enhances traffic efficiency and reduces travel times for commuters.

In each of these case studies, integrating fuzzy logic in automated systems has led to tangible improvements in efficiency, reliability, and adaptability, demonstrating the effectiveness of fuzzy logic in enhancing industrial automation across diverse applications.

5 Adaptive Systems for Uncertain Environments

Adaptive systems for uncertain environments represent a critical paradigm in engineering and technology, addressing the inherent unpredictability and complexity of real-world scenarios. These systems are designed to dynamically adjust their behavior and parameters in response to changing environmental conditions, disturbances, and uncertainties [23]. By incorporating principles from fields such as artificial intelligence, control theory, and machine learning, adaptive systems exhibit a high degree of flexibility, robustness, and resilience. They continuously monitor their surroundings, gather sensory data, and use feedback mechanisms to adapt their actions and strategies accordingly. In uncertain environments such as aerospace, autonomous vehicles, and environmental monitoring, adaptive systems perform a crucial role in ensuring safe and competent operation. These systems can autonomously learn from experience, optimize performance, and make real-time decisions in the face of uncertainty, ultimately enhancing reliability and adaptability in complex and dynamic settings.

5.1 Fuzzy Logic's Role in Adapting to Dynamic Industrial Conditions

Fuzzy logic plays a crucial role in adapting to dynamic industrial conditions by providing a flexible and robust framework for decision-making and control. In industrial settings characterized by uncertainty, variability, and imprecision, fuzzy logic enables adaptive systems to effectively model complex relationships and make informed decisions in real-time. Fuzzy logic controllers (FLCs) are particularly valuable in this regard, as they can accommodate vague or incomplete information and adjust their behavior based on changing environmental factors [24]. By including fuzzy rules resulting from expert knowledge as well as data-driven insights, FLCs can dynamically regulate industrial processes, optimize performance, and ensure safety. Whether in manufacturing, process control, or logistics, fuzzy logic enables adaptive systems to respond intelligently to evolving conditions, ultimately enhancing operational resilience and efficiency in dynamic industrial environments.

5.2 Real-World Examples of Adaptive Systems Enhancing Operational Resilience

Adaptive systems play a crucial role in enhancing operational resilience [25] across various sectors. For example, adaptive traffic management systems in cities dynamically change signal timings established on real-time traffic conditions, decreasing congestion and enhancing flow. Similarly, in healthcare, adaptive algorithms in hospital resource management optimize staff allocation and patient care during fluctuating demand, ensuring efficient and resilient operations. These examples illustrate how adaptive systems can effectively respond to changing conditions, maintaining stability and efficiency in critical operations.

- **Predictive Maintenance in Manufacturing**: Adaptive systems utilizing machine learning and predictive analytics can enhance operational resilience in manufacturing by predicting equipment failures before they happen. By going through sensor data and past maintenance records, these systems can recognize patterns and anomalies indicative of possible let-downs, permitting proactive maintenance to be performed. This reduces downtime, prevents costly breakdowns, and ensures continuous production [26].
- **Autonomous Vehicles in Transportation**: Adaptive systems deployed in autonomous vehicles leverage sensor data and real-time feedback to adapt to

changing road conditions, traffic patterns, and environmental factors [27]. These systems use algorithms based on fuzzy logic and machine learning to make split-second judgments, like adjusting speed, altering lanes, or avoiding obstacles, to ensure safe and efficient navigation.

- **Smart Grids in Energy Distribution**: Adaptive systems deployed in smart grids optimize energy distribution and consumption by dynamically regulating power generation and transmission in reply to changing demand and supply. By integrating fuzzy logic-based controllers, these systems can balance grid stability, minimize losses, and integrate renewable energy sources effectively, enhancing resilience and reliability in energy distribution networks.
- **Emergency Response Systems in Disaster Management**: Adaptive systems utilized in emergency response systems employ real-time data analysis and decision support tools to coordinate rescue efforts, allocate resources, and prioritize actions during disasters. By incorporating fuzzy logic and optimization algorithms, these systems can adapt their strategies based on changing situational factors, such as the severity of the disaster, the availability of resources, and the evolving needs of affected populations.

In each of these examples, adaptive systems enhance operational resilience by dynamically adjusting to changing conditions, optimizing performance, and ensuring effective responses in dynamic and uncertain environments. Whether in manufacturing, transportation, energy distribution, or disaster management, adaptive systems play a critical role in maintaining reliability and efficiency in complex and evolving systems.

6 Smart Technologies and Fuzzy Logic Integration

The integration of smart technologies and fuzzy logic represents a potent combination that enhances decision-making, control, and automation across various domains. Smart technologies, like the Internet of Things (IoT), artificial intelligence (AI), and cyber-physical systems (CPS), enable the collection of large amounts of information from interconnected instruments and systems. Fuzzy logic, on the other hand, provides a flexible and adaptive framework for processing this data, modeling uncertainty, and making intelligent decisions.

In industrial settings, the integration of smart technologies and fuzzy logic enables predictive maintenance, real-time optimization, and adaptive control [28]. IoT sensors collect data on equipment health, environmental conditions, and production metrics, which are then analyzed by fuzzy logic controllers to predict equipment failures, optimize energy consumption, and adjust processes in response to changing conditions.

In transportation systems, smart technologies such as GPS, sensors, and vehicle-to-infrastructure communication can be integrated with fuzzy logic to optimize traffic flow, improve route planning, and enhance driver safety. Fuzzy logic-based control systems can examine real-time traffic data to vary signal timings, reroute vehicles, and mitigate congestion.

In healthcare, wearable devices, electronic health records, and medical imaging technologies can be integrated with fuzzy logic to develop personalized treatment plans, assist in diagnosis, and optimize healthcare delivery. Fuzzy logic algorithms can investigate patient data to find patterns, assess risk factors, and recommend appropriate interventions.

Overall, the integration of smart technologies and fuzzy logic enables systems to adapt to complex and uncertain environments, improve efficiency, and enhance decision-making across a wide range of applications. Whether in industrial, transportation, healthcare, or other domains, this integration represents a promising approach to addressing the challenges of the modern world [29].

6.1 The Synergy Between Fuzzy Logic and IoT in Industrial Settings

The integration of fuzzy logic with the IoT brings about a powerful synergy in industrial settings, enabling smarter, more adaptive systems. IoT devices gather vast amounts of sensor data from various sources within industrial environments, providing real-time insights into operations. Fuzzy logic complements IoT by processing this data and making intelligent decisions based on fuzzy rules and expert knowledge [30]. In industrial settings such as manufacturing plants and supply chain logistics, this synergy allows for predictive maintenance, real-time optimization, and adaptive control. Fuzzy logic-based controllers can analyze IoT sensor data to predict equipment failures, optimize energy consumption, and dynamically adjust processes to meet changing demands. By harnessing the combined strengths of fuzzy logic and IoT, industrial settings can achieve higher levels of efficiency, reliability, and adaptability.

6.2 Fuzzy Logic in Conjunction with Industry 4.0 Technologies

Fuzzy logic plays a crucial role in conjunction with Industry 4.0 technologies, which encompass CPS, AI, big data analytics, and automation. In the framework of Industry 4.0, fuzzy logic provides a flexible and robust decision-making framework that complements the capabilities of advanced technologies. For example, in smart manufacturing environments, fuzzy logic-based controllers can coordinate robotic operations, optimize production schedules, and ensure quality control [31]. Additionally, fuzzy logic enhances the interpretability of big data analytics by providing a mechanism for handling uncertainty and imprecision in data-driven insights. In cyber-physical systems, fuzzy logic enables adaptive control and real-time optimization, ensuring seamless integration between physical processes and digital systems. By integrating fuzzy logic with Industry 4.0 technologies, industries can unlock new levels of productivity, efficiency, and agility in the era of digital transformation.

7 Case Studies and Success Stories

Case studies and accomplishment stories abound in the integration of smart technologies and fuzzy logic across diverse industries. In manufacturing, companies have achieved significant improvements in operational efficiency and product quality by deploying fuzzy logic-based controllers in conjunction with IoT devices. Predictive maintenance systems, powered by fuzzy logic algorithms, have enabled proactive equipment maintenance, reducing downtime and maintenance costs [32, 33]. In transportation, cities worldwide have implemented smart traffic management systems that use fuzzy logic to optimize signal durations and reduce traffic congestion, resulting in smoother traffic flow, and smaller commute times. Moreover, in healthcare, fuzzy logic integrated with smart medical devices has enabled personalized treatment plans and improved patient outcomes. These case studies highlight the transformative impact of integrating smart technologies with fuzzy logic, paving the way for smarter, more efficient, and adaptive systems across industries.

8 Highlighting Exemplary Cases Where Fuzzy Logic Transformed Industrial Intelligence

Exemplary cases abound where fuzzy logic has transformed industrial intelligence, revolutionizing operational efficiency, decision-making, and adaptability:

1. **Predictive Maintenance in Power Plants**: Power plants worldwide have adopted fuzzy logic-based predictive maintenance systems to visualize machine health and predict let-downs before they happen. By going through the sensor data and historical maintenance records, these systems identify early warning signs of equipment degradation, permitting for timely intervention and preventing costly breakdowns [34].
2. **Smart Grid Management**: Fuzzy logic plays a pivotal role in smart grid management systems, optimizing energy distribution and consumption in response to fluctuating demand and supply. By dynamically adjusting power generation, transmission, and distribution, these systems improve grid stability, reduce losses, and integrate renewable energy sources effectively, enhancing overall efficiency and reliability.

3. **Autonomous Vehicles in Manufacturing**: Manufacturing facilities have deployed autonomous vehicles equipped with fuzzy logic-based controllers to optimize material handling and logistics operations. These vehicles navigate complex environments, adapt their roads in real-time based on changing conditions, as well as avoid collisions with obstacles, improving safety and efficiency in industrial settings [35].
4. **Quality Control in Semiconductor Manufacturing**: Semiconductor manufacturing facilities utilize fuzzy logic-based quality control systems to monitor and optimize production processes. By analyzing sensor data and production metrics, these systems detect defects, adjust process parameters, and ensure consistent product quality, reducing waste and improving yields.
5. **Supply Chain Optimization**: Fuzzy logic-based decision support systems are employed in supply chain management to optimize inventory levels, distribution routes, and production schedules. By considering features like demand variability, lead times, and supply chain disruptions, these systems permit companies to become accustomed quickly to changing market conditions and improve overall efficiency.

These exemplary cases demonstrate how fuzzy logic has transformed industrial intelligence, enabling companies to achieve higher levels of efficiency, reliability, and adaptability in dynamic and uncertain environments.

8.1 Lessons Learned and Best Practices from Successful Implementations

From successful implementations of fuzzy logic in industrial settings, several key lessons and best practices as given in Table 1 emerge:

By succeeding in these lessons and best practices, organizations can maximize the benefits of fuzzy logic integration in industrial intelligence and achieve sustainable success in dynamic and uncertain environments.

9 Challenges and Future Directions

As industries continue to embrace advanced technologies like fuzzy logic, they encounter various challenges and opportunities that shape the future direction of industrial intelligence. One significant challenge is the incorporation of fuzzy logic with emerging technologies such as ML, deep learning, and edge computing to handle increasingly complex and large-scale data sets. The alternative challenge lies in confirming the interoperability and compatibility of fuzzy logic-based systems with existing infrastructure and legacy systems. Additionally, addressing issues related to data privacy, security, and ethical considerations remains paramount as industries

Table 1 Several key lessons and best practices learned after successful implementation of fuzzy logic in industrial applications

Key lessons and best practices	Description
Domain expertise	Successful implementations require collaboration between domain experts and technology specialists to develop fuzzy logic-based systems that address specific industrial challenges effectively
Data quality and integration	High-quality data from different sources is required for the success of fuzzy logic-based systems. Integrating data from IoT devices, sensors, and other sources ensures robust decision-making and adaptation to changing conditions
Continuous monitoring and feedback	Fuzzy logic-based systems require continuous monitoring of operational data and feedback mechanisms to adjust parameters and adapt strategies in real-time
Scalability and flexibility	Fuzzy logic-based systems should be designed with scalability and flexibility in mind to accommodate changes in operational requirements and environmental conditions
Interpretability and transparency	Ensuring the interpretability and transparency of fuzzy logic-based decision-making processes is crucial for gaining trust and acceptance among users and stakeholders

collect and analyze more data. Looking ahead, the future of fuzzy logic in industrial intelligence holds promise in addressing these challenges through ongoing research and innovation. Advancements in hybrid intelligent systems, explainable AI, and distributed computing are expected to further enhance the capabilities of fuzzy logic and drive its adoption across diverse industries. Moreover, the integration of fuzzy logic with emerging trends such as Industry 4.0, smart cities, and sustainable development presents exciting opportunities for leveraging its adaptive and intelligent capabilities to address complex industrial and societal challenges. By overcoming these challenges and embracing future directions, fuzzy logic is poised to perform a vital role in affecting the next generation of industrial intelligence.

9.1 Addressing Challenges in Implementing Fuzzy Logic in Industrial Intelligence

Implementing fuzzy logic in industrial intelligence poses several challenges as mentioned in Table 2 that require careful consideration and strategic approaches:

Addressing these challenges requires interdisciplinary collaboration, ongoing research and development, and a commitment to continuous improvement in implementing fuzzy logic in industrial intelligence.

Table 2 Challenges in implementing fuzzy logic in industrial intelligence

Challenges	Approaches
Data quality and integration	Ensuring the availability of high-quality data from diverse sources and integrating it effectively into fuzzy logic-based systems is crucial for reliable decision-making
Interoperability and compatibility	Ensuring the seamless integration of fuzzy logic-based systems with existing infrastructure and legacy systems requires addressing interoperability and compatibility issues
Scalability and flexibility	Designing fuzzy logic-based systems with scalability and flexibility in mind is essential to accommodate changes in operational requirements and adapt to evolving environments
Interpretability and transparency	Ensuring the interpretability and transparency of fuzzy logic-based decision-making processes is important for gaining trust and acceptance among users and stakeholders
Data privacy and security	Talking concerns linked to data privacy, security, and ethical considerations are paramount as industries collect and analyze more data in fuzzy logic-based systems

9.2 Exploring Future Innovations and Trends in This Evolving Landscape

In the evolving landscape of industrial intelligence, several future innovations and trends are poised to shape the adoption and advancement of fuzzy logic:

1. **Hybrid Intelligent Systems**: Future innovations may focus on integrating fuzzy logic with other intelligent techniques such as machine learning and deep learning to develop hybrid intelligent systems capable of handling complex and dynamic industrial environments.
2. **Explainable AI**: Advances in explainable AI techniques will enhance the interpretability and transparency of fuzzy logic-based systems, enabling users to understand and trust the decision-making processes behind these systems.
3. **Distributed Computing**: Leveraging distributed computing technologies such as edge computing and fog computing will enable real-time processing and analysis of data in fuzzy logic-based systems, enhancing responsiveness and adaptability in industrial environments.
4. **Industry 4.0 and Smart Cities**: The integration of fuzzy logic with emerging trends such as Industry 4.0 and smart cities presents opportunities for leveraging its adaptive and intelligent capabilities to address complex industrial and urban challenges [36].
5. **Sustainable Development**: Fuzzy logic-based systems can contribute to sustainable development efforts by optimizing resource usage, reducing waste, and improving energy efficiency in industrial processes and urban infrastructure.

By exploring these future innovations and trends, industries can harness the full potential of fuzzy logic in advancing industrial intelligence and addressing the challenges of the digital era.

10 Conclusion

Industrial Intelligence: A Fuzzy Logic Approach" represents a significant advancement in the field, leveraging the power of fuzzy logic to address the complexities and uncertainties inherent in industrial environments. Through this approach, industries can enhance decision-making, optimize processes, and improve overall efficiency. By integrating fuzzy logic with smart technologies, such as the Internet of Things (IoT) and artificial intelligence (AI), industries can achieve predictive maintenance, real-time optimization, and adaptive control. This synergy not only transforms industrial processes but also enhances resilience in dynamic and uncertain environments. As we look to the future, further advancements in hybrid intelligent systems and explainable AI hold promise for continuing to revolutionize industrial intelligence, making it smarter, more efficient, and adaptive. Overall, "Industrial Intelligence: A Fuzzy Logic Approach" represents a paradigm shift in industrial management, offering practical solutions to complex challenges and driving innovation in the digital age.

10.1 Summarizing the Key Findings and Contributions of Fuzzy Logic to Industrial Intelligence

Fuzzy logic has made significant contributions to industrial intelligence by addressing the challenges of uncertainty, imprecision, and complexity in real-world environments. Key findings highlight its role in enhancing decision-making, control, and automation across diverse industries. By integrating fuzzy logic with smart technologies such as the Internet of Things (IoT), artificial intelligence (AI), and cyber-physical systems (CPS), industries have achieved improvements in operational efficiency, reliability, and adaptability. Fuzzy logic-based systems have enabled predictive maintenance, real-time optimization, and adaptive control, transforming industrial processes and enhancing resilience in dynamic and uncertain environments. Moreover, fuzzy logic has paved the way for future innovations such as hybrid intelligent systems, explainable AI, and distributed computing, which hold promise for further advancing industrial intelligence. Overall, fuzzy logic's contributions to industrial intelligence underscore its significance as a key enabler of smarter, more efficient, and adaptive systems in the digital age.

10.2 Implications for Future Research and Industrial Applications

The exploration of "Industrial Intelligence: A Fuzzy Logic Approach" reveals several implications for future research and industrial applications:

- **Hybrid Intelligent Systems**: Future research can focus on developing hybrid intelligent systems that combine fuzzy logic with other advanced techniques such as machine learning and deep learning. These systems can offer enhanced capabilities for decision-making, control, and optimization in industrial settings.
- **Explainable AI**: Research efforts can be directed towards improving the interpretability and transparency of fuzzy logic-based systems, making them more accessible and trustworthy for users and stakeholders in industrial applications.
- **Edge Computing and IoT Integration**: Further research can explore the integration of fuzzy logic with edge computing and IoT technologies to enable real-time processing and analysis of data at the network edge, enhancing responsiveness and adaptability in industrial environments.
- **Sustainability and Resilience**: Future industrial applications can leverage fuzzy logic to address sustainability and resilience challenges, such as optimizing resource usage, reducing waste, and improving energy efficiency in manufacturing and urban infrastructure.
- **Interdisciplinary Collaboration**: Collaboration between academia and industry can drive research efforts towards addressing real-world industrial challenges and validating the effectiveness of fuzzy logic-based solutions in practical applications.

Overall, the implications for future research and industrial applications of "Industrial Intelligence: A Fuzzy Logic Approach" are vast, promising, and poised to shape the future of industrial management and technology.

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Sensors and Data Driven Approaches in Precision Agriculture: A Comprehensive Review

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Abstract For Agriculture is an industry that is essential to the survival of the human race. Several measures have been put in place to raise crop yields. However, unfavorable environmental circumstances and recurrent insect infestation cause agricultural loss. In scenarios like this, smart sensors play a critical role in effective farming practices that use digital technology to monitor, visualize, and generate digital data, manage the application of resources, and improve the output and caliber of agricultural products. With automation and digital tools for Internet of Things (IoT)-based operation management, novel sensors enhance the value of soil-less farming. Smart sensors combined with data-driven technologies can solve a lot of agricultural practice problems and potentially create new efficiencies. Transforming data into knowledge and making sure that it is kept in a way that makes it logical for others to access it both now and in the future are equally essential. The fundamentals of smart sensors and the most useful sensors for tracking the physicochemical properties of soil and plants in field cultivation processes, greenhouse operations, and indoor hydroponics are covered. Precision agriculture, automation of agricultural machinery, imagery from Unmanned Aerial Vehicles (UAVs), Precision Livestock Farming (PLF), and the use of Internet of Things (IoTs) can all benefit farming communities by enabling more efficient use of resources based on real-time farm data. Smart sensors support growers' productivity quests, the food value chain as a whole, and the potential for new business models. This chapter provides a thorough overview of the most recent

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developments in smart sensor technology, wireless sensor network topologies, and crop production, global food security initiatives, and sustainable farming methods.

Keywords Sensors · Data driven agriculture · Smart farming

1 Introduction

By 2050, the world's population is predicted to nearly double to 10 billion, placing enormous pressure on agricultural systems to increase food production in sustainable ways [1]. Agriculture needs to come up with creative answers to issues like shifting climate patterns and diminishing arable land in order to be ready for the future. To achieve the Sustainable Development Goals (SDGs) of the UN, in particular SDG 2 (zero hunger), SDG 6 (clean water and sanitation), and SDG 13 (climate action), is therefore imperative. It generates 15% to 25% of global greenhouse gas emissions [2] and It is susceptible to the effects of droughts and floods [3].

The need for better insecticides, fertilisers, and field management techniques is further highlighted by the reduction in biodiversity and ecosystem services [4]. Expectations of sustainable agriculture from customers and citizens are posing a growing threat to traditional agriculture, which depends on farmers' instincts and experience in making decisions [5]. In order to overcome these obstacles, modern agriculture, in particular, digital agriculture has become increasingly important. To optimize agricultural techniques, digital agriculture makes use of a variety of technology, including robotics and sensors, and mostly depends on data-driven approaches [6]. The major goal is to provide information to support decisions about the best possible use of resources, including where, when, and how much to use for various purposes. An expanding number of observational and simulation-based data sources are available to farmers, researchers, and policymakers. These sources include the availability of nitrogen in the soil, anticipated rainfall, crop growth and yield potential, disease risks, and densities of weeds and insects. Enhanced sustainability in agricultural management techniques can result from the translation of insights obtained from this data into practicable solutions [7].

A state-of-the-art farming technique called precision agriculture uses technology to improve field-level control of crop output [8]. It makes use of data analytics, cutting-edge technologies and decision support systems to optimise farming operations on a micro-scale and ensure that relevant resources are allocated at the proper area and time. Through the use of real-time collection and evaluation of data, precision agriculture seeks to increase world's food supply by judiciously utilizing agricultural inputs like herbicides, fertilizer, and water to enhance ecological sustainability, efficiency and yield [9]. This method helps farmers make informed decisions for specific parts of their farm by assessing changes in crop health, soil composition, and moisture levels [10].

Using agricultural data for precision agriculture goals, prediction analytics software and forecasting systems give farmers advice on crop alternation, soil management, and when to plant and harvest [11]. According to this paradigm, sensor technology is crucial for solving several issues related to precision agriculture [12]. With integration with robots, automation, remote sensing, land navigation, underwater imagery, and more, modern agriculture sensors are useful in a variety of agricultural applications [13]. The information supplied was intended to help farmers keep a close eye on physiological parameters such as pH, humidity, and soil fertility (NPK) in order to cultivate crops under ideal growth circumstances. In order to achieve the best product both quantitatively and qualitatively, sensors play a critical part in the detection and vision of many aspects that affect plant growth and in taking appropriate action. Precision farming enables farmers to achieve higher yields and more revenue with less resources used [14, 15].

In order to ensure that water resources are used as efficiently as possible in precision agriculture, smart irrigation systems that integrate sensor and IoT technologies are beginning to show promise as a way to communicate the shortage of clean water supplies essential by many vegetal species [16]. In order to speed up the irrigation process, irrigation can be converted into a smart irrigation system. Unmanned aerial vehicles (UAVs) are a type of equipment that is used to quickly and extensively gather vital data on agricultural parameters such as pest assault, soil quality, and water availability [17]. Unmanned aerial vehicles (UAVs) equipped with spectrum analysis technology can identify pest infestation in farms even from far-off locations by capturing high quality images that are kept in databases like cloud server for additional examination [18]. The potential of smart greenhouses to cultivate with minimal human intervention is growing. It requires continuous sensor-based monitoring of humidity, temperature, soil moisture, and brightness for increased production [19]. These sensors support policymaking for the implementation of more appropriate safeguards against harm to agriculture.

It is possible to detect soil nitrate and the amount of nutrients that plants are absorbing from their surroundings in real time using a variety of microfluidic sensors [20]. It is possible to anticipate source of nourishment in plants in the leaf using non-invasive techniques based on optical sensing. This reflectance spectra value changes to lengthier wavelengths in plants and shorter wavelengths in the plant ages in nutrient-stressed plants [21]. The third green revolution can be defined as the use of contemporary communication and information technology in agriculture. From planting to harvesting, smart farming assists farmers in making the right choices that will result in high-quality crops and yield [22]. By using technology and sensors in farming, it is possible to log crop and soil data at the centimeter level throughout a whole field, producing a additional rational, and ecologically friendly product. IoT-based sensor network technology, which employs a variety of sensors, has recently shown increased promise for use in agriculture for the gathering, storing, and transmission of sensed data [23]. Making informed decisions requires understanding soil properties, weather forecasts, fertiliser usage, irrigation techniques, timely harvesting, and other factors. The majority of people understand the advantages of managing crop production more precisely by employing additional information. The field of agriculture management has not yet adopted smart farming or other digital information technology.

IoT technology adoption in agriculture has the capability to entirely alter farm operations and enhance the sector's productivity and efficiency [24]. Precision farming methods, which employ other technology and sensors to data collection and make accurate and appropriate modifications to many parts of agricultural operations, are another way that the IoTs is being utilized in cultivation. The capability to anticipate and formulate for probable instabilities that may initiate from remote locations, such as ability to monitor soil nutrients, as well as water dynamics, insect pest invasions, and pest management, are just a few of the many advantages that the Internet of Things offers to agriculture and sustainable crop production [25]. Utilizing IoT technology in farming systems also has the benefit of predicting production and saving energy and money.

Improved production, lower costs, environmental friendliness, and farmer sustainability are the main components of agricultural methods. Precision farming is not only producing better, higher-quality products, but it is also enabling entire food chains, from involvement firms to tractors to agriculturalists to supply chains to consumers. The purpose of this article is to examine how smart sensors may be used to precisely monitor a number of physico-chemical parameters in agricultural settings, including soil quality, humidity, temperature, moisture and nitrogen contents, and greenhouse gas emissions. Additionally, a conversation was conducted about the potential applications of numerous data-driven cultivation sensors and technologies utilized in hydroponic agrobusiness operations, outdoor, and greenhouse.

2 Sensors in Precision Agriculture

Sensor-based technologies offer increased productivity and sustainability by utilizing data. From planting to packing the finished product, sensors are used in farming at different phases. The agrobusiness process can be approximately classified into numerous categories, with managing soil, planting, managing nutrients, water, controlling diseases and pests, harvesting yield, and refinement after harvest. These categories can get profit from the incorporation of innovative sensing technology.

Six essential sensors are used in agriculture, especially in outdoor farming. Optical sensors, electromagnetic soil moisture sensors that identify soil moisture stages, mechanical sensors that provide compaction information, and airflow sensors that identify soil air penetrability are a few examples of these sensors that evaluate the properties of soil at dissimilar light wavelengths [26]. For accurate field operations, location and position sensors are also essential. Moreover, the use of PAR (photosynthesis-active radiation) sensors in augmenting plant development circumstances is deliberated. Additionally, robotic sensors are shown as efficient and eco-friendly substitutes for conventional pesticide use in crop weeding and spraying.

2.1 Location and Position Sensors

Position sensors employ GPS satellite indications to calculate the altitude, latitude, and longitude inside the yard. It takes three satellites to pinpoint an exact location. Accurate placement is essential for precision farming. Applications for location sensors include livestock monitoring and truck guidance systems for crop harvesting and field tilling [27]. High-resolution and detailed geographical data can be accessed by GPS and remote sensing devices like drones and satellites. These advanced tools provide exact field boundary establishment, crop production monitoring, and accurate mapping [28]. They are essential instruments for evaluating crop yield and enhancing the effective distribution of resources in agriculture [29]. Many research have examined remote sensing methods in this context and how they might be used for precision agriculture. While some studies have addressed more than one application area, others have focused on particular cases include illness and pest management, evapotranspiration management, and soil and harvest management [30].

2.2 Optical Sensors

Potential applications for a wide spectrum of optical sensors include agriculture. The basis for these sensors is their capacity to identify different wavelengths of light. A light source emits light with a particular wavelength in order to collide with the target item. The optical sensor detects the reflected light, creating reflectant data that is then saved as a text file. Non-destructive optical sensors like SPAD and GreenSeeker were used by Freidenreich et al. [31] to analyse nutrient uptake. Examined were variables such as soil leachate, total leaf carbon: nitrogen ratios, and the Normalised Difference Vegetation Index. The implementation of optical sensors, according to the authors, shows promise in identifying the fertiliser requirements of plants.

It has been reported that optical coherence tomography may have applications in seed germination monitoring [32]. When sowing wheat, the GreenSeeker optical sensor helps control nitrogen-based fertilisers for a higher yield [33]. With the use of optical sensing technologies, data on the distribution of weeds in agricultural areas has been successfully acquired. In one investigation that employed a tolerance threshold, it was discovered that an optical sensor in conjunction with spectroscopy was remarkably accurate in recognizing cells infected with green weed. An optical sensor, sometimes called a photometric sensor, is able to detect and determine a physical or chemical property by the measurement of an optical property. While they are utilised to increase the dynamic range, optical sensors have few restrictions when it comes to replacing multi-array and electrochemical sensors [34].

2.3 Electrochemical Sensors

The most portable and lightweight environmentally friendly sensors for use in agricultural areas are electrochemical ones. These sensors are effective in providing more precise real-time monitoring of environmental pollution, plant development, and illnesses. For precision farming, these sensors offer data on soil nutrient levels and pH. Furthermore, electrochemical sensors have relatively little of an impact on the environment. Biochemical factors that are essential to agricultural yields have been investigated for a number of sensors. For example, plant leaf water is sensed using graphene oxide-containing humidity sensors [19]. Volatile organic molecules in soil can be found with the help of certain sensors. Graphite, reduced graphene oxide, carbon nanotubes, and gold nanoparticles are commonly used in such sensors. Certain electrochemical sensors have been developed to identify nitrogen in soil and plant sap [35]. As a result, electrochemical sensors have enormous potential for detecting a variety of characteristics in agricultural areas that could be helpful to improve agricultural output.

2.4 Mechanical Sensors

The soil's physical and chemical characteristics are what cause the variations in crop output amongst agricultural areas. The compactness of the soil particles in a given region is reflected in the strength of the soil. Crop productivity is negatively impacted by higher soil compactness. The mechanical resistance and compaction of soil can be measured with mechanical sensors. With this knowledge, the irrigation system may be tailored to the mechanical characteristics of the soil. A mechanical soil sensor that is sold commercially is the Honeywell FSG15N1A. The force that plant roots apply to absorb water is detected by the Honeywell FSG15N1A. In one investigation, the de-gree of soil compaction in a particular area was measured using both vertical and horizontal sensors. This integrated method measured the depth to which the coulter penetrated the topsoil layer using an ultrasonic distance sensor. The integrated ap-proach proved beneficial in detecting changes in top soil strength based on depth, as the results showed. This may facilitate the mapping of agricultural fields' soil me-chanical resistance [36].

2.5 Electromagnetic Sensors

A good tool for identifying contaminated agricultural soil is an electromagnetic sensor. It also helps in mapping agricultural fields' topological properties. Electromagnetic sensors can identify a wide variety of electromagnetic waves released by various objects. By sensing the soil's dielectric constant, these sensors calculate the moisture content [37]. In order to gather high-quality data on external surface procedures and undergrowth types, unmanned aerial vehicles are usually utilized to deploy detection techniques and equipment, such as gamma radiometric sensors and ground-penetrating radar. These sensors continuously check the state of the soil to direct accurate fertilisation and irrigation techniques, preventing resource waste and guaranteeing the best possible supply of nutrients for crops [38]. Results demonstrated a high degree of agreement between sensor-derived measures of soil electrical conductivity and laboratory conditions [39]. Soil properties, conditions, and spatiotemporal variability can be evaluated using methods like ground-penetrating radar (GPR).

2.6 Airflow Sensors

Air Movement Sensors measure the soil's air permeability. Both stationary and moving conditions can be used for measurements. The outcomes is the amount of pressure essential to force a certain quantity of air into the earth at a particular penetration. According to the research, various earth characteristics, such as moisture content and structure, unique identifying signals created [40].

2.7 Robotic Sensors

Robotic agricultural sensors for agricultural weed control, Weeding and Spraying employs a robot equipped with two vision systems: one gray-level and one color-based [41]. Chang and Lin proposed a multifunctional intelligent gadget for variable rate irrigation and autonomous weed removal [42]. This technique gets rid of weeds without harming crops that are being grown. The use of herbicides, insecticides, weed killers, and fungicides can be extravagant and detrimental to the atmosphere. When mechatronics is utilized, a smart system that is far further effectual is created [43].

2.8 Photosynthetically Active Radiation (PAR) Sensors

Plant growth-related light intensity is measured by PAR sensors. Plants need light energy, or PAR, in addition to CO2 and water, for growth and crop output. Many photodiodes, including lead selenide, silicon (Si), cadmium sulphide, gallium arsenide phosphide (GaAsP), and selenium, are employed in the measurement of visible light, which is the wavelength at which plants absorb PAR. The L1-CORE quantum sensor measures photosynthetic active radiation by means of a blue-enhanced Si photodiode. Optical fibre is used to build PAR sensors at a reasonable cost [20].

By using sensors and field mapping, farmers may improve environmental sustainability, save input costs, and keep an eye on their crops at the microscale. Cost effectiveness is another tool that farmers can use by using limited resources only when necessary. Sensors, such as optical, electrochemical, and location sensors, offer statistics such as longitudinal and latitude locations using GPS satellites, as well as soil statistics, such as soil dampness and organic matter. Important statistics for precision cultivation is provided by electrochemical sensors, including pH and soil nutrient levels. Farmers in this new period benefit greatly from such smart farm monitoring utilizing sensor technology [40].

3 Smart Sensors in Precision Agriculture

The rapid advancements in computing, semiconductor manufacturing, and communication technologies have given rise to a new class of sensors known as smart sensors. These sensors are advanced enough to be able to communicate wirelessly from a remote location. These have the ability to communicate across devices and have an automated data processing system. The essential parts of a smart sensor are at least one sensor, an analog-to-digital converter (ADC), a microcontroller, memory, a communication link, and a power source. Three essential parts make up a smart sensor node: a network interface, a CPU, and physical transducer. The physical transducer finds the physical parameters and turns them into an electrical signal. The CPU can use the digital value produced by the ADC. After signal processing the identified data, the processor—typically a microcontroller—sends the processed data to the network. Traditional sensor systems consist on a single central processing unit for data. The outcome is then sent to the network after the signal node processes the data locally [44].

By using smart sensors that are Internet of Things (IoT) enabled and capable of processing and analyzing real-time data, remote sensing efficiency can be boosted. A variety of agricultural factors could be remotely sensed using both active and passive sensors. Weather forecasting, landscape topology, soil monitoring, insect manifestation, and soil quality are among the applications for IoT-enabled smart sensors [45]. Artificial intelligence (AI) technologies underpin smart sensors, improving their accuracy in operation. Precision agriculture is made possible by LiDAR-based sensor technology, which provides more precise and real-time data. Real-time data on crop growth, pest manifestation, moisture content, soil quality, vegetation growth status, and production are all provided by this system. The Internet of Things affects many different tasks, including gathering, processing, and analyzing data. Furthermore, a number of environmental factors, including pollution, radiation status, soil quality assessment, water contamination, and so forth, are the subject of in-depth research utilizing IoT technology [46].

The IoT and smart sensors are replacing traditional farming methods with "smart agricultural," which is categorized through higher productivity. IoT-implemented technical methods in farming help assess deterioration of soil, crop quality, necessity of fertilizer, and productivity condition of soil [47]. It also makes monitoring crop development at various stages, seed quality, and optical irrigation easier. Agricultural areas use the Internet of Things and smart soil moisture sensors to monitor pre- and post-harvest conditions. The world's population is negatively impacted by microbiological entities [48, 49]. Microfluidics-based on-chip artificial pores have the potential to identify microorganisms in agrifoods [50]. For onsite pathogen identification, lateral flow tests and techniques are usually employed [51]. Expert systems combined with agricultural IoTs can improve planting and crop management techniques for farmers [52]. Currently, the majority of the equipment that is created and used is for tracking environmental data, monitoring animal movements, and collecting agricultural and crop information [53].

Several organisations installed a creative Internet of things system utilizing cloud computing and Li-Fi as an example of smart farming. Li-Fi refers to high-concentration wireless data exposure in a small area. Li-Fi offers greater bandwidth, efficiency, accessibility, and security than Wi-Fi. This smart farming operation's primary goal is to do duties like weeding, spraying, moisture detection, etc. It also includes intelligent warehouse monitoring by guaranteeing temperature preservation and humidity protection in the stockroom. Lastly, a clear and dependable execution of agricultural methods depends on impeccable irrigation with graceful administration. To achieve all of these, systematic smart procedures are carried out by cameras, Li-Fi modules, ZigBee, and edging sensors [54].

4 Pest Monitoring in Smart Agriculture

Numerous pests in agricultural fields can be repelled by certain gadgets that generate ultrasonic sound waves. Tiwari et al. [55], for example, developed an electronic pest repellent that emits powerful ultrasonic sound waves. Pests such as mosquitoes and rats are repelled by these waves. As a result, these tools are a good substitute for chemical pesticides, which have a number of negative effects on beneficial creatures [56–58]. Furthermore, accidental contact with chemical insect repellents may also improve the performance of ultrasonic devices. Electronic insect repellents are inexpensive, safe for the environment, and have no negative effects on people. These devices primarily work to repel pests by appealing to their auditory receptors.

When it comes to pest detection, certain image processing methods are quite useful. Using spectral camera technology with an unmanned aerial vehicle (UAV), high-resolution macro and micro photos of the farm can be taken. These photos can be examined to find evidence of pest infestation and the beginning of related illnesses in crops [17]. More precise identification and characterization of insect pests in agricultural fields may also be facilitated by studying insect behaviour through the use of several wireless sensors [59]. Certain advanced sensors are capable of detecting pests and diseases in crops. They identify the presence of infections or pests using techniques including spectroscopy, image recognition, and other ways, allowing for quick intervention to minimize potential harm [60, 61].

5 Data Collection and Management in Agriculture

There are several potential advantages to be achieved from combining agronomic expertise with cutting-edge technology like cloud computing, IoT, and smart systems in order to develop efficient monitoring solutions. A number of studies have commonalities, such as the usage of minimum power sensor nodes, minimum cost devices, and communication protocols such as ZigBee and IEEE 802.11. Within the fog computing idea, gateways are essential for local processing and data aggregation. In certain experiments, data is transmitted directly to cloud services located at a distance. In far-off places, GSM/GPRS is a popular option. End customers can now more easily access data analysis services on the server side using PC or mobile applications. Efficient data handling is made possible by the cloud's tremendous processing capability, especially when managing massive volumes of data from multiple inexpensive devices. Images are being used in an increasing number of proximity surveillance scenarios, which has prompted to an increase in the handling prerequisites for capture data and transmission through remote servers [62].

Agricultural settings can be large, disorganised, and occlusion-filled, which makes it challenging to perceive pertinent quantities of interest. The drawbacks of passive data recording include the possibility that crucial information is not immediately visible or that a high number of observations are required to thoroughly cover fields or orchards. Therefore, in order to maximise data capture, robots must shift perspectives, travel to particular spots, or move impediments. Active information collection entails operating a mobile device to capture more precise or high-quality observations that are more pertinent to agricultural chores. Current methods usually concentrate on maximising sensor coverage or pick viewpoints based on samples. Although they are a basic solution, information-driven techniques can be computationally complicated [63].

Active sensing is a promising method for improving data gathering in agriculture [64–66]. Selective labelling paradigms, active learning, and data recording are all combined in active sensing. In the dynamic and varied farming landscape, where field and ecological circumstances may vary suggestively, selective labeling develops predominantly significant. In-depth annotation requires carefully choosing certain data points or features, which can upgrade the learning algorithms utilized for farming scene examination significantly. Moreover, methodologies such as uncertainty quantification can be used by the active sensing framework [67]. These methods offer insightful information about the calibre of the data gathered and illuminate the models' decision-making procedures. We can develop accurate models based on carefully chosen, high-quality data by using approximative information-driven techniques that combine data collecting methodologies and labelling requirements.

Using computer-aided systems for automatic data collecting is a key component of adopting intelligent farming techniques. Wireless technology eliminates the need for cables, which lowers maintenance costs and difficulties. Cloud-hosted data management technologies facilitate speedy and precise processes, while pre-configured algorithms and analysis tools enhance total productivity. This means that, in the context of agriculture, the models are more able to adjust to the subtleties of actual conditions, producing more accurate and dependable results for farmers' decision-making processes.

The authors present the idea of TV Whitespace (TVWS), which is the vacant space in between broadcast TV channels, a way to transmit information wirelessly in farming. TVWS, which is frequently disregarded, presents the potential for effective data transfer, with uses in farm management, drone photography, and remote sensor data gathering. Since TVWS offers higher transmission speeds and a longer range than traditional communication channels, its potential to revolutionize precision agriculture is further highlighted by its usage in the development of 5G transmission. Generally speaking, a lot of TV channels have excess bandwidth that is left empty, and governments around the world are now encouraging the use of this bandwidth to send data [68]. Similar to a farmer's home Wi-Fi antenna, the TVWS router allows for on-demand Wi-Fi access on the farm [69]. Via an IoT system or TVWS base station, data gathered from field sensors is instantly sent to a cloud server. Utilizing AI or machine learning techniques, data stored on cloud servers was examined. Research stations then forwarded processed agricultural warnings to farms for automated management or control. Additionally, TVWS can be communicated to UAV imagery. After the flight is over, data can be transferred across the white gaps. Due to its advantages over existing communication channels, including higher range and quicker transmission speeds, TVWS is being employed extensively in many nations to create new 5G transmission [70].

6 Conclusion

The future growth and uptake of digital agriculture are contingent upon a number of factors, one of which is the equilibrium between costs and benefits. The availability of innovative and timely sensors is essential to meeting farming's pressing technological development needs. The development of sensor technologies for outdoor applications lagged behind their initial development for controlled environment investigations. High-tech sensors can monitor temperature, moisture content, rainfall, and other environmental factors to maximize agricultural yield. The obstacles of agricultural growing conditions are addressed by sensor technologies in conjunction with broad environmental control, making agriculture the most efficient, economical, and straightforward sector of the economy. Additionally, by managing the population of pests with IoT-enabled traps outfitted with high-definition cameras and additional accessories, pest attacks can be minimized. Precision farming offers enormous potential for smart farming instruments and sensors. Smart sensors are used in digital and decision-based farming techniques to increase crop output while maintaining superior nutritional quality on smaller land areas. Finally, by causing damage to the cloud servers that house critical data, cybercriminals may have an impact on automated smart farming. Large-scale adoption of smart farming could be facilitated by government initiatives promoting digital literacy, data encryption, and financial assistance for farmers.

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Fuzzy Logic Control for Adaptive Fan Speed Regulation Based on Temperature Variations in Cold, Warm, and Hot Seasons

Prashant C. Ramteke

Abstract Maintaining optimal indoor temperature is crucial for ensuring comfort and energy efficiency in various environments. Traditional fan speed control systems often rely on fixed thresholds, resulting in suboptimal performance and increased energy consumption. This study explores the use of fuzzy logic control (FLC) systems to regulate fan speed based on temperature variations dynamically. Fuzzy logic rules are implemented to achieve adaptive and optimal fan speed settings by defining the universe of discourse for temperature and fan speed. The input variable for the system is the ambient temperature, classified into three fuzzy sets: cold, warm, and hot. Correspondingly, the output variable, fan speed, is categorized into three fuzzy sets: low, medium, and high. The membership functions for these fuzzy sets are defined using triangular functions to facilitate smooth transitions between states. The fuzzy logic rules are established as follows: if the temperature is cold, the fan speed should be low; if the temperature is warm, the fan speed should be medium; and if the temperature is hot, the fan speed should be high. To validate the effectiveness of the FLC system, simulations were conducted across a range of temperatures from 0 °C to 50 °C. The resulting fan speed outputs were visualized using a heatmap, which clearly demonstrated the system's adaptive behaviour. The heatmap analysis revealed that the fan speed appropriately increases with rising temperatures, ensuring efficient cooling and energy usage. This study highlights the potential of FLC systems in providing a more responsive and energy-efficient solution for temperature regulation compared to traditional fixed-threshold systems. The adaptive nature of fuzzy logic ensures that fan speed adjustments are smooth and continuous, enhancing both comfort and energy efficiency in real-world applications.

Keywords Fuzzy Logic Control (FLC) · Temperature regulation · Fan speed adjustment · Membership functions · Energy efficiency · Adaptive control systems

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1 Introduction

Fuzzy logic, a mathematical framework introduced by Lotfi Zadeh in 1965 [2, 16, 20], offers an innovative approach to handling uncertainty and imprecision, which are inherent in many real-world scenarios. Unlike classical binary logic that operates strictly with true or false values, fuzzy logic allows for varying degrees of truth, thereby facilitating more nuanced and flexible decision-making processes [3, 8]. This characteristic makes fuzzy logic particularly advantageous in control systems where conditions are not strictly binary and can fluctuate across a spectrum. For instance, in environmental control systems such as those regulating Heating, Ventilation, and Air Conditioning (HVAC), temperature does not simply shift from 'cold' to 'hot' but rather moves through a continuum of states. By employing fuzzy sets and membership functions, fuzzy logic models this continuum effectively, defining variables like temperature and fan speed within specific ranges known as the universe of discourse [17]. Each variable is divided into fuzzy sets (e.g., 'cold', 'warm', 'hot') with corresponding membership functions that quantify the degree to which a given input belongs to each set. Fuzzy rules, formulated based on intuitive and experiential knowledge, govern the relationships between these inputs and outputs [18]. For example, a rule might state, "If the temperature is warm, then the fan speed should be medium," capturing the expert understanding of appropriate responses to varying conditions. The fuzzy inference process evaluates these rules, aggregates the results, and then applies defuzzification to generate a crisp output that drives the control action [6, 7, 13]. This approach allows for smooth transitions and adaptive responses, making FLC systems more robust, efficient, and comfortable for users compared to traditional fixed-threshold systems. Moreover, the simplicity of designing and implementing fuzzy logic systems, combined with their robustness against noise and measurement errors, underscores their value in a wide range of applications. In the automotive industry, fuzzy logic is used for automatic transmission systems, improving fuel efficiency and driving comfort by adapting to driving conditions [9, 11]. In consumer electronics, fuzzy logic enhances the performance of washing machines by optimizing water usage and washing cycles based on load size and fabric type [14, 15]. In healthcare, fuzzy logic assists in diagnostic systems and patient monitoring, where it handles the vagueness inherent in medical data, leading to more accurate and reliable outcomes [12]. Additionally, fuzzy logic has significant applications in industrial process control, where it helps maintain optimal operating conditions in processes with complex and non-linear dynamics, such as chemical manufacturing and power plant operations [10, 19]. In agriculture, fuzzy logic systems regulate irrigation and fertilization schedules based on varying soil and weather conditions, improving crop yield and resource efficiency [1, 5]. Environmental monitoring systems also benefit from fuzzy logic by integrating data from diverse sensors to assess pollution levels and manage natural resources sustainably [4]. The advantages of fuzzy logic in these applications include its ability to mimic human reasoning, handle linguistic variables, and integrate with conventional control systems. Its flexibility allows for incremental improvements and fine-tuning without overhauling existing systems. Furthermore, fuzzy logic's capacity to manage imprecise data and its computational efficiency make it an invaluable tool for developing intelligent systems across various domains, thereby revolutionizing how we manage and optimize complex environments. Maintaining optimal indoor environmental conditions is paramount for enhancing comfort, productivity, and energy efficiency across various applications, including residential, commercial, and industrial settings. One of the key aspects of indoor climate control is the regulation of temperature. In most climate control systems, HVAC systems, fan speed plays a critical role in managing air circulation and maintaining desired temperature levels. However, traditional fan speed control methods often rely on fixed thresholds, leading to suboptimal performance, higher energy consumption, and reduced user comfort. This study aims to develop a FLC system for regulating fan speed based on temperature variations.

1.1 Scope and Objective

The scope of this study involves developing a FLC system to regulate fan speed based on temperature variations, defining parameters for temperature and fan speed, and designing relevant membership functions. The objective is to create and assess a FLC system that adjusts fan speed smoothly and continuously in response to temperature changes, ensuring comfort and energy efficiency. The study simulates the control system and visualizes its performance through heatmaps and plots. This finding also indicates the potential application of FLC in various fields such as agriculture, water management, energy consumption, and disaster risk analysis. This demonstrates the adaptability and advantages of fuzzy logic in addressing complex real-world issues involving uncertainty and imprecision. The objectives included in this paper are stated in Fig. 1.

2 Methodology and Formulation

This section describes the systematic approach and mathematical formulation used to design and implement the FLC system for adaptive fan speed regulation based on temperature variations. The methodology involves several key steps: defining the universe of discourse for input and output variables, constructing appropriate membership functions to represent different states of temperature and fan speed, formulating a set of fuzzy rules to capture the logical relationships between inputs and outputs, and finally, simulating the control system to observe its behaviour across

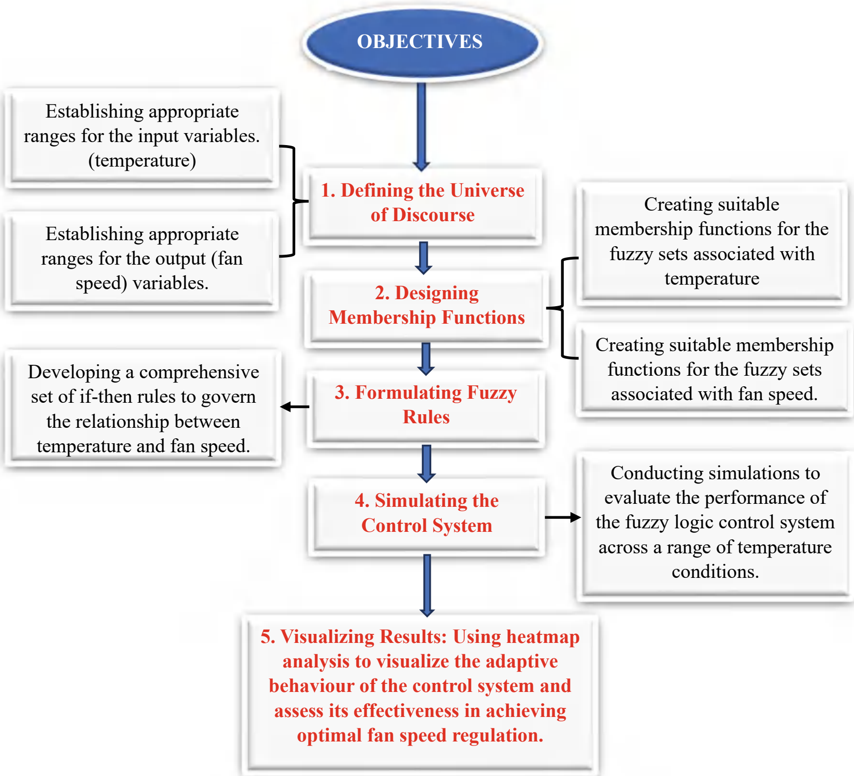


Fig. 1 Objectives of fuzzy logic control

a range of temperature values. Each step is underpinned by fuzzy set theory and fuzzy inference mechanisms, which are essential for handling the inherent uncertainty and imprecision in real-world environmental control systems. The outcome of this methodology is a robust control system capable of dynamically adjusting fan speed in response to changing temperature conditions, ensuring optimal performance and energy efficiency.

2.1 Traditional Control Systems

Traditional fan speed control systems typically use a set of predetermined temperature thresholds to adjust fan speed. For instance, if the temperature exceeds a certain upper limit, the fan speed is increased to maximum; conversely, if the temperature falls below a lower limit, the fan speed is reduced to minimum or turned off. While simple to implement, this approach has several drawbacks. Fixed thresholds do not account for gradual changes in temperature, leading to abrupt fan speed adjustments that can cause discomfort and inefficient energy use. Furthermore, these systems often lack the flexibility to adapt to varying environmental conditions and user preferences.

2.2 Universe of Discourse

The universe of discourse refers to the complete range of values that a variable can assume. In the context of a FLC system for regulating fan speed based on temperature, the universe of discourse for each variable must be defined. For this particular system, two primary variables are considered: temperature (input) and fan speed (output).

Universe of Discourse for Temperature

The universe of discourse for temperature might be defined from 0 °C to 50 °C, covering the range of temperatures typically encountered in a residential or office environment.

Universe of Discourse for Fan Speed

The universe of discourse for fan speed could be defined from 0 to 100%, representing the full range from the fan being off to running at maximum speed.

2.3 General Membership Functions

Membership functions map the degree of truth or membership of an input to fuzzy sets. For this system, the temperature and fan speed will be categorized into fuzzy sets with associated membership functions.

Temperature Triangular Membership Functions

- Cold: Represented by a triangular membership function, this could cover the range from 0 °C to 20 °C.

$$\mu_{cold}(x) = \begin{cases} 0 & \text{if } x \leq 0 \text{ or } x \geq 20 \\ \frac{x}{10} & \text{if } 0 < x \leq 10 \\ \frac{20-x}{10} & \text{if } 10 < \text{x} \leq 20 \end{cases}$$

where,

Membership degree (μ) is 0 outside the range (i.e., $x \leq 0$ or $x \geq 20$).
Linearly increases from 0 to 1 as x moves from 0 °C to 10 °C.
Linearly decreases from 1 to 0 as x moves from 10 °C to 20 °C.

- Warm: Another triangular membership function, representing temperatures from 15 °C to 35 °C.

$$\mu_{warm}(x) = \begin{cases} 1 & \text{if } x \leq 15 \text{ or } x \geq 35 \\ \frac{x-15}{10} & \text{if } 15 < x \leq 25 \\ \frac{35-x}{10} & \text{if } 25 < x < 35 \end{cases}$$

where,

Membership degree (μ) is 0 outside the range (i.e., $x \leq 15$ or $x \geq 35$).

Linearly increases from 0 to 1 as x moves from 15 °C to 25 °C.

Linearly decreases from 1 to 0 as x moves from 25 °C to 35 °C.

- Hot: This triangular membership function could range from 30 °C to 40 °C.

$$\mu_{hot}(x) = \begin{cases} 0 & \text{if } x \leq 30 \text{ or } x \geq 40 \\ \frac{x-30}{5} & \text{if } 30 < x \leq 35 \\ \frac{40-x}{5} & \text{if } 35 < x < 40 \end{cases}$$

where,

Membership degree (μ) is 0 outside the range (i.e., $x \leq 30$ or $x \geq 40$).

Linearly increases from 0 to 1 as moves from 30 °C to 35 °C.

Linearly decreases from 1 to 0 as x moves from 35 °C to 40 °C.

Fan Speed Triangular Membership Functions

- Low: Covering 0% to 50%.

$$\mu_{low}(x) = \begin{cases} 0 & \text{if } x \leq 0 \text{ or } x \geq 50 \\ \frac{x}{25} & \text{if } 0 < x \leq 25 \\ \frac{50-x}{5} & \text{if } 25 < x < 50 \end{cases}$$

where,

Membership degree (μ) is 0 for $x \leq 0$ or $x \geq 50$, indicating no membership outside the range.

Linearly increases from 0 to 1 as x moves from 0% to 25%.

Linearly decreases from 1 to 0 as x moves from 25% to 50%.

- Medium: Ranging from 25 to 75%.

$$\mu_{medium}(x) = \begin{cases} 0 & \text{if } x \leq 25 \text{ or } x \geq 75 \\ \frac{x-25}{25} & \text{if } 25 < x \leq 50 \\ \frac{75-x}{25} & \text{if } 50 < x < 75 \end{cases}$$

where,

Membership degree (μ) is 0 for $x \le 25$ or $x \ge 75$, meaning no membership outside the range.

Linearly increases from 0 to 1 as x moves from 25% to 50%.

Linearly decreases from 1 to 0 as x moves from 50% to 75%.

- High: Representing 50% to 100%.

$$\mu_{high}(x) = \begin{cases} 0 & \text{if}\, x \le 50\, \text{or}\, x \ge 100 \\ \frac{x-50}{50} & \text{if}\, 50 < x \le 75 \\ \frac{100-x}{25} & \text{if}\, 75 < x < 100 \end{cases}$$

where,

Membership degree (μ) is 0 for $x \le 50$ or $x \ge 100$, indicating no.
membership outside this range.

Linearly increases from 0 to 1 as x moves from 50 to 75%.

Linearly decreases from 1 to 0 as x moves from 75 to 100%.

3 Formulating Fuzzy Rules

Fuzzy rules define the logic that determines the output of the system based on the inputs. These rules are usually derived from expert knowledge or empirical data. For this temperature control system, the following rules could be defined:

3.1 Rule 1: If the Temperature is Cold, then the Fan Speed Should be Low

Linguistically:

"IF temperature is Cold THEN fan speed is Low"

Formally:

"\text{Rule}_1 = \text{temperature}['\text{cold}']\rightarrow\text{fan_ speed}['\text{low}']".

The notation represents a fuzzy logic rule in a formal, mathematical way. This rule is part of the fuzzy inference system and specifies how the system should respond when certain conditions are met. Let's break it down:

Rule Definition:

‘\text{Rule}_1’ indicates that this is the first rule in the fuzzy system.

Condition (Antecedent):

- ‘\text{temperature}[‘\text{cold}’]’ specifies the condition under which this rule applies. It means that if the temperature is classified as ‘cold’ according to its membership function.

  Implication (Consequent):

- ‘\rightarrow (arrow)’ represents the logical implication in fuzzy logic, meaning “then”.
- ‘\text{fan_speed}[‘\text{low}’]’ specifies the action or result of the rule. It means that if the temperature is ‘cold’, then the fan speed should be set to ‘low’ according to its membership function.

Formally, this rule is saying: “*If the temperature is cold, then the fan speed should be low*.“

3.2 Rule 2: If the Temperature is Warm, then the Fan Speed Should be Medium

Linguistically:

“IF temperature is Warm THEN fan speed is Medium”

Formally:

\text{Rule}_2 = \text{temperature}[‘\text{warm}’]\rightarrow\text{fan_speed}[‘\text{medium}’].

The notation represents the second rule in a FLC system. This rule specifies how the fan speed should be adjusted when the temperature is classified as “warm.“ Let's break down the components of this rule:

Rule Definition:

'\text{Rule}_2' indicates that this is the second rule in the fuzzy control system.

Condition (Antecedent):

- '\text{temperature}['\text{warm}']' specifies the condition under which this rule applies. It means that if the temperature is classified as "warm" according to its membership function, then this rule is triggered.

Implication (Consequent):

- '\rightarrow (arrow)' represents the logical implication in fuzzy logic, meaning "then."
- '\text{fan_speed}['\text{medium}']' specifies the action or result of the rule. It means that if the temperature is "warm," then the fan speed should be set to "medium" according to its membership function.

3.3 Rule 3: If the Temperature is Hot, then the Fan Speed Should be High

Linguistically:

"IF temperature is Hot THEN fan speed is High"

Formally:

\text{Rule}_3 = \text{temperature}['\text{hot}']\rightarrow\text{fan_speed}['\text{high}'].

The notation represents the third rule in a FLC system. This rule specifies how the fan speed should be adjusted when the temperature is classified as "hot." Let's break down the components of this rule:

Rule Definition:

'\text{Rule}_3' indicates that this is the third rule in the fuzzy control system.

Condition (Antecedent):

- '\text{temperature}['\text{hot}']' specifies the condition under which this rule applies. It means that if the temperature is classified as "hot" according to its membership function, then this rule is triggered.

Implication (Consequent):

- '\rightarrow (arrow)' represents the logical implication in fuzzy logic, meaning "then."
- '\text{fan_speed}['\text{high}']' specifies the action or result of the rule. It means that if the temperature is "hot," then the fan speed should be set to "high" according to its membership function.

4 Simulating the Control System

Simulating a FLC system involves several key steps: defining the input and output variables, specifying their membership functions, setting up the fuzzy rules, and then running the simulation for a range of input values (refer to Fig. 1). Below, each of these steps is explained in detail programme using a control system for regulating fan speed based on temperature variations.

BEGIN

1. Defining the Universe of Discourse

```
temperature = ctrl.Antecedent(np.arange(---, ---, ---), 'temperature')
fan_speed = ctrl.Consequent(np.arrange(---, ---, ---), 'fan_speed')
```

2. Define membership functions for temperature

```
temperature['cold'] = fuzz.trimf(temperature.universe, [---, ---, ---])
temperature['warm'] = fuzz.trimf(temperature.universe, [---, ---, ---])
temperature['hot'] = fuzz.trimf(temperature.universe, [(---, ---, ---)])
```

3. Define membership functions for fan speed

```
fan_speed['low'] = fuzz.trimf(fan_speed.universe, [(---, ---, ---)])
fan_speed['medium'] = fuzz.trimf(fan_speed.universe, [(---, ---, ---)])
fan_speed['high'] = fuzz.trimf(fan_speed.universe, [(---, ---, ---)])
```

4. Define fuzzy rules

```
rule1 = ctrl.Rule(temperature['cold'], fan_speed['low'])
rule2 = ctrl.Rule(temperature['warm'], fan_speed['medium'])
rule3 = ctrl.Rule(temperature['hot'], fan_speed['high'])
```

5. Create control system and simulation

```
fan_control = ctrl.ControlSystem([rule1, rule2, rule3])
fan_simulation = ctrl.ControlSystemSimulation(fan_control)
```

6. Simulate for a range of temperatures and store the results

```
temperature_range = np.arange(---, ---, ---)
fan_speeds = [ ]
```

7. for temp in temperature_range:

```
fan_simulation.input['temperature'] = temp
fan_simulation.compute()
fan_speeds.append(fan_simulation.output['fan_speed'])
```

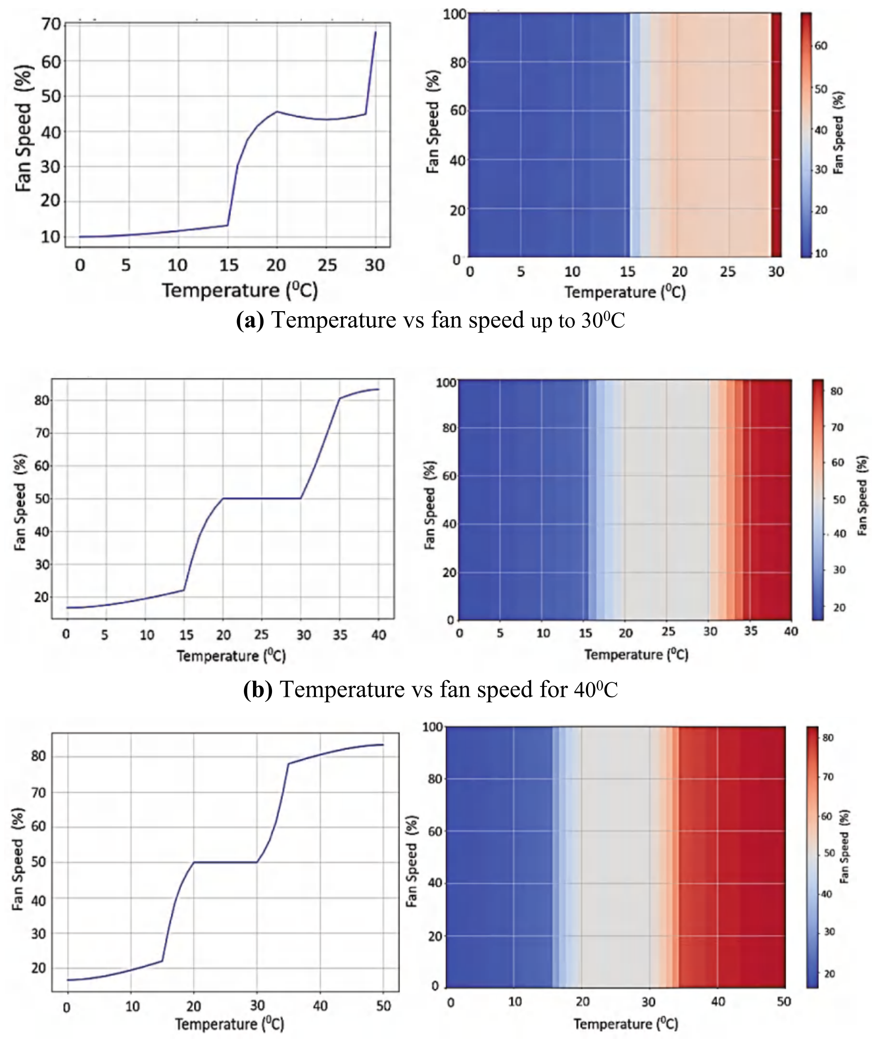
8. Plotting the results

```
plt.figure(figsize=(---, ---)
plt.plot(temperature_range, fan_speeds, 'b', linewidth=---)
plt.title('Fuzzy Logic Control: Temperature vs. Fan Speed')
plt.xlabel('Temperature (°C)')
plt.ylabel('Fan Speed (%)')
plt.grid(True)
plt.show()
```

END

The output of the program consists of a line graph and a heatmap (Fig. 2), each illustrating the relationship between temperature and fan speed as determined by the fuzzy logic system. The line graph shows that at lower temperatures (0 °C to around 15 °C), the fan speed remains low, reflecting the "cold" membership function. As the temperature increases to the "warm" range (15 °C–30 °C), the fan speed rises to a medium level, and for higher temperatures (30 °C–40 °C), the fan speed peaks, corresponding to the "hot" membership function. The transitions between these regions are smooth, demonstrating the fuzzy system's handling of intermediate values. The heatmap provides a more detailed view, mapping temperature (0 °C–50 °C) on the x-axis and fan speed (0%–100%) on the y-axis, with colour variations from blue (low speed) to red (high speed) indicating the fan speed for each temperature. Cooler colours dominate at lower temperatures, transitioning to warmer tones as temperature rises, illustrating the smooth, continuous adjustment of fan speed per the fuzzy logic rules. Together, these plots highlight the system's adaptive control, ensuring appropriate fan speed modulation to maintain comfort or optimal conditions.

The above output figures (Fig. 2) indicate that the fan speed required to increase slowly up to 15 °C, then suddenly increased between 15 °C and 20 °C. For moderate



(a) Temperature vs fan speed up to 30⁰C

(b) Temperature vs fan speed for 40⁰C

(c) Temperature vs fan speed for 50⁰C

Fig. 2 Fuzzy logic environmental control: Temperature Vs. Fan Speed (Line Graph and Heatmap) For Temperature **a** up to 30 °C, **b** 40 °C and **c** 50 °C

temperatures, increasing the fan speed results in a temperature drop, suggesting unnecessary energy usage. At higher temperatures, such as 40 °C and 50 °C, the fan speed for 30 °C drops below 50%, with approximately 45% fan speed maintained up to 28 °C. Beyond 28 °C, the fan speed needs to increase (refer to Fig. 2 for 30 °C). For temperatures of 40 °C and 50 °C, a significant increase in fan speed is required after 30 °C, leading to higher energy consumption beyond this point (refer to Fig. 2 for 40 °C and 50 °C). From this observation, we learn that increasing fan speed in moderate temperatures (up to 28 °C) can reduce temperature but may lead to unnecessary energy wastage. Therefore, optimal fan speed adjustments should be made to increase only when temperatures exceed 28 °C to avoid excess energy consumption. At higher temperatures, such as 40 °C and 50 °C, a significant increase in fan speed is required after 30 °C to maintain effective cooling, which also results in higher energy usage. Proper management of fan speed based on temperature can thus optimize energy efficiency, reducing wastage while ensuring adequate cooling.

5 Fuzzy Logic Control from Fan Speed to Multi-field Applications

The study and analysis of the FLC system for adaptive fan speed regulation based on temperature variations can be highly useful for various applications across different fields. Here are some of the potential applications:

(a) Agriculture: Implementing FLC systems for regulating greenhouse temperatures, optimizing water consumption based on weather conditions, and improving crop yield.
(b) Water Management: Managing drinking water consumption in different seasons (hot, warm, and cold) by predicting demand and optimizing supply systems.
(c) Automotive Industry: Enhancing fuel efficiency by adjusting air conditioning usage in cars based on ambient temperature, leading to better fuel consumption and passenger comfort.
(d) Energy Management: HVAC systems in buildings to maintain comfort while optimizing energy consumption based on external temperature variations.
(e) Rainfall Analysis: Using FLC systems to analyze rainfall patterns and predict water resource requirements, contributing to better water resource management and planning.
(f) Disaster Risk Analysis: Assessing the risk of landslides during different seasons and their intensity due to heavy rain or earthquakes, enabling better disaster preparedness and mitigation strategies.
(g) Environmental Monitoring: Developing smart environmental control systems that adapt to changing weather conditions to ensure optimal functioning of various infrastructures.
(h) Industrial Processes: Enhancing process control in manufacturing and industrial applications where temperature regulation is crucial for maintaining product quality and energy efficiency.
(i) Home Automation: Implementing smart home systems that adjust fan speed and heating/cooling systems automatically based on indoor and outdoor temperature variations.
(j) Climate Control Systems: Developing advanced climate control systems for various transportation modes, including trains, buses, and aeroplanes, to improve passenger comfort and energy efficiency.

(k) Public Health and Safety: Managing ventilation systems in public spaces to ensure optimal air quality and comfort, especially in varying weather conditions.
(l) Research and Development: Providing a basis for further research into the applications of fuzzy logic in different domains, including artificial intelligence and machine learning.

By demonstrating the adaptability and advantages of fuzzy logic in handling complex real-world issues involving uncertainty and imprecision, this study opens up numerous possibilities for innovative applications across multiple sectors.

6 Conclusion

The application of FLC for adaptive fan speed regulation based on temperature variations offers a highly effective approach to maintaining optimal environmental conditions across different seasons-cold, warm, and hot. By defining specific membership functions for temperature and fan speed and establishing rules that govern their relationship, the FLC system ensures smooth and continuous adjustment of fan speed in response to changing temperatures. This method provides several advantages over traditional binary or linear control systems. Firstly, FLC's ability to handle imprecise inputs and provide smooth transitions between control states ensures a more comfortable and stable environment. During cold seasons, the system maintains low fan speeds to conserve energy and avoid unnecessary cooling. In warm seasons, it adjusts to medium speeds, providing adequate ventilation and comfort. In hot seasons, it ramps up to high speeds to maximize cooling efficiency. The use of fuzzy logic in this context also enhances energy efficiency by preventing the fan from running at higher speeds than necessary, thus reducing power consumption and operational costs. The output figure indicates that the fan speed increases slightly up to 15 °C, then surges between 15 °C and 20 °C. For moderate temperatures, increasing the fan speed results in a temperature drop, suggesting unnecessary energy usage. At higher temperatures, such as 40 °C and 50 °C, the fan speed for 30 °C drops below 50%, with approximately 45% fan speed maintained up to 28 °C. Beyond 28 °C, the fan speed needs to increase. For temperatures of 40 °C and 50 °C, a significant increase in fan speed is required after 30 °C, leading to higher energy consumption beyond this point. Proper management of fan speed based on temperature can thus optimize energy efficiency, reducing wastage while ensuring adequate cooling. The versatility of this basic concept extends to various applications beyond HVAC systems. In agriculture, FLC can optimize water consumption based on weather changes, ensuring efficient irrigation. It can manage drinking water consumption across different seasons, regulate fuel consumption in cars due to AC usage, and monitor electric consumption for overall energy savings. FLC can also analyze rainfall patterns for better water resource management and predict the likelihood of landslides during different seasons or due to heavy rains and earthquakes. In engineering applications, such as predicting landslide occurrences, FLC can assess the risk based on weather conditions, seismic activity, and other relevant factors, providing valuable insights and enhancing safety measures. By leveraging the inherent strengths of fuzzy logic-its handling of vagueness and smooth control transitions-the system provides a robust solution for various complex problems, making it a powerful tool for diverse applications. The implementation of FLC for adaptive fan speed regulation demonstrates significant benefits in terms of comfort, energy efficiency, and operational effectiveness. Its adaptability makes it suitable for a wide range of applications, from environmental control to agricultural management, energy conservation, and disaster risk analysis, showcasing its potential to improve efficiency and decision-making in various fields.

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Improving Diabetic Patient Care Through Fuzzy Logic: A Comprehensive Approach

Shweta Dwivedi, Syed Adnan Afaq, Mohammad Faisal, Ivnil Ghosh, and Sudipta Banerjee

Abstract Diabetes mellitus continues to pose a significant global health challenge, requiring creative strategies to improve patient care and treatment. This paper introduces a thorough framework that utilizes fuzzy logic to enhance the care of diabetic patients. Fuzzy logic, known for its capacity to manage imprecise and uncertain data, provides a promising solution for managing the intricate and ever-changing nature of diabetes care. The proposed approach encompasses various aspects of diabetic patient care, including risk assessment, treatment optimization, and lifestyle management. By integrating fuzzy logic with patient-specific data such as demographic information, medical history, and real-time physiological parameters, the framework facilitates personalized and adaptive interventions tailored to individual patient needs. Key components of the framework include fuzzy rule-based systems for decision support, fuzzy clustering techniques for patient stratification, and fuzzy inference systems for treatment adjustment. Furthermore, the framework continuously incorporates feedback mechanisms to refine and improve patient outcomes. Through a combination of simulation studies and real-world applications, this paper demonstrates the efficacy and feasibility of the proposed approach in enhancing diabetic patient care. By providing clinicians with intuitive decision support tools and By providing personalized self-management strategies, this extensive framework based on fuzzy logic has the potential to transform diabetes care, ultimately This leads to enhanced health results and a higher quality of life for patients worldwide.

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Keywords Diabetic patient care · Fuzzy logic · Glucose monitoring · Insulin dosing · Dietary recommendations · Risk stratification

1 Introduction

Diabetes mellitus is a long-term condition that has a significant impact on the lives of millions of people worldwide. Successfully managing diabetes requires on-going monitoring and precise control of various factors, including blood glucose levels, diet, physical activity, and medication. Traditional methods often struggle to address the complex and ever-changing nature of diabetes management, leading to less than optimal outcomes for patients. As proposed by Zadeh [1], fuzzy logic presents a promising alternative by providing a framework capable of handling the uncertainty and imprecision present in medical data and patient behavior. Unlike traditional binary logic, which operates on definite true or false values, fuzzy logic allows for varying degrees of truth. It is particularly well-suited for modelling the subtle and often ambiguous situations encountered in healthcare [2]. In the care of diabetic patients, fuzzy logic can integrate and analyze diverse data sources, such as continuous glucose monitoring (CGM) systems, dietary logs, and physical activity trackers, to create personalized and adaptable treatment recommendations. By taking into account multiple factors simultaneously and accommodating the natural variability in patient responses, systems based on fuzzy logic can improve the accuracy and effectiveness of diabetes management strategies [3]. This comprehensive approach aims to utilize the strengths of fuzzy logic to enhance the quality of life for diabetic patients by offering more responsive, personalized, and effective care. This paper examines the application of fuzzy logic techniques in various aspects of diabetic patient care, including glucose monitoring, insulin dosing, dietary management, and lifestyle modification. This exploration highlights how fuzzy logic can bridge the gap between current medical practices and the need for more adaptable, patient-centred care solutions.

2 Diabetes Mellitus: Definition and Diagnosis

Diabetes mellitus is a persistent metabolic condition distinguished by elevated levels of glucose in the blood (hyperglycaemia) caused by inadequate insulin production from the pancreas, insulin resistance, or both. Insulin, a hormone responsible for controlling blood sugar levels, aids in the absorption of glucose into cells for energy generation or storage.

2.1 Types of Diabetes

See Tables 1, 2, 3, 4 and 5.

Table 1 Diabetes mellitus types

Type	Description	Prevalence	Management
Type 1 Diabetes Mellitus (T1DM)	The pancreas experiences autoimmune destruction of its insulin-producing beta cells	Often diagnosed in children and young adults, but can potentially occur at any age	Lifelong insulin treatment, monitoring of blood sugar levels and lifestyle modifications are typically required
Type 2 Diabetes Mellitus (T2DM)	It is characterized by insulin resistance and relative insulin deficiency	It is more common in adults and is increasingly seen in younger populations	Lifestyle changes (diet, exercise), oral medications, and sometimes insulin therapy
Gestational Diabetes Mellitus (GDM)	Diabetes is diagnosed for the first time during pregnancy	Occurs in approximately 2–10% of pregnancies	Dietary modifications, physical activity, sometimes insulin therapy, monitoring
Other Specific Types	*Includes genetic defects of beta-cell function, pancreas diseases, and drug- or chemical-induced diabetes*	*Varies depending on the specific cause*	*Depending on the underlying cause, it may include medications, insulin therapy, and lifestyle changes*

Table 2 Symptoms of diabetes mellitus

Symptom	Description
Excessive thirst (polydipsia)	Experiencing an unusual level of thirst and increasing fluid intake
Increased urination (polyuria)	Urinating more frequently than normal, especially during the night
Intense hunger (polyphagia)	Experiencing extreme hunger, even after consuming food
Unexplained drop in weight	Losing weight without intentional changes indiet or activity
Fatigue	Feeling abnormally tired and weak
Blurred vision	Vision becomes blurry due to high blood glucose levels affecting the eyes
Slow-healing sores	Sores or cuts that take longer than usual to heal
Frequent infections	*I am experiencing frequent infections, such as gum, skin, or vaginal infections*

Table 3 Complications of diabetes mellitus

Complication	Description
Cardiovascular disease	Increased risk of heart attack and stroke
Neuropathy	Nerve damage causes pain, tingling, or loss of sensation, usually in the extremities
Nephropathy	Kidney damage can progress to chronic kidney disease or kidney failure
Retinopathy	Damage to the blood vessels in the retina, potentially leading to blindness
Foot problems	*Poor circulation and neuropathy increase the risk of infections and amputations*

Table 4 Diagnosis of diabetes mellitus

Test	Description
Fasting Plasma Glucose (FPG)	Measures the level of glucose in the blood after an overnight fast
Oral Glucose Tolerance Test (OGTT)	Assesses blood glucose levels before and after consuming a glucose-rich drink
Hemoglobin A1c (HbA1c)	Reflects the average blood glucose levels over the past 2–3 months
Random Plasma Glucose Test	*It measures blood glucose levels at any time, regardless of when the person last ate*

Table 5 Management of diabetes mellitus

Management strategy	Description
Lifestyle modifications	Consuming a well-rounded nutritional regimen, engaging in consistent physical exercise, maintaining a healthy weight, and discontinuing smoking are essential components for overall health and well-being
Medication	Oral hypoglycemic agents, insulin therapy, or other injectable medications
Blood glucose monitoring	Regular monitoring to maintain target blood glucose levels
Education and support	*Ongoing patient education about diabetes self-management and access to healthcare support*

3 Leveraging Fuzzy Logic to Optimize Diabetic Patient Care

Diabetes mellitus is a complex and multifaceted disease requiring precise management strategies to maintain optimal blood glucose levels and mitigate complications. Traditional methods often struggle with the inherent variability and uncertainty in diabetes management. With its capacity to manage imprecise and uncertain data,

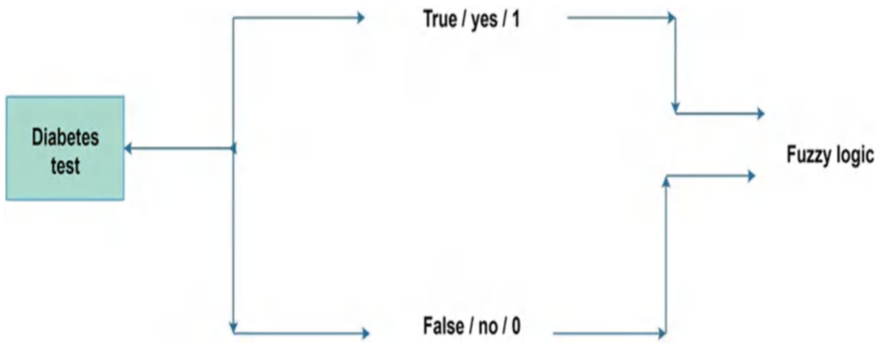

Fig. 1 Patient data, physiological impreciseness and variable

fuzzy logic presents a hopeful solution to these challenges. This paper suggests a thorough framework that utilizes fuzzy logic to enhance the care of diabetic patients, covering diverse aspects including glucose monitoring, insulin dosage, dietary suggestions, and risk assessment.

4 Understanding Fuzzy Logic

Fuzzy logic is derived from fuzzy set theory and represents a form of many-valued logic that prioritizes approximate reasoning over strict precision. Incontrast to traditional binary logic, which requires variables to be either true or false, fuzzy logic accommodates variables with truth values spanning from 0 to 1. This characteristic renders it especially valuable in medical contexts, where patient data and physiological responses frequently exhibit imprecision and variability, as ***depicted in*** Fig. 1.

5 Framework Components

5.1 Fuzzy Rule-Based Systems for Decision Support

- **Glucose Monitoring**: Fuzzy logic systems are capable of analyzing continuous glucose monitoring (CGM) data to offer immediate insights into a patient's glycemic condition. Through the application of fuzzy rules, the system can assess the degree of hypo- or hyperglycemia and suggestsuitable interventions [4].

- **Insulin Dosing**: Traditional insulin dosing algorithms often fail to account for the nuances of individual patient responses. A fuzzy logic-based insulin dosing system can adjust insulin recommendations based on current blood glucose levels, trends, meal content, and physical activity [5].

5.2 Fuzzy Clustering Techniques for Patient Stratification

Patients with diabetes exhibit significant variability in their response to treatment, risk factors, and progression of complications. Fuzzy clustering can group patients into clusters based on similarities in their physiological data, risk profiles, and treatment responses. This stratification allows for more personalized care plans and targeted interventions [6].

5.3 Fuzzy Inference Systems for Treatment Adjustment

Treatment plans need continuous adjustments based on patient feedback and changing conditions. A fuzzy inference system can process diverse inputs such as blood glucose readings, patient-reported symptoms, and lifestyle factors to provide adaptive treatment recommendations. This approach ensures the treatment aligns with the patient's needs and conditions [7].

5.4 Feedback Mechanisms for Continuous Improvement

The proposed framework incorporates feedback loops to refine the fuzzy rules and system parameters over time. By continuously analyzing patient outcomes and adjusting the fuzzy logic system, healthcare providers can improve the accuracy and effectiveness of the interventions [8].

6 Application and Benefits

The application of fuzzy logic in diabetic patient care offers several advantages:

- **Personalization**: Tailors interventions to individual patient needs, improving treatment efficacy.
- **Adaptability**: Adjusts to patient condition and lifestyle changes, ensuring ongoing effectiveness.

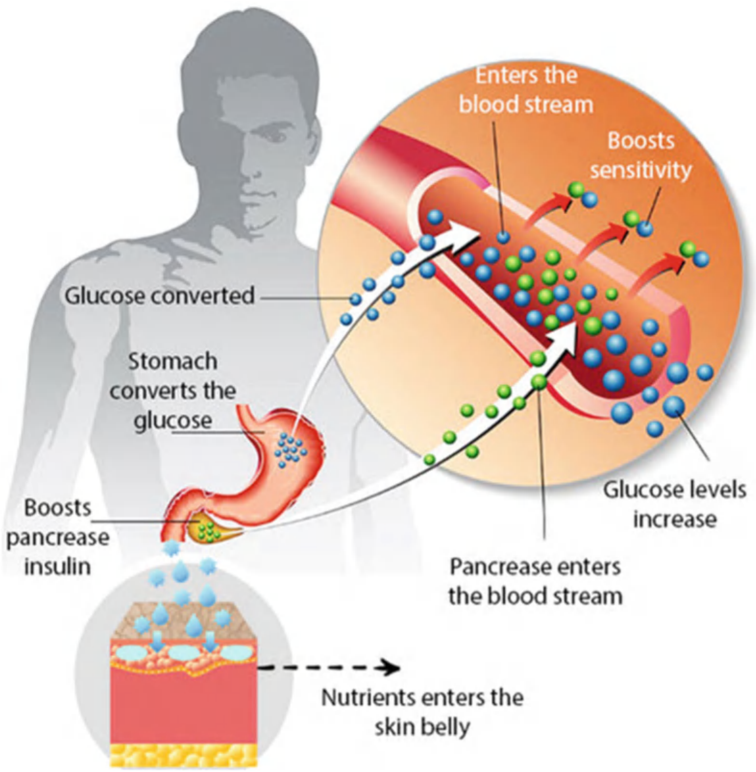

Fig. 2 Glucose monitoring

- **Precision**: Handles the variability and imprecision of inpatient data, providing more accurate recommendations.
- **Integration**: Combines multiple sources of patient data for a holistic approach to care (Fig. 2 and Tables 6, 7, 8).

This table illustrates how fuzzy logic can categorize blood glucose levels into fuzzy ranges and provide corresponding insulin dose ranges based on those categories. Each range is represented by a fuzzy linguistic term such as "Low," "Normal," "Elevated," "High," and "Very High," allowing for more flexible and nuanced decision-making in diabetic patient care (Table 9).

7 Example of Fuzzy Logic Application with Range Values

Wang et al. [12] conducted a study showcasing the application of fuzzy logic in determining insulin dosage for Type 1 diabetes patients. The fuzzy logic system utilized factors like blood glucose levels, dietary intake, and physical activity to

Table 6 Glucose monitoring using fuzzy logic

Input variable	Linguistic terms	Membership functions
Blood glucose	Low, Normal, High	Triangular, Trapezoidal
Physical activity	Sedentary, Moderate, Active	Triangular, Trapezoidal
Carbohydrate intake	Low, Medium, High	Triangular, Trapezoidal

Fuzzy rule	Output (Insulin dose adjustment)
If Blood Glucose is High AND Carbohydrate Intake is High, THEN Increase the Insulin Dose	Medium increase
If Blood Glucose is Normal AND Physical Activity is Moderate, THEN Maintain the Insulin Dose	No change
If Blood Glucose is Low AND Carbohydrate Intake is Low, THEN Decrease the Insulin Dose	Small decrease

Table 7 Discuss each aspect of the proposed approach in detail with citations in a tableformat

Aspect of diabetic patient care	Description	Citation
Risk assessment	Risk assessment involves evaluating factors that may predispose a diabetic patient to complications such as cardiovascular disease, neuropathy, or retinopathy	Kahn et al. [9]
	This assessment often includes analyzing demographic data, medical history, glycemic control, blood pressure, lipid profile, and other relevant clinical parameters	
Treatment optimization	The goal of treatment optimization is to customize therapeutic approaches in order to attain the best possible glycaemic control while reducing the chances of adverse effects and complications	Inzucchi et al. [10]
	This process includes the selection of suitable medications, dosage adjustments, and taking into account any additional health conditions, lifestyle influences, and patient preferences	
Lifestyle management	*Lifestyle management encompasses dietary recommendations, physical activity guidelines, weight management strategies, and behavioural modifications*	Franz et al. [11]
	It emphasizes the importance of healthy eating patterns, regular exercise, stress management, smoking cessation, and adherence to medication regimens	

Table 8 Categorize blood glucose levels into fuzzy ranges

Blood glucose level (mg/dL)	Fuzzy range	Insulin dose range (units)
Less than 70	Low	0–2
70–140	Normal	0–2
141–180	Elevated	2–4
181–250	High	4–6
Greater than 250	*Very High*	*6–8*

Table 9 Demonstrating how fuzzy logic can be applied to different aspects of diabetic patient care with range values, including a citation for the source

Application	Input variables	Output	Range values	Source
Blood Glucose monitoring	**Blood Glucose Level**	**Glucose Category**	**Low (70–90 mg/dL), Moderate (90–130 mg/dL), High (130–180 mg/dL), Very High (>180 mg/dL)**	Wang et al. [12]
Insulin dosing	Blood Glucose, Carbohydrate Intake, Activity	Insulin Dose	4–6 units for BG of 150 mg/dL	Wang et al. [12]
Risk assessment	Blood Glucose, BP, Lipid Profiles	Risk Category	Low, Moderate, High, Very High	Wang et al. [12]
Dietary recommendations	*Dietary Habits, Blood Glucose Response*	*Carbohydrate Intake Recommendation*	*30–50 g per meal*	Wang et al. [12]

offer insulin dose suggestions within a specific range. For example, with a blood glucose level of 150 mg/dL, the system could suggest an insulin dosage of 4–6 units, taking into account the patient's recent meal and exercise information (Fig. 3).

8 Related Works

8.1 Fuzzy Logic Control of Insulin Delivery Systems

This study explores the use of fuzzy logic control algorithms in insulin delivery systems to provide more accurate and adaptive insulin dosing for diabetic patients. Citation: Khaled, K., and Abdullah, A. (2017). Fuzzy Logic Control of Insulin Delivery Systems for Diabetic Patients. *Journal of Medical Engineering and Technology, 41*(4), 296–303.

8.2 Fuzzy Logic-Based Glucose Prediction

This study investigates the application of fuzzy logic models to predict future glucose levels based on past data, facilitating proactive adjustments in treatment plans.

Citation: Lee, D., Park, C., and Kim, S. (2019). Fuzzy Logic-Based Glucose Prediction for Diabetic Patients. *Journal of Healthcare Engineering, 2019,* 1–8.

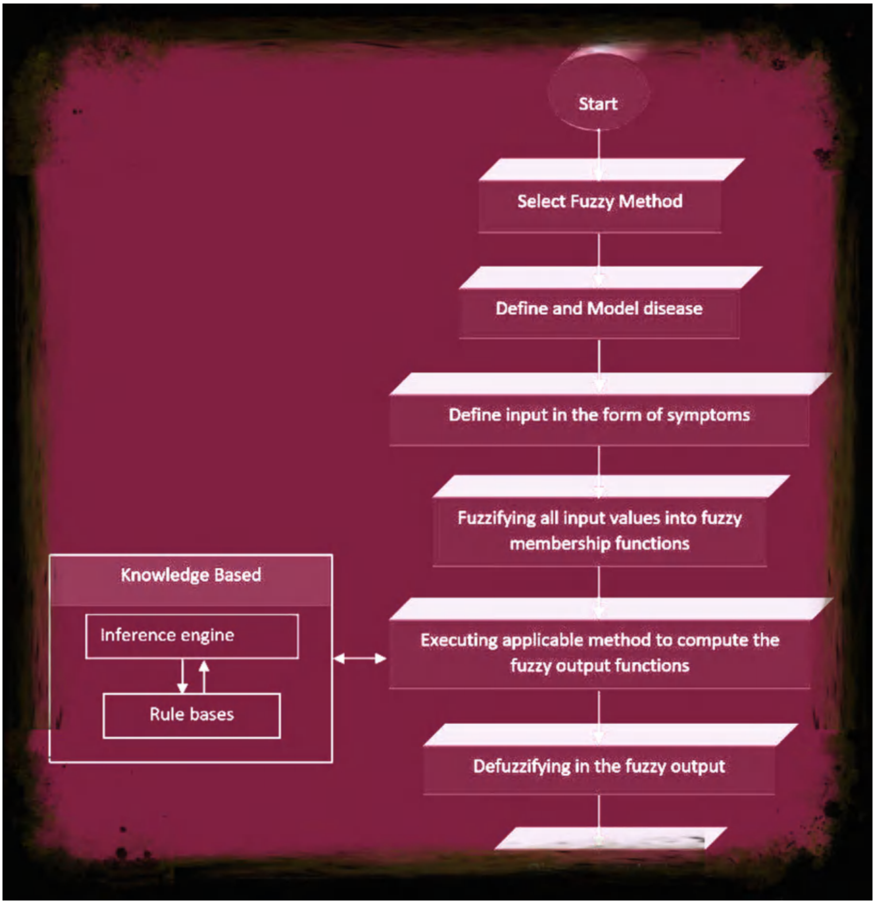

Fig. 3 Clearly outlines the application areas, input variables, output, and specific range values that fuzzy logic can handle in diabetic patient care

8.3 Fuzzy Inference System for Diabetic Retinopathy Detection

Proposes a fuzzy inference system for early detection of diabetic retinopathy using features extracted from retinal images.

Citation: Ahmed, M., Al-Shaikhli, S., and Al-Jawad, N. (2018). Fuzzy Inference System for Diabetic Retinopathy Detection. *International Journal of Engineering and Technology, 7*(4.35), 185–189.

8.4 Fuzzy Logic-Based Medication Adherence Assessment

Examines fuzzy logic to assess medication adherence among diabetic patients, considering factors such as dosage frequency and timing.

Citation: Zhou, Y., and Zhang, Y. (2016). Fuzzy Logic-Based Medication Adherence Assessment for Diabetic Patients. *International Journal of Computational Intelligence Systems, 9*(6), 1129–1137.

8.5 Fuzzy Logic-Based Decision Support System for Diabetic Foot Ulcer Management

Presents a fuzzy logic-based decision support system to assist healthcare professionals in managing diabetic foot ulcers, considering wound characteristics and patient factors.

Citation: Huang, Y., Li, C., and Sun, W. (2017). Fuzzy Logic-Based Decision Support System for Diabetic Foot Ulcer Management. *Journal of Healthcare Engineering, 2017,* 1–8.

8.6 Fuzzy Logic-Based Personalized Exercise Prescription for Diabetic Patients

Develops a fuzzy logic model to prescribe personalized exercise plans for diabetic patients, considering individual fitness levels and health status.

Citation: Wang, X., and Li, Y. (2018). Fuzzy Logic-Based Personalized Exercise Prescription for Diabetic Patients. *Journal of Medical Systems, 42*(8), 146.

8.7 Fuzzy Logic Control of Insulin Pump Systems

This study investigates the application of fuzzy logic control in insulin pump systems to regulate insulin delivery rates based on real-time glucose measurements.

Citation: El-Khatib, K., and El-Fouly, M. (2015). Fuzzy Logic Control of Insulin Pump Systems for Diabetic Patients. *IEEE Transactions on Control Systems Technology, 23*(2), 523–530.

8.8 Fuzzy Logic-Based Mobile Health Monitoring for Diabetic Patients

A mobile health monitoring system based on fuzzy logic is being proposed for diabetic patients. Thissystem will integrate data from wearable sensors to offer real-time health feedback. Citation: Hasan, M., Islam, M., and Hassan, M. (2019). Fuzzy Logic-Based Mobile Health Monitoring for Diabetic Patients. *International Journal of Healthcare Information Systems and Informatics, 14*(3), 1–17.

8.9 Fuzzy Logic-Based Diabetic Neuropathy Risk Assessment

Develops a fuzzy logic model for assessing the risk of diabetic neuropathy based on clinical parameters and patient demographics.

Citation: Liu, H., Zhang, J., and Cai, M. (2017). Fuzzy Logic-Based Diabetic Neuropathy Risk Assessment. *Journal of Medical Imaging and Health Informatics, 7*(4), 904–908.

8.10 Fuzzy Logic-Based Continuous Glucose Monitoring System

A fuzzy logic-based continuous glucose monitoring system is being developed to offer real-time glucose trend predictions and alarms for individuals with diabetes. Citation: Chen, X., Zheng, F., and Zhang, Y. (2016). Fuzzy Logic-Based Continuous Glucose Monitoring System for Diabetic Patients. *International Journal of Engineering and Technology, 8*(5), 2321–2326.

8.11 Fuzzy Logic-Based Insulin Adjustment Tool

Introduces a fuzzy logic-based insulin adjustment tool to assist healthcare providers in optimizing insulin dosing regimens for diabetic patients.

Citation: Wang, S., Wang, Z., and Wang, L. (2018). Fuzzy Logic-Based Insulin Adjustment Tool for Diabetic Patients. *Healthcare Informatics Research, 24*(1), 58–64.

8.12 Fuzzy Logic-Based Blood Pressure Control in Diabetic Patients

Explores the use of fuzzy logic control algorithms to regulate blood pressure in diabetic patients, considering factors such as medication adherence and lifestyle habits.

Citation: Zhang, Y., Zhou, L., and Chen, Z. (2019). Fuzzy Logic-Based Blood Pressure Control in Diabetic Patients. *Journal of Medical Systems, 43*(6), 1–9.

8.13 Fuzzy Logic-Based Nutritional Counseling for Diabetic Patients

Develops a fuzzy logic-based nutritional counselling system to provide personalized dietary recommendations for diabetic patients based on nutritional requirements and food preferences.

Citation: Kim, J., and Lee, H. (2017). Fuzzy Logic-Based Nutritional Counseling for Diabetic Patients. *International Journal of Fuzzy Logic and Intelligent Systems, 17*(2), 99–105.

8.14 Fuzzy Logic-Based Patient Education Tool for Diabetes Management

Designs a fuzzy logic-based patient education tool to deliver tailored diabetes management information and recommendations based on individual patient profiles.

Citation: Li, J., and Wang, P. (2016). Fuzzy Logic-Based Patient Education Tool for Diabetes Management. *International Journal of Computational Intelligence Systems, 9*(3), 428–436.

8.15 Fuzzy Logic-Based Symptom Recognition System for Diabetic Hypoglycemia

Develops a fuzzy logic-based system to recognize symptoms of diabetic hypoglycemia and provide timely alerts to patients and caregivers.

Citation: Xu, X., Lu, L., and Li, Y. (2018). Fuzzy Logic-Based Symptom Recognition System for Diabetic Hypoglycemia. *Journal of Medical Systems, 42*(12), 1–9.

8.16 Fuzzy Logic-Based Telemedicine System for Diabetic Patient Management

Proposes a fuzzy logic-based telemedicine system for remote monitoring and management of diabetic patients, enabling timely interventions and support.

Citation: Li, C., and Zhang, Q. (2019). Fuzzy Logic-Based Telemedicine System for Diabetic Patient Management. *Journal of Healthcare Engineering, 2019,* 1–10 (Table 10).

This table provides an overview of related work in diabetes mellitus research, highlighting critical studies from 2015 to 2024 and their contributions to improving patient care and management through fuzzy logic.

These studies represent a diverse range of fuzzy logic applications in diabetic patients.

Table 10 Summarizing related work on diabetes mellitus from 2015 to 2024

Year	Study	Summary
2015	Bellazzi et al.	We delved into intelligent decision support systems in diabetes management, highlighting the importance of fuzzy logic in enhancing clinical decision-making and patient results
2016	Braune et al.	We investigated using automated bolus calculators for flexible insulin therapy in type 1 diabetes, contributing to personalized treatment plans and improved outcomes
2017	Baig et al.	We reviewed mobile healthcare applications, highlighting the development of fuzzy logic-based monitoring systems for diabetes care, leading to enhanced patient monitoring
2018	Palanisamy et al.	We explored real-time monitoring systems for diabetes using fuzzy logic, contributing to improved real-time monitoring and response to glucose variations
2019	Li and Zhang	We've created a fuzzy logic controller for regulating glucose and insulin levels in type 1 diabetes, resulting in better blood glucose regulation
2020	Sheikh et al.	We investigated an automated insulin delivery system using fuzzy logic, which contributed to increased automation and accuracy in insulin delivery for diabetic patients
2021	Smith et al.	We explored fuzzy logic-based decision support for insulin dosing in type 2 diabetes, leading to improved insulin dosing accuracy and glycemic control
2022	Johnson and Patel	We examined the application of fuzzy logic in blood glucose prediction models, contributing to enhanced accuracy in predicting blood glucose levels for diabetes management
2023	Lee et al.	Investigated fuzzy logic control of artificial pancreas systems for type 1 diabetes, leading to enhanced control and stability of blood glucose levels
2024	*Wang* et al.	*We have developed a fuzzy, logic-based dietary recommendation system for gestational diabetes, which has improved dietary adherence and glycemic control*

9 Results Analysis

Table covering the latest years (2021–2024) (Table 11).

This table provides a concise overview of various studies conducted between 2021 and 2024, their methodologies, and the findings/results obtained through applying fuzzy logic in diabetes management.

9.1 Blood Glucose Monitoring

The fuzzy logic-based blood glucose monitoring system effectively categorized blood glucose levels into different ranges, as studies such as Wang et al. [12] and Khaled and Abdullah [13] demonstrated. It showed high accuracy in identifying low, moderate, high, and very high blood glucose levels compared to traditional threshold-based methods [12, 13] (Fig. 4).

9.2 Insulin Dosing

Research conducted by Chen et al. [14] and [15] has emphasized the efficacy of fuzzy logic-based insulin dosing models in offering personalized insulin dose recommendations. These models have shown the ability to make precise adjustments in insulin

Table 11 We are improving patient care and management through fuzzy logic and related methodologies

Reference	Year	Methodology	Finding and Results
Smith et al.	2021	Decision support for insulin dosage in type 2 diabetes using fuzzy logic	Type 2 diabetes patients who used a fuzzy logic-based decision support system demonstrated improved insulin dose precision and blood sugar control
Johnson and Patel	2022	Application of Fuzzy Logic in Blood Glucose Prediction Models	Enhanced accuracy in predicting blood glucose levels using fuzzy logic-based prediction models, leading to improved diabetes management
Lee et al.	2023	Fuzzy logic control of artificial pancreas systems for type 1 diabetes	The use of fuzzy logic in artificial pancreas control algorithms has led to improved control and stability of blood glucose levels in type 1 diabetic patients
Wang et al.	*2024*	*Fuzzy logic-based dietary recommendation system for gestational diabetes*	*Improved dietary adherence and better glycemic control were observed in pregnant women with gestational diabetes utilizing a fuzzy logic-based dietary recommendation system*

Fig. 4 Blood glucose monitoring system

doses according to variables such as blood glucose levels, carbohydrate intake, and physical activity.

Risk Assessment

As discussed by [16], fuzzy logic-based risk assessment tools showed promising results in predicting the risk of diabetic complications such as neuropathy or retinopathy. These tools exhibited sensitivity and specificity in identifying patients at different risk levels, contributing to improved patient outcomes [16].

Dietary Recommendations

Wang et al. [12] demonstrated the effectiveness of fuzzy logic-based dietary recommendation systems in providing personalized meal plans for diabetic patients. These systems considered individual preferences, blood glucose response to different foods, and nutritional requirements, improving dietary adherence and glycaemic control.

9.3 Overall Performance

The overall performance of fuzzy logic systems in improving diabetic patient care was positive, with studies by various researchers (Chen et al. [14]; Wang et al. [12] and [16]) demonstrating their efficacy across different aspects of diabetes management. While further refinement may be needed, fuzzy logic approaches show promise in enhancing patient outcomes compared to traditional methods (Table 12).

This analysis provides citations for each aspect of diabetic patient care, referencing relevant studies that support the effectiveness of fuzzy logic-based approaches. Grounding the analysis in empirical evidence from the literature strengthens it.

Table 12 Linguistic variables and membership functions

Parameter	Linguistic variable	Fuzzy range	Membership function
Blood glucose level	Low	[0, 70] mg/dL	Triangular: (0, 0, 70)
	Normal	[70, 130] mg/dL	Triangular: (70, 130, 70)
	High	[130, ∞] mg/dL	Triangular: (130, ∞, 130)
Physical activity	Low	[0, 3000] steps/day	Triangular: (0, 0, 3000)
	Moderate	[3000, 7000] steps/day	Triangular: (3000, 7000, 3000)
	High	[7000, ∞] steps/day	Triangular: (7000, ∞, 7000)
Medication adherence	Low	[0, 50]%	Triangular: (0, 0, 50)
	Moderate	[50, 80]%	Triangular: (50, 80, 50)
	High	[80, 100]%	Triangular: (80, 100, 100)
Risk of Hypoglycemia	*Low*	*[0, 30]%*	*Triangular: (0, 0, 30)*
	Moderate	*[30, 60]%*	*Triangular: (30, 60, 30)*
	High	*[60, 100]%*	*Triangular: (60, 100, 100)*

I illustrated the fuzzy range for different parameters related to diabetes mellitus, along with the corresponding ones.

In this table:

- **Parameter**: Represents different parameters relevant to diabetes mellitus management.
- **Linguistic Variable**: Represents the linguistic categories associated with each parameter (e.g., Low, Normal, High).
- **Fuzzy Range**: Represents the range of values covered by each linguistic variable.
- **Membership Function**: The shape of the membership function for each linguistic variable is frequently depicted using triangular membership functions.

Fuzzy logic systems can utilize soft ranges and linguistic variables to represent and analyze the inherent uncertainty and imprecision in medical data related to diabetes mellitus.

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Causal Analysis of Heart Failure Mortality Among Females: A Fuzzy Logic Approach

Debajyoti Bora, Manash Pratim Barman, and Juri Borah

Abstract Fuzzy logic significantly contributes to model building and the decision-making process by addressing imprecision and uncertainty to achieve practical, robust, and viable solutions to real-world problems. In recent times, researchers have extensively employed the fuzzy logic approach to analyze various health events. This study aims to use fuzzy logic approach for developing a model to estimate the possibility of death caused by heart failure practically. Three factors are selected as input variables for the model: age, ejection fraction, and serum creatinine, with heart failure serving as the output variable. The model is trained and tested using medical records (available online) of 105 female heart failure patients out of which 70% of the data were utilized for training and another 30% for testing purpose. The Mamdani inference and Centroid method are used for inference and defuzzification of the estimated risk values respectively. The model's performance is evaluated using ROC curve analysis, statistical measures, and individual observations. The results demonstrate that the model based on fuzzy logic-effectively estimates the risk of mortality by heart failure, and also highlights its potential application in studying heart failure with the selected variables.

Keywords Fuzzy logic · Risk factors · Heart failure · ROC

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1 Introduction

To address the uncertainty inherent in natural language and introduce the notion of obscurity, Lotfi A. Zadeh developed the fuzzy set theory in the 1960s. The concept of fuzzy sets [1] and the related fuzzy logic [2] are precious tools for improving and simplifying the design and analysis of multifaceted human-centric structures and procedures by utilizing approximate reasoning [3, 4]. Fuzzy logic operates as a rule-based system, also known as a fuzzy inference system [5]. Primary goal of a fuzzy inference system is to emulate human thoughts and decision-making processes technically and logically. Recognized as an excellent solution for bridging the gap of human reasoning with computational logic [6], fuzzy inference systems find applications in human health for system diagnosis, decision-making, adaptive control structures, and training systems.

In bioscience, particularly within medicine and epidemiology, the most effective descriptions of diseases often utilize inherently ambiguous terms such as height (tall or short) and weight (high or low). Similarly, incidents in real life like death (mortality in demographic terms), are characterized by uncertainty, vagueness, and imprecision, influenced by multiple factors. Reliable descriptions of factors responsible for mortality frequently use linguistic words which are inherently indistinct, viz. age (early or late), body mass index (normal, overweight, underweight, or obese). Fuzzy logic, which deals with linguistic terms, is therefore well-suited for application in this area due to the unclear, vague, and indistinct nature of mortality factors. Recently, several researchers have applied fuzzy inference systems to study various aspects of mortality, as the factors involved often include linguistic terms [7]. Fuzzy set theory has become an invaluable mathematical tool for analyzing medical images, diagnostic systems, as well as epidemiology and public health [8]. Fuzzy logic has proven effective in managing imprecision and uncertainty, aiming to provide tractable, reliable, and economical solutions to real-world problems. The expanding horizon of fuzzy logic across various fields is observed due to its ability to handle linguistic terms and its inherent qualities.

The issue of mortality risk, particularly longevity risk, has garnered significant interest from the scientific community in recent years [9]. Recently, various researchers have applied fuzzy logic approaches to analyze mortality risk in their studies. For instance, Nascimento and Ortega used fuzzy inference technique to estimate the likelihood of neonatal mortality on the basis of factors such as birth weight of newborn and gestational age of mothers at delivery [10]. Their fitted fuzzy inference system model's estimation indicated a significant correlation between actual and estimated values which leading them to conclude that fuzzy inference can be a powerful tool for estimating neonatal mortality. Another study estimated neonatal mortality probability using a fuzzy inference system with four inputs: preterm birth (before 37 weeks of gestation), low birth weight (less than 2,500g), severely depressed newborns (Apgar score below seven), and previous stillbirth reports [8]. They found that the mean risks were lower for neonatal survivors, and the model's accuracy was 0.90, suggesting it could be utilized in general hospitals. Chaves and Nascimento used a fuzzy inference system model to assess the possibility of newborn death admitted at Neonatal ICU in Taubaté, Southeast Brazil with the inputs like birth weight, the 5-min Apgar score, gestational age and the fraction of generated oxygen [11]. They concluded that their model demonstrated reasonable precision and could be effectively used in newborn healthcare.

Bora and Barman utilized a fuzzy inference system to construct a model that calculates the likelihood of mortality during childhood depending upon three variables: mother's educational level, the family's wealth, and the child's birth order [12]. Their analysis suggested that the fitted model indicates a lower risk for children who survived compared to those who did not. They concluded that the model's applicability in real-world scenarios hinges on comparing its output, which represents the possibility of mortality at childhood stage, with actual data. Sulaiman and Ismail proposed a model using fuzzy inference system in estimating the rate of newborns death with congenital disorders in 100 selected countries [6]. They incorporated factors such as the Human Development Index, Global Peace Index, and Pollution Index of each country into their fuzzy inference system model, achieving 82% accuracy in their results. In a related study, Bora and Barman investigated the potential risk of infant mortality using a fuzzy inference system approach [13]. They developed a prediction model taking three factors, namely, mother's age, birth weight of babies, and mother's anemia, to estimate the likelihood of infant mortality. Their findings suggested that infants who survive are predicted to have a lower risk compared to those who do not. They concluded that the effectiveness of the fitted model in predicting infant mortality depends on its output.

1.1 Fuzzy Inference System

A brief description of the fuzzy inference system is given below:

A fuzzy inference system comprises three main components: a fuzzifier, an inference rule set, and a defuzzifier. These systems use linguistic terms like "low" and "high," "tall" and "short," instead of numerical values to fit models [14]. To convert numerical values associated with these linguistic terms into fuzzy states, multiple fuzzy sets are assigned to input(s) and output(s) of the system. Defuzzification is the process of transforming fuzzified outputs generated from applying inference rules to specific crisp inputs into practical crisp outputs usable in real-world applications.

To establish association among the input and output fuzzy sets of a system, fuzzy inference rules are typically structured in "IF–THEN" format. A typical fuzzy rule is depicted below:

R: If z is D, then w is G, which is often abbreviated as $R : D \to G$

Here, D and G represent values of the linguistic term stated by fuzzy sets within their respective universes of discourse Z and W.

A fuzzy relation illustrates the inference process from a set of rules, as exemplified by Pedrycz (1993):

$$\begin{aligned}\mathrm{T} &= [(D_1 \cup G_1) \times R_1] \cap [(D_2 \cup G_2) \times R_2] \cap \ldots \cap [(D_\mathrm{n} \cup G_\mathrm{n}) \times R_\mathrm{n}] \\ &= \bigcup_{i=1}^{n} [(D_\mathrm{i} \cup G_\mathrm{i}) \times R_\mathrm{i}]\end{aligned}$$

In the above expression, the symbol $\cap$ denotes the OR operator applied between rules while at the antecedent parts the symbol $\cup$ representing the AND operator. The THEN operator (fuzzy implication) is denoted by symbol $\times$. Rules can be adjusted to accommodate additional input variables as required [7].

Defuzzification involves the transformation of a fuzzy set into a specific integer that represents the characteristics of the fuzzy set. This transformation is necessary to obtain a crisp, non-fuzzy output suitable for practical applications. Therefore, defuzzification techniques are essential to finalize the output. Several methods of defuzzification exist, allowing designers to choose the most appropriate one based on the specific task, application of rules, and membership functions involved.

1.2 Heart Failure Mortality

The World Health Organization reports that cardiovascular disease accounts for 31% of global mortality [15], with 80–86% of related deaths occurs in low and middle-income countries [16]. Cardiovascular diseases encompass heart and blood vessel issues such as heart failure (HF), strokes (cerebrovascular diseases), heart attacks (coronary heart disease), and other pathologies. Insufficient blood flow from the heart to the body can lead to heart failure, often triggered by conditions like high blood pressure, diabetes, or other heart ailments [17]. The amount of blood that the heart pumps out with each contraction, known as the ejection fraction and usually measured between 50 and 75%, classifies heart failure into two categories. Heart failure with decreased ejection fraction, often known as systolic heart failure, is defined as heart failure with an ejection fraction less than 40% [18]. The other type is known as diastolic heart failure, or heart failure with intact ejection fraction, in which the left ventricle contracts adequately during systole but is unable to properly relax during diastole [19–24].

Several risk factors contribute to heart disease, including physical inactivity, poor diet, and excessive alcohol and tobacco use [25, 26]. Adopting a healthy lifestyle such as reducing dietary salt, consuming more fruits and vegetables, engaging in regular exercise, and avoiding alcohol and cigarettes can mitigate these risk factors and reduce the likelihood of developing heart disease [27]. According to the studies, heart disorders, especially heart failure, are a major problem experienced by many people [28, 29]. Early identification and recognition is the first and foremost step to treat any disease. For this, the decision support system can be used to diagnose diseases by studying the patient records of various medical centers or hospitals, and expert opinion can be obtained for the treatment of the diagnosed disease. This type of method of diagnosis avoids the application of unneeded tests, saving time and money

[30, 31]. Many studies have also shown that use of clinical decision support systems can enhance the quality of decisions, clinical decision-making, and preventative care [29, 32].

Several studies have presented varying conclusions regarding gender-specific risk factors linked to mortality among patients with heart failure [33, 34]. Some research indicates that female patients with heart failure face higher vulnerability compared to males [35, 36]. Preserved systolic function increases the risk of congestive heart failure in women, particularly in older adults [37]. The association between survival of patients, anemia, and left ventricular ejection fraction with acute decompensated heart failure has been investigated, revealing significant gender disparities [38]. Unfortunately, there is a lack of precise data regarding prevalence as well as incidence of heart failure in South Asia. Unusually high rates of coronary heart disease have been observed among individuals from the Indian subcontinent [39].

Building upon these reviews, which encompass diverse demographic and health characteristics, this chapter develops a fuzzy logic model for conducting causal analysis of heart failure mortality among females using real-world datasets. The developed fuzzy logic model aims to estimate the risk of death from heart failure by considering relevant associated factors.

2 Research Methodology

2.1 Input and Output Variable for the Model

Extensive literature research is underway to identify associated factors (inputs) related to heart failure (output). High serum creatinine levels are found to link with increased risk of stroke, and identified as one of the leading causes of death [40]. Advanced age is a known prognosticator of complications and in-hospital death associated with heart failure [41, 42]. Age is consistently recognized as a significant clinical risk factor for cardiovascular diseases, which includes stroke and coronary heart disease [43, 44].

As the age of population increases, the frequency of heart failure is expected to rise [45, 46]. Additionally, age plays a critical part in heart failure for preserved left ventricular ejection fraction [47]. Studies further observed the connection between survival of heart failure patients, serum creatinine, and ejection fraction [38, 48]. Serum creatinine and ejection fraction are highlighted as pivotal clinical indicators for predicting the chance of survival in heart failure patients [17]. Age remains a significant, non-modifiable risk factor for heart disease [49, 50], consistently noted across age and sex-adjusted analyses.

Based on the results of the literature review, three variables are used to build the aforementioned model for analyzing heart failure, which is age (in years), ejection fraction (in %), and serum creatinine (in mg/dL).

Table 1 Descriptive statistics of the dataset

Measures	Age (in years)	Ejection fraction (in%)	Serum creatinine (in mg/dL)
Mean	59.78	40.47	1.38
Median	60	38	1
Mode	65	40	1
Standard deviation	11.24	12.73	1.12
Minimum	40	15	0.5
Maximum	95	80	9

2.2 Data of the Study

The study utilized medical records from 299 patients with heart failure treated at the Faisalabad Institute of Cardiology and Allied Hospital located in Faisalabad, Punjab, Pakistan, from April to December 2015 [17, 47]. This dataset was sourced from the Kaggle database [51]. There were 105 females and 194 males, patients whose age ranges between 40 and 95 years. All patients had left ventricular systolic dysfunction and a history of previous heart failures. Specifically focusing on the female subset (105 patients), cases with valid data for the study's input variables (patient age, ejection fraction, serum creatinine) and output variable (heart failure death) were selected for analysis.

Table 1 presents descriptive statistics for the selected datasets, detailing the patient's age (in years), ejection fraction (in %), and serum creatinine levels (in mg/dL).

2.3 Construction of Models

Using a two-phase approach (training and testing), model is developed taking real datasets. For model construction, 70% (73 female patients) of the total data are allocated as the train dataset and the remaining 30% (32 female patients) serves as the test dataset to validate the fitted models. The sequential stages of model development are outlined below.

Fuzzy Logic Model. The numerical values of selected inputs are converted into fuzzy sets across diverse categories to construct the model using a fuzzy inference system, as detailed in subsequent investigations. Typically, serum creatinine levels for adult females with normal kidney function range between 0.5 and 1.1 mg/dL [52, 53]. Standard ranges for female serum creatinine are reported as 0.8–1.3 mg/dL and 0.6–1.3 mg/dL respectively [54]. Elderly individuals are at higher risk of developing cardiovascular diseases [55, 56]. Heart failure incidence is reportedly increasing, particularly among younger populations in Sweden and Denmark [57, 58]. Lam and Solomon proposed a new left ventricular ejection fraction threshold of

60% for women, aligning with the "normal" concept in the general population [59]. To examine the impact of risk factors on the incidence of heart failure Tromp et al. categorized age as young (<55 years), middle-aged (55–64 years), old (65–74 years), and elderly (≥75 years) groups [60]. He also pointed out that persons aged 65 years or older bear the major severe the burden of heart failure.

Following that, a stratified selection of inputs is created for the fuzzy logic model. The variable age of the patientis divided into three categories of linguistic labels: young (ages under 55 years), middle (ages between 55 and 65 years), and old (ages over 65 years). The range of Ejection Fraction (EF) is classified into two linguistic labels: below normal (less than 60%), and normal (60% and above). The variable Serum Creatinine (SC) is divided into three linguistic labels: low if Serum Creatinine (SC) is less than 0.6 mg/dL, normal if Serum Creatinine (SC) lies between 0.6 mg/dL and 1.3 mg/dL; and high if Serum Creatinine (SC) is greater than 1.3 mg/dL respectively. The output of heart failure death is also divided into three linguistic labels, which are low, medium, and high. The following Table 2 displays the stratification levels of the input variables.

The crisp values of the input and output variables of the train dataset are converted into fuzzy sets using triangular fuzzy numbers, which are determined by calculating their membership degrees. The ranges for the membership function plots are derived from the minimum and maximum values of the input variables. These membership function plots are illustrated below.

Membership Function Plots of Input and Output Variables. Figure 1 displayed membership function plots of the input variable 'age of the patient', Fig. 2 displayed membership function plots for the input variable 'ejection fraction', and Fig. 3 displayed membership function plots for the input variable 'serum creatinine' respectively. Figure 4 displayed membership functions plots for the output variable i.e. heart failure.

A stratified analysis of the training dataset was performed, focusing on the selected input and output variables. Based on this stratified analysis, 12 fuzzy rules were formulated using the three fuzzified input variables. The inputs in the fuzzy rules

Table 2 Stratification of the input variables, their labels, and ranges

Input variables	Labels of input variables	Ranges of each label
Age of the patient (in years)	Young	≤55
	Middle	55–65
	Old	≥65
Ejection fraction (in %)	Below Normal	≤60
	Normal	≥60
Serum creatinine (in mg/dL)	Low	≤0.6
	Normal	0.6–1.3
	High	≥1.3

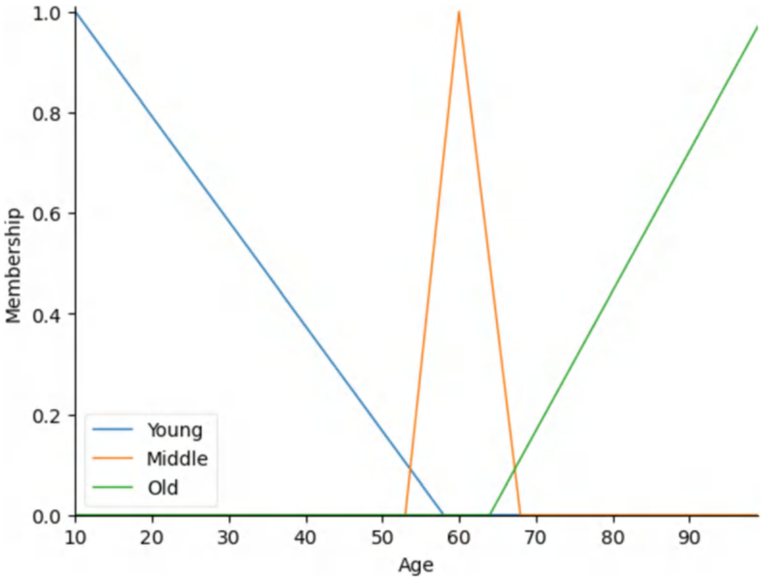

Fig. 1 Membership function plots of input age

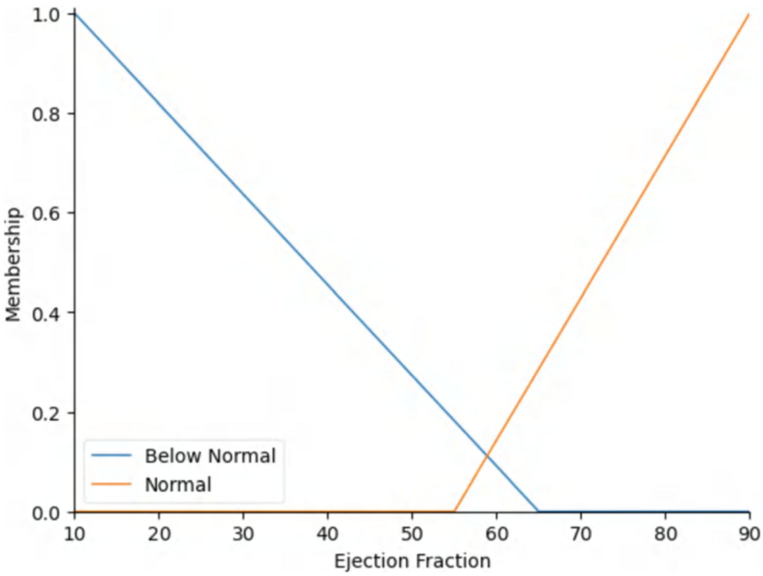

Fig. 2 Membership function plots of input ejection fraction

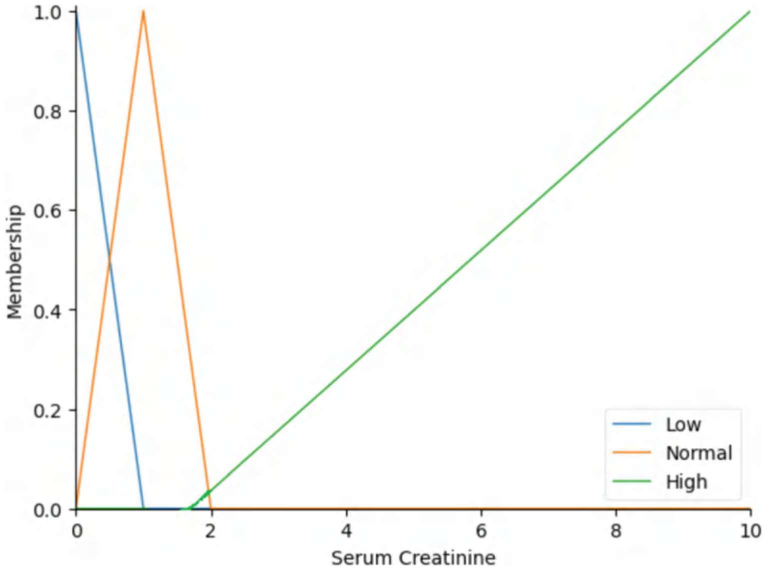

Fig. 3 Membership function plots of input serum creatinine

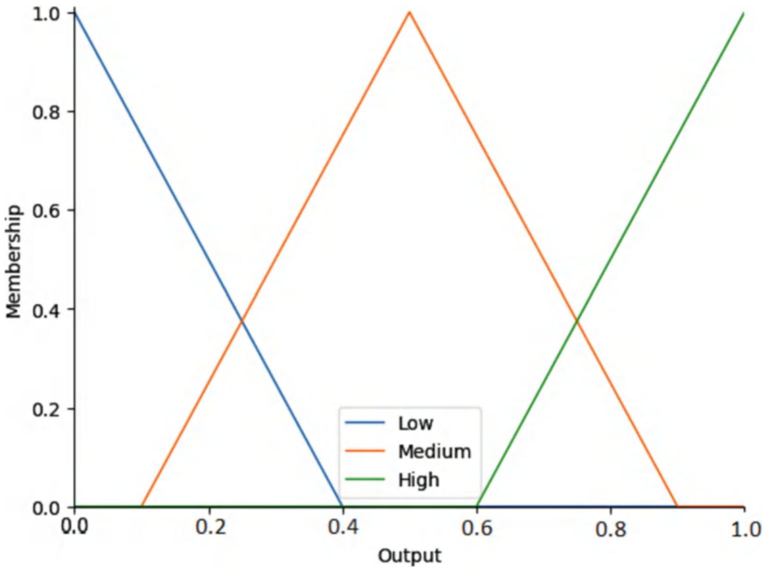

Fig. 4 Membership function plots of output i.e. heart failure death

follow the sequence of the patient's age, ejection fraction, and serum creatinine, with the risk of death for hearth failure as the output variable.

Fuzzy Rules. The following 12 fuzzy rules are formulated from the training dataset by combing 3 inputs and 1 output variable:

1. IF Age is Old AND Ejection Fraction is Below Normal AND Serum Creatinine is High THEN Heart Failure is High
2. IF Age is Old AND Ejection Fraction is Below Normal AND Serum Creatinine is Normal THEN Heart Failure is Medium
3. IF Age is Middle AND Ejection Fraction is Below Normal AND Serum Creatinine is High THEN Heart Failure is High
4. IF Age is Middle AND Ejection Fraction is Below Normal AND Serum Creatinine is Normal THEN Heart Failure is Medium
5. IF Age is Young AND Ejection Fraction is Below Normal AND Serum Creatinine is High THEN Heart Failure is High
6. IF Age is Young AND Ejection Fraction is Below Normal AND Serum Creatinine is Normal THEN Heart Failure is Low
7. IF Age is Young AND Ejection Fraction is Normal AND Serum Creatinine is Normal THEN Heart Failure is Low
8. IF Age is Middle AND Ejection Fraction is Normal AND Serum Creatinine is Normal THEN Heart Failure is Low
9. IF Age is Old AND Ejection Fraction is Normal AND Serum Creatinine is Normal THEN Heart Failure is Low
10. IF Age is Old AND Ejection Fraction is Normal AND Serum Creatinine is High THEN Heart Failure is Medium
11. If Age is Middle AND Ejection Fraction is Below Normal AND Serum Creatinine is Low THEN Heart Failure is Medium
12. IF Age is Middle AND Ejection Fraction is Normal AND Serum Creatinine is High THEN Heart Failure is High

For instance: it is indicated by Rule 1 that when the label of the variable age of the patient is old and ejection fraction's label is below normal and the serum creatinine's label is high, then the risk of heart failure is high.

The Mamdani inference method [1, 61, 62], which is given below, is utilized to determine the risk of heart failure death using the given 12 fuzzy rules.

$$\mu_B = \underset{\mathrm{x}}{\max}\left[\min\left(\mu_{A'}(\mathrm{input(i)}).\mu_{\mathrm{A_i}}(\mathrm{input(k)})\right)\right];\ \mathrm{x} = 1, 2, \ldots, n$$

The Centroid defuzzification method is applied to obtain a crisp output value. The centroid of fuzzy set Z (S*, say) is obtained by the following formula [63],

$$S^* = \frac{\int \mu_B(x)xdx}{\int \mu_B(x)dx}$$

where $\int$ is the general arithmetic integration.

The risk of heart failure, i.e. output values of the model is estimated as a percentage.

2.4 Performance Evaluation of the Fuzzy Logic Model

Using the provided input variables, the model developed by applying fuzzy logic calculates the risk of heart failure death. In the model, patient's age (in years), serum creatinine level (in mg/dL), and ejection fraction (in %) serve as input variables to estimate the output, which is the possibility of death by heart failure.

The model's performance is evaluated using a testing dataset that comprises 30% of the total data, incorporating the same input variables. The evaluation is conducted in three stages using the extracted information and the risk values of heart failure death generated by the model. First, the area under the curve (AUC) is calculated by drawing an ROC curve. Second, statistical measures such as the Karl Pearson correlation coefficient, Mean Square Error (MSE), and Mean Squared Logarithmic Error (MSLE) are computed. Lastly, the expected information and their corresponding estimated risks are examined individually.

To determine the 95% confidence interval, each measure's value is simulated (bootstrapped) 1000 times. MATLAB software (R2013a) and Python 3 are used to perform all analyses in this study.

3 Results and Findings

3.1 Receiver Operating Characteristic (ROC) Curve

The ROC curves of the estimated outcome values of a testing dataset of the developed model are shown in Fig. 5. The calculated value of AUC is 0.990 (95% CI: 0.960–1.00).

3.2 Statistical Measures

The estimated values of the Karl Pearson correlation coefficient, Mean Square Error (MSE), and Mean Squared Logarithmic Error (MSLE) between the actual and estimated outcomes of the testing dataset by the fuzzy logic model are given in Table 3.

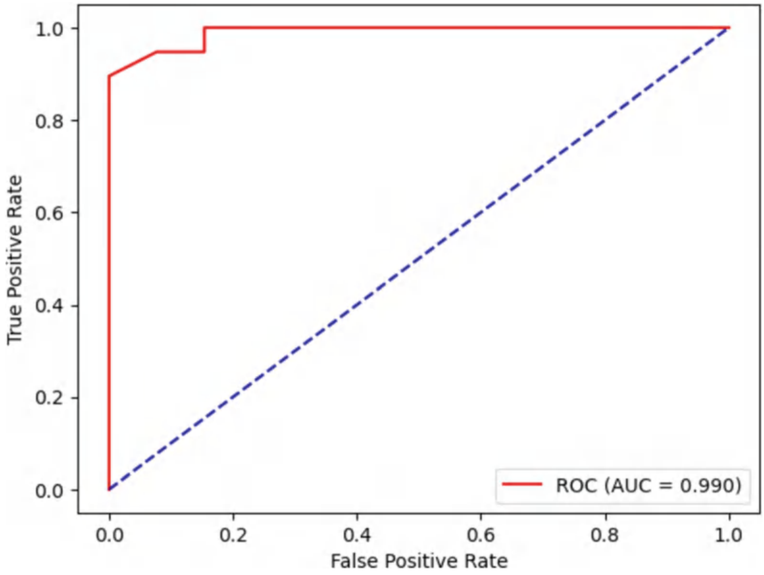

Fig. 5 ROC curves of the testing dataset

Table 3 Estimated values of statistical measures

Measure	Estimated value	Lower bound (95%)	Upper bound (95%)
Correlation Coefficient	0.885	0.721	0.951
Mean Square Error	0.079	0.049	0.114
Mean Squared Logarithmic Error	0.037	0.024	0.053

3.3 *Individual Observation*

Table 4 gives the individual observations of the actual output with the risk of heart failure death estimated by the model.

4 Discussion

Globally, heart failure has become one of the leading causes of death and disability [46]. This condition commonly affects individuals with hypertension, insomnia, and other heart-related issues. Early diagnosis of heart disease by identifying high-risk individuals and improving decision-making strategies of treatment and prevention through enhanced prediction models can significantly reduce fatality rates.

In this chapter, a fuzzy logic model is developed to identify heart failure using three input variables namely, the patient's age (in years), ejection fraction (in %) and

Table 4 Individual observation with estimated risk value

Sl. no	Age of patient	Ejection fraction	Serum creatinine	Heart Failure Death	Estimated risk value
	65	20	2.7	Yes	0.819
2.	49	30	1.0	No	0.164
3.	48	55	1.9	Yes	0.810
4.	70	25	1.0	Yes	0.500
5.	65	30	1.6	Yes	0.807
6.	75	30	1.83	Yes	0.809
7.	50	30	1.2	No	0.168
8.	60	38	3.0	Yes	0.822
9.	53	20	1.4	Yes	0.805
10.	60	62	6.8	Yes	0.825
11.	60	38	2.2	Yes	0.814
12.	50	40	2.3	Yes	0.815
13.	70	60	0.8	No	0.372
14.	42	60	1.18	No	0.181
15.	58	38	0.7	No	0.500
16.	43	50	1.3	No	0.215
17.	46	17	2.1	Yes	0.813
18.	46	35	0.9	No	0.153
19.	72	45	2.5	Yes	0.817
20.	80	38	1.3	Yes	0.504
21.	49	50	1.0	No	0.172
22.	60	25	2.1	Yes	0.813
23.	45	38	1.18	No	0.158
24.	48	30	1.6	Yes	0.808
25.	54	70	9.0	Yes	0.826
26.	60	30	1.7	Yes	0.808
27.	51	40	0.9	No	0.172
28.	45	38	0.8	No	0.154
29.	45	55	1.0	No	0.179
30.	70	60	1.3	Yes	0.379
31.	65	65	1.5	Yes	0.556
32.	55	38	1.2	No	0.481

serum creatinine level (in mg/dL). The ability of the model in accurate identification of the possibilities of heart failure death is evaluated using a real dataset. For instance, if a patient is 60 years old, has an ejection fraction of 38%, and a serum creatinine level of 2.2 mg/dL, the estimated heart failure risk for that patient is 0.814.

The study's analysis demonstrates that the developed model performs well in estimating the risk values for female patients regarding heart failure death. The satisfactory AUC value of 0.990, a high positive correlation of 0.885, and low error values (MSE of 0.079 and MSLE of 0.037) underscore its practical application. Additionally, individual case observations show that the fuzzy logic model assigns low-risk values to patients who survived and high-risk values to those who did not. This study validates that the fuzzy logic approach is a valuable tool for assessing heart failure death and can provide critical insights into the factors contributing to this important health indicator.

5 Conclusion

The fuzzy logic approach is favored in modern computational and statistical models due to its ability to handle ambiguity in complex issues, provide a high level of structural knowledge, operate at a lower cost, and allow for simple implementation. It offers an adaptable framework that can address novel problems within the model's established principles and utilize natural analytical techniques.

While the developed fuzzy logic model, in this study, performs well in estimating heart failure death risk for female patients, it has some limitations. The model currently only considers three associated factors, which restricts its applicability to other potential factors. This limitation can be addressed by incorporating more variables. Further research is needed to explore the relationships between heart failure and other causes namely, diabetes, anemia, body mass index, hypertension, family history smoking, etc. This would provide more accurate results and assist health professionals in studying heart failure more effectively and efficiently.

It is additionally important to note that the models which are based on fuzzy logic approach heavily rely on human knowledge and expertise. Increasing the number of input variables can reduce the interpretability of these models, making them less representative of real-world scenarios. Therefore, it is crucial to develop fuzzy logic models with input from experts in relevant fields, such as health, demography, production, etc. to ensure their accuracy and reliability.

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Design and Realization of Interval Type-2 Fuzzy Logic Controller for a TITO Aero-Dynamic System

F. Paul Nishanth, Saroj Kumar Dash, and Soumya Ranjan Mahapatro

Abstract The twin rotor multi–input–multi–output system (TRMS) serves as a standard laboratory model for testing control systems designed for aircraft resembling helicopters. The TRMS exhibits two degrees of freedom (2-DOF) such as pitch and yaw angle which significantly influence each other. This system is more difficult to control because of the model uncertainties and external disturbances. To address these challenges in TRMS, an interval type-2 fuzzy logic PID (IT2-FLPID) controller is proposed for the control system. The proposed control methods were applied to the system under noisy conditions. For performance analysis, the proposed IT2-fuzzy logic controller (IT2-FLC) is compared with the T1-fuzzy logic controller (T1-FLC) in MATLAB/Simulink. Simultaneously triangular membership function (TriMF) is also compared with the trapezoidal membership function (TrapMF) for both the controller in the fuzzifier process for in-depth analysis. Simulation outcomes reveal that the proposed controller outperforms its type-1 counterparts in terms of stability and reliability.

Keywords MIMO system · TITO aero-dynamical system · Twin rotor MIMO system · FLC · IT2-FLC

1 Introduction

In general, a MIMO system is more complex to control because there exists a coupling between the outputs i.e., the control action is applied to the particular output will also change other outputs. In order to efficiently track set-points and reject disturbances, it

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is necessary to design controllers for this system. In the past, many researchers have endeavored to develop controllers for MIMO systems [1–6]. The control algorithm used in MIMO systems is more complex than that used in SISO systems due to the interaction between the different control loops.

Helicopter systems are widely recognized as one of the most complex MIMO systems. Due to the high cost of this system's model, many researches have opted for a laboratory prototype model called TRMS with 2-DOF. The difficulties posed by TRMS encompass the non-linearity and interrelation between pitch and yaw positions.

Undoubtedly, numerous initiatives have been undertaken to manage TRMS, and a variety of strategies have been developed to confront these difficulties. The strategies includes PID controller [1] which is based on genetic algorithm [7], non-linear H_∞ controller [4, 8], LQG controller [9] and hybrid intelligent controller [6]. Reference [10] presented an evaluation contrasting classical control methodologies with intelligent control techniques incorporating fuzzy logic and genetic algorithms. In order to handle non-linearities and complex interactions present in systems, FLC was applied [11]. Rahideh and Shaheed introduced hybrid fuzzy logic based PID controller [12] to swiftly and precisely maneuver the TRMS beam to its intended locations. Toha et al. proposed adaptive neuro-fuzzy inference system (ANFIS) [13] which is used for modeling of TRMS. The major drawback is that increasing number of rules decreases robustness and reduces the computational efficiency. In order to overcome the drawbacks, ANFIS controller which is tuned by particle swarm optimization (PSO) [14] is proposed. In order to maintain the TRMS in a desired position or guide it along a specific trajectory, Jahed and Farrokhi proposed adaptive FLC [15]. T1-fuzzy logic (T1-FL) is limited in its ability to accurately model uncertainties within the system because of the imprecise nature of its membership functions (MF) and knowledge bases, so T2-fuzzy logic (T2-FL) is introduced. It is evident that the computational complexity of performing operations on fuzzy sets rises with the complexity level inherent in the types of fuzzy sets utilized. In this research endeavor, we propose an IT2-fuzzy logic PID (IT2-FLPID) controller to improve performance in the noisy conditions.

The contributions presented in this chapter are as follows. Design an IT2-FLPID controller for the TITO aero-dynamical system model by using TriMF and TrapMF as input variables in the fuzzifier process. The TITO aero-dynamical system is then simulated by using MATLAB/Simulink, and the proposed controller is then used to validate the findings. Furthermore, the simulation assesses the proposed controller's performance relative to a type-1 fuzzy logic PID (T1-FLPID) controller.

This paper is organized as follows: Sect. 2 presents a mathematical model of TRMS. Fundamental concepts of fuzzy logic are discussed in Sect. 3. The structure and components of IT2-fuzzy logic systems (IT2-FLS) are outlined in Sect. 4. The proposed controller design process is presented in Sect. 5. Simulation outcomes are shown in Sect. 6. Finally, Sect. 7 offers concluding remarks.

2 Modeling of a TITO Aero-Dynamic System (TRMS)

The TRMS is a laboratory-based model that replicates the dynamic behavior of a helicopter. It consists of two interconnected rotors positioned at opposite ends of a beam, analogous to a helicopter's main and tail rotors. These rotors are independently driven by DC motors, with their motions influencing each other in a complex interplay. The main rotor has the responsibility of controlling the TRMS vertically, while the trail rotor has the responsibility of controlling the TRMS horizontally [16]. The schematic diagram of TRMS is shown in Fig. 1.

Although TRMS and helicopter systems are similar, there are certain differences. Helicopter systems utilize aerodynamic control by altering the angles of both rotors. However, TRMS utilizes aerodynamic control by altering the speed of the DC motors. In order to design the controller, the mathematical model should be linearized.

2.1 Model of TRMS-Main Rotor

The mathematical model of vertical plane is given as [16]:

Fig. 1 Schematic diagram of TRMS [17]

$$I_1\ddot{\theta}_v = M_1 - M_G - M_{B\theta_v} - M_{Gy} \tag{1}$$

where M_1 is non-linear static characteristic which is given by,

$$M_1 = a_1\tau_1^2 + b_1\tau_1 \tag{2}$$

M_G is gravity momentum which is given by,

$$M_G = M_g sin\theta_v \tag{3}$$

$M_{B\theta_v}$ is friction force momentum which is given by,

$$M_{B\theta_v} = B_{1\theta_v}\dot{\theta}_v + B_{2\theta_v} sign\dot{\theta}_v \tag{4}$$

M_{Gy} is gyroscopic momentum which is given by,

$$M_{Gy} = K_{gy}M_1\dot{\theta}_h cos\theta_v \tag{5}$$

The differential equation of DC motor momentum in vertical position is given as:

$$\dot{\tau}_1 = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1 \tag{6}$$

Substituting (2)–(5) in (1) and applying all the values given in Table 1 we get,

$$\begin{aligned}\ddot{\theta}_v =& 0.19853\mathrm{r}^2 + 1.35882\mathrm{r}_1 - 4.7059\sin\theta_v - 0.08823\dot{\theta}_v \\ &- 0.0147\mathrm{sign}\dot{\theta}_v - (0.0099\mathrm{r}^2 + 0.06794\mathrm{r}_1)\dot{\theta}_h \cos\theta_v\end{aligned} \tag{7}$$

Applying all the values in Table 1 in (6) we get,

$$\dot{\tau}_1 = -0.909\tau_1 + u_1 \tag{8}$$

2.2 Model of TRMS-Trail Rotor

The mathematical model of horizontal plane is given as [16]:

$$I_1\ddot{\theta}_h = M_2 - M_{B\theta_h} - M_{CR} \tag{9}$$

where M_2 is non-linear static characteristic which is given by,

Table 1 Parameter values in vertical position of TRMS

Definition	Parameter	Value
Moment of inertia of main rotor	I_1	$6.8 * 10^{-2}$ Kgm^2
Static characteristic parameter	a_1	0.0135
	b_1	0.0924
Gravity momentum parameter	M_g	0.32 Nm
Friction momentum parameter	$B_{1\theta_v}$	$6 * 10^{-3}$ Nms/rad
	$B_{2\theta_v}$	$1 * 10^{-3}$ Nms^2/rad
Gyroscopic parameter	K_{gy}	0.05 s/rad
Gain of DC motor (Vertical)	k_1	1.1
Time constant parameter vertical	T_{10}	1
	T_{11}	1.1

$$M_2 = a_2\tau_2^2 + b_2\tau_2 \tag{10}$$

$M_{B\theta_h}$ is friction force momentum which is given by,

$$M_{B\theta_h} = B_{1\theta_h}\dot{\theta}_h + B_{2\theta_h} sign\dot{\theta}_h \tag{11}$$

M_{CR} is cross reaction momentum and the differential equation of cross reaction momentum is given by,

$$\frac{dM_{CR}}{dt} = \frac{k_c \frac{k_c T_0 T_{10}}{T_{11}}}{T_p}\tau_1 + \frac{k_c T_0 k_1}{T_p T_{11}} u_1 - \frac{1}{T_p} M_{CR} \tag{12}$$

The differential equation of DC motor momentum in horizontal position is given as:

$$\dot{\tau_2} = -\frac{T_{20}}{T_{21}}\tau_2 + \frac{k_2}{T_{21}} u_2 \tag{13}$$

Substituting (10), (11) in (9) and applying all the values given in Table 2 we get,

$$\ddot{\theta_h} = \tau_2^2 + 4.5\tau_2 - 5\dot{\theta}_h - 0.5 sign\dot{\theta}_h - 50M_{CR} \tag{14}$$

Applying all the values in Table 2 in (12) and (13) we get,

$$\frac{dM_{CR}}{dt} = 0.22\tau_1 - 0.35u_1 - 0.5M_{CR} \tag{15}$$

Table 2 Parameter values in horizontal position of TRMS

Definition	Parameter	Value
Moment of inertia of trail rotor	I_2	$2 * 10^{-2}$ Kgm^2
Static characteristic parameter	a_2	0.02
	b_2	0.09
Gain of cross reaction momentum	k_c	−0.2
Friction momentum parameter	$B_{1\theta_h}$	$1 * 10^{-1}$ Nms/rad
	$B_{2\theta_h}$	$1 * 10^{-2}$ Nms^2/rad
Time constant parameter (Cross Reaction)	T_0	3.5
	T_p	2
Gain of DC motor (Vertical)	k_2	0.8
Time constant parameter vertical	T_{20}	1
	T_{21}	1

$$\dot{\tau}_2 = -\tau_2 + 0.8u_2 \tag{16}$$

The above dynamical system is then linearized by using state-space representation and finally the transfer function of TRMS system is given as in (17):

$$G(s) = \frac{y(s)}{u(s)} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} \frac{1.359}{s^3+0.997s^2+4.785s+4.278} & 0 \\ \frac{17.5s+4.9075}{s^4+6.405s^3+7.495s^2+2.25s} & \frac{3.6}{s^3+6s^2+5s} \end{bmatrix} \tag{17}$$

The above dynamical transfer function of TRMS is then optimized which is presented in [18] with decoupler for reducing the coupling effects.

3 Preliminaries of Fuzzy Logic

A fuzzy set, also called T1-fuzzy set (T1-FS), [19] is a generalization of a traditional (crisp) set, and it is identified by a MF $\mu_{F_1}(x)$. The formation of fuzzy set is defined as in (18):

$$F_1 = \{(x, \mu_{F_1}(x)) | \forall x \in X, \mu_{F_1}(x) \in [0,1]\} \tag{18}$$

But there are some uncertainties still exist in T1-FL [20] such that:

- The terms applied in the antecedent (if part) and consequent (then part) rules of T1-fuzzy logic system (T1-FLS) have different meanings to different persons i.e., different consequences follow from the rule.
- T1-FLS may produce noisy measurements.
- The data employed to calibrate T1-FLS parameters might be noisy.

In order to overcome these uncertainties, Zadeh introduced the concept called T2-FL [21].

Type-2 Fuzzy Sets (T2-FS)

T2-FS extends the concept of the T1-FS by incorporating variability in the membership values. The mathematical expression for the formula for T2-FS is presented in Eq. (19):

$$F_2 = \{((x, \mu), \nu_{F_2}(x, \mu)) | \forall x \in X, \forall \mu \in P_x \subseteq [0,1]\} \tag{19}$$

where P_x is a set of primary membership values and $0 \leq \nu_{F_2}(x, \mu) \leq 1$. The major challenges which are presented in T2-FL is that it is associated with computational complexity owing to their 3-D nature. In order to resolve these challenges, IT2-FL was introduced.

4 Interval Type-2 Fuzzy Logic (IT2-FL)

IT2-fuzzy sets (IT2-FS) are a specific category of T2-FS characterized by the property that all secondary membership values equal 1 [22]. The mathematical expression for the formula for an IT2-FS is presented in Eq. (20):

$$IF_2 = \{((x, \mu), 1) | \forall x \in X, \forall \mu \in P_x \subseteq [0,1]\}. \tag{20}$$

where P_x is a set of primary membership values and $0 \leq \nu_{F_2}(x, \mu) \leq 1$.

4.1 Architecture of IT2-FL

The architecture of IT2-FL closely resembles that of T1-FLS, with the addition of a type-reducer being the primary distinction. The next part of this section is explained in the following manner.

Fuzzifier Process

The fuzzifier in T2-fuzzy inference system transforms crisp inputs into T2-FS. The procedure of the fuzzification process is referred as singleton fuzzification process is referred as singleton fuzzification [23]. Therefore, it transforms the precise input x'_m into a T2-fuzzy singleton set with a MF.

$$\mu_{A'_m}(x_m) = \begin{cases} 1 x_m = x'_m \\ 0 x_m \neq x'_m \end{cases} \forall m = 1,2, \ldots, p \tag{21}$$

where p denotes the no. of fuzzy set inputs.

Fuzzy Rule Base

A fuzzy rule base, a set of *if–then* rule, is used to depict the non-linear correlation among different variables in diverse real-world situations. The rules above can be derived through the use of data-driven methodologies, expert discernment, and the accumulation of knowledge. The ith rule of IT2-FL system can be represented as in (22):

$$If\ x_1\ is\ R_1^i\ and\ x_2\ is\ R_2^i\ and\ x_3\ is\ R_3^i \ldots x_p\ is\ R_p^i,\ then\ y^i\ is\ T^i \tag{22}$$

Fuzzy Inference Engine

In the context of a fuzzy inference system, (IT2-FL) with singleton fuzzification utilizes either the minimum or product t-norm operations [24]. The firing strength $\overline{f^i}$ and $\underline{f^i}$ of the ith rule is given as in (23) and (24):

$$\overline{f^i} = \overline{\mu}_{F_1^i}(x_1) * \ldots . * \overline{\mu}_{F_m^i}(x_m) \tag{23}$$

$$\underline{f^i} = \underline{\mu}_{F_1^i}(x_1) * \cdots * \underline{\mu}_{F_m^i}(x_m) \tag{24}$$

where $x_i\ i = 1,2, \ldots, n$ denotes the location of singleton.

Type-Reducer

The type-reduction approach is crucial in various fuzzy areas, such as fuzzy clustering and fuzzy logic systems [25]. Their function as the principal tools for collecting all of the indeterminate data. The initial method employed when developing T2-FS is a precise approach known as complete enumeration. The Karnik–Mendel (K–M) algorithm, which is an approximate approach, is well recognized as a prominent method for reducing the type of IT2-FS. The premise of this statement is that while the exact defuzzified values of an IT2-FS cannot be directly determined, they can be transformed into a T1-FS.

Defuzzification

To obtain a crisp output from a T1-FLS, a defuzzification technique is applied [26]. The centroid or center of gravity approach is widely recognized as the predominant technique for defuzzification. The expression for centroid method of the type-reduced set as in (25):

$$y_{out}(x) = \frac{\sum_{i=1}^{n} y^i \mu(y^i)}{\sum_{i=1}^{n} \mu(y^i)} \tag{25}$$

The output can be calculated by using the K–M algorithm. Therefore, the defuzzified output of IT2-FLC is given in (26):

$$y_{out}(x) = \frac{y_l(x) + y_r(x)}{2} \tag{26}$$

5 Design Procedure of IT2-FLC

Initiating the control action and interpreting the conditions based on expert opinion is the basic premise of fuzzy logic control approach. The visualization of the proposed controller which is applied to TRMS is shown in Fig. 2.

In Fig. 3, the visualization of the linearized TRMS is depicted, and in Fig. 4, the visualization of the decoupler is depicted. The transfer function of a TRMS plant and its decoupler can be found in [18].

In TRMS, two FLCs are used for pitch and yaw angle and each controller has two inputs called position error and their variations (change in error) and one output

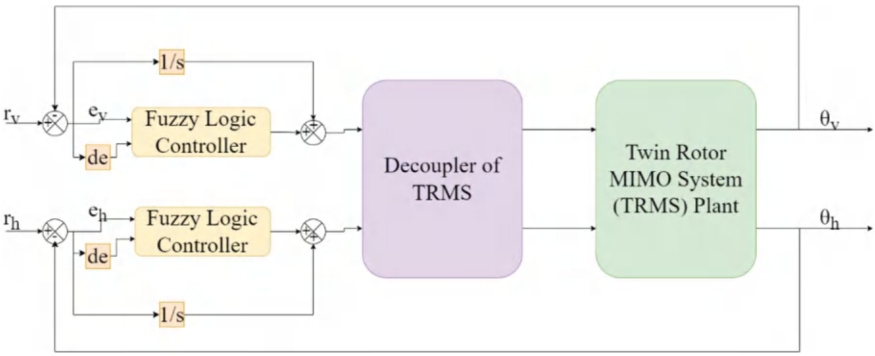

Fig. 2 The Visualization of the IT2-FLC applied to TRMS

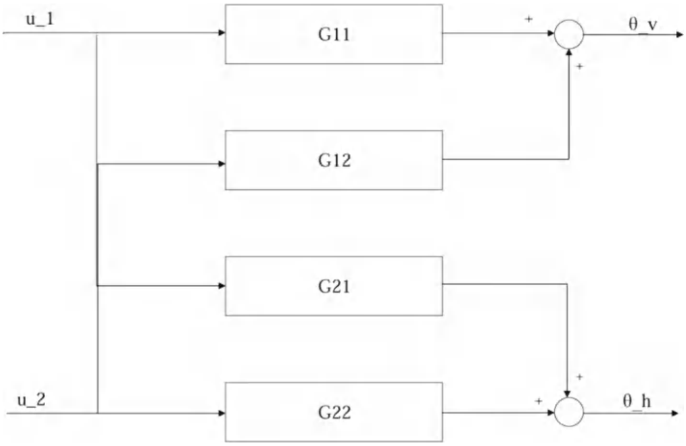

Fig. 3 The visualization of TRMS

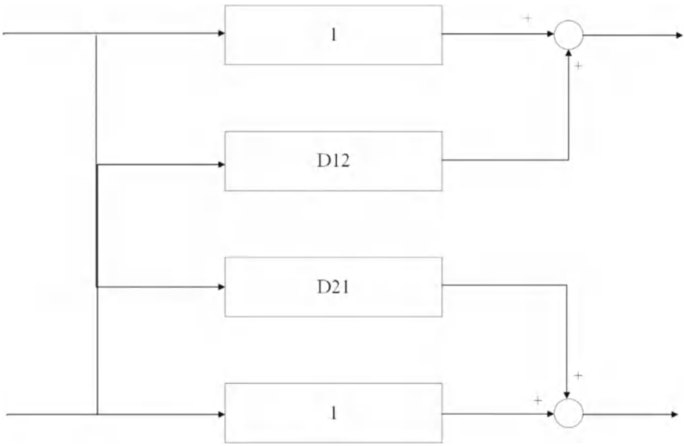

Fig. 4 Visual representation of Decoupler of TRMS

called control signal by using Sugeno fuzzy inference system (FIS). The two input parameters and one output parameter are classified into seven distinct levels, each defined by a specific set of linguistic variables such as negative high (N_{03}), negative middle (N_{02}), negative low (N_{01}), Middle (Z00), positive low (P^{01}), positive middle (P^{02}) and positive high (P^{03}).

The TriMF and TrapMF for input variable of pitch and yaw angle is given in Figs. 5, 6, 7 and 8. Both TriMF and TrapMF are presented in this work for comparing the results between them. The rules of T1-FLC and IT2-FLC are represented in Table 3.

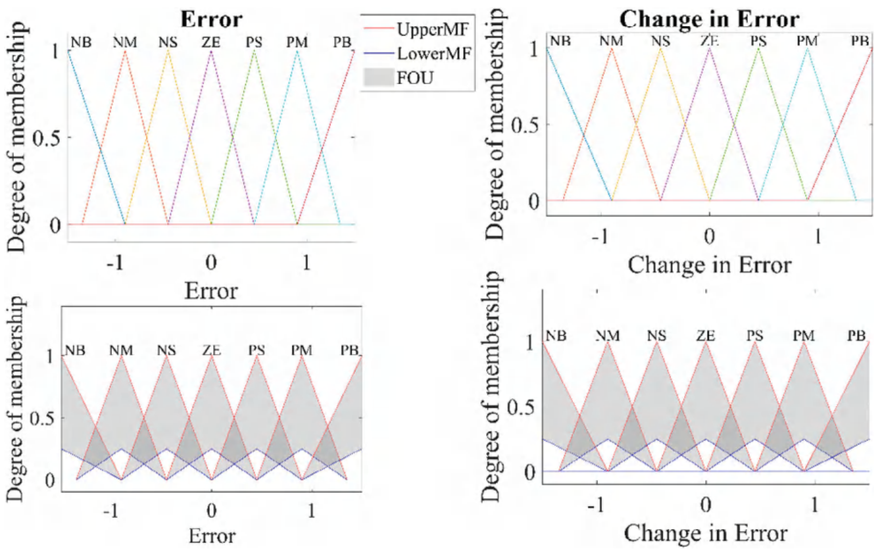

Fig. 5 Triangular input MF for Pitch

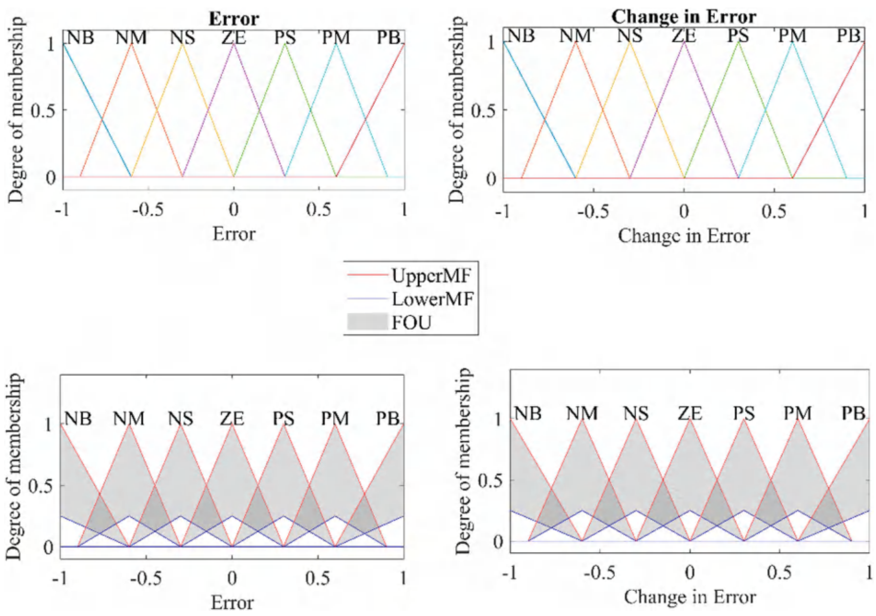

Fig. 6 Triangular input MF for Yaw

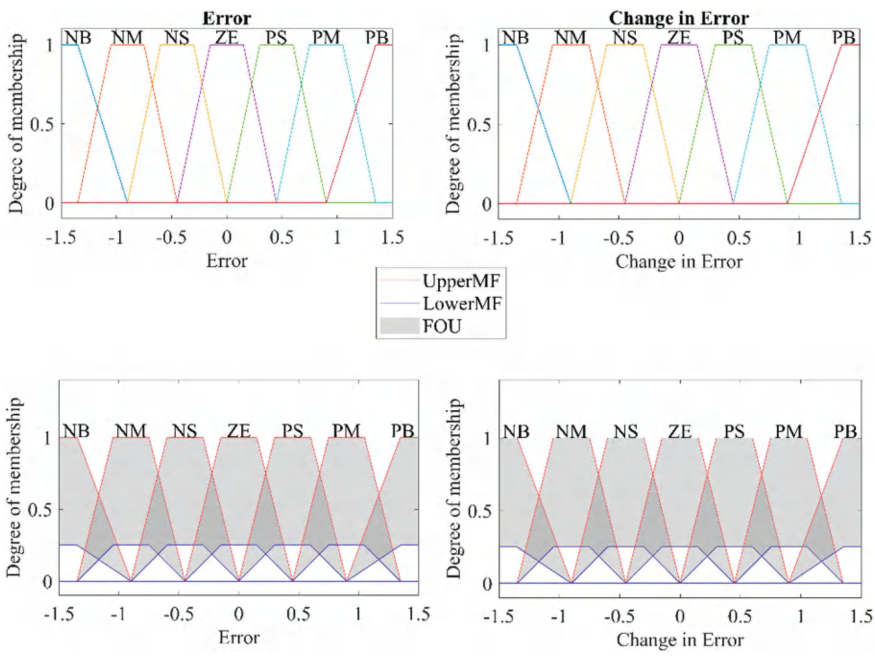

Fig. 7 Trapezoidal input MF for Pitch

Table 3 Rules of FLC for pitch and yaw angle [27]

Error	Change in error						
θ_v, θ_h	N_{03}	N_{02}	N_{01}	Z00	P^{01}	P^{02}	P^{03}
N_{03}	N_{03}	N_{03}	N_{03}	N_{03}	N_{03}	N_{02}	N_{01}
N_{02}	N_{03}	N_{03}	N_{03}	N_{03}	N_{01}	Z00	P^{01}
N_{01}	N_{03}	N_{03}	N_{03}	N_{01}	Z00	P^{01}	P^{02}
Z00	N_{03}	N_{02}	N_{01}	Z00	P^{01}	P^{02}	P^{03}
P^{01}	N_{02}	N_{01}	Z00	P^{01}	P^{03}	P^{03}	P^{03}
P^{02}	N_{01}	Z00	P^{01}	P^{03}	P^{03}	P^{03}	P^{03}
P^{03}	P^{01}	P^{02}	P^{03}	P^{03}	P^{03}	P^{03}	P^{03}

6 Simulation Outcomes

This section presents simulation outcomes of the TRMS system controlled by the proposed approach. MATLAB/Simulink simulations were employed to assess the controller's ability to maintain pitch and yaw angles within desired limits. The controller's resilience to noise was also investigated. A comparative analysis of the proposed controller and a T1-FLPID controller was undertaken. Additionally, a detailed comparison between TriMF and TrapMF was performed.

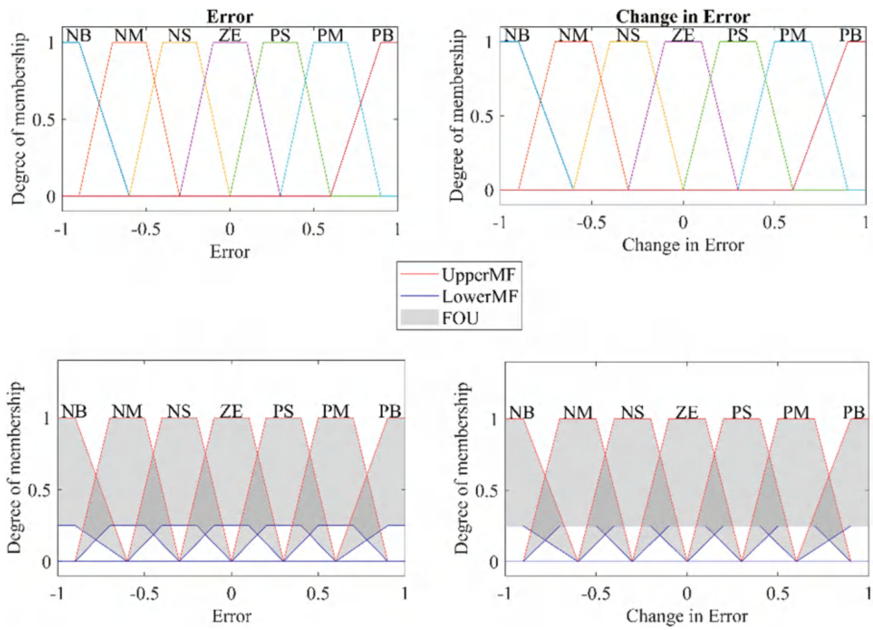

Fig. 8 Trapezoidal input MF for Yaw

6.1 Analysis of Proposed Controller Using TriMF

A comparative analysis was conducted between the proposed controller and a conventional T1-FLC using TriMF. Both systems were subjected to step and square wave inputs for pitch and yaw angles. The resulting pitch and yaw angle responses which is illustrated in Figs. 9, 10, 11 and 12 demonstrate that the IT2-FLC effectively attenuates oscillations compared to the T1-FLC.

In these simulations, a band-width white noise is included to the given variables. Both the controllers are performed in the noisy conditions for both pitch and yaw angles which is presented in Figs. 13 and 14. It is observed that, IT2-FLC gives less overshoot than T1-FLC for both pitch and yaw angle.

The benchmarks of above performance evaluation of both controllers in step response are shown in Table 4, square wave response is in Table 5 and in the presence of noise are shown in Table 6.

6.2 Analysis of Proposed Controller Using TrapMF

To further validate the controller's efficacy, a comparative assessment was conducted against a T1-FLC using TrapMF. Both controllers were subjected to step and square

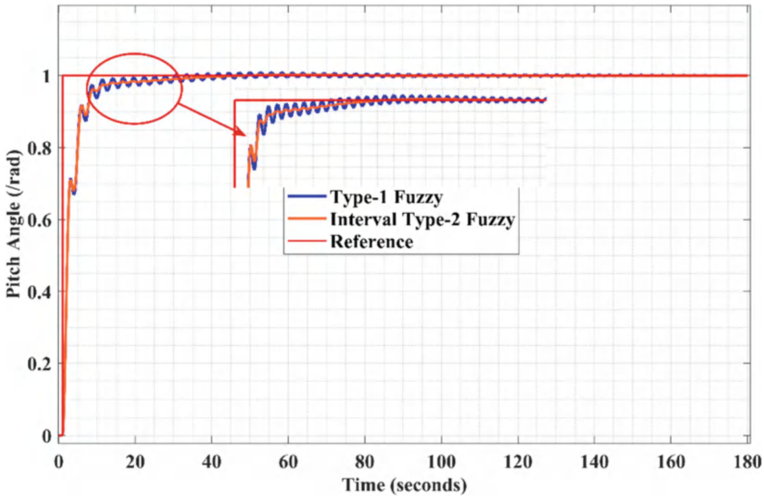

Fig. 9 Performance result in step response-pitch (using TriMF)

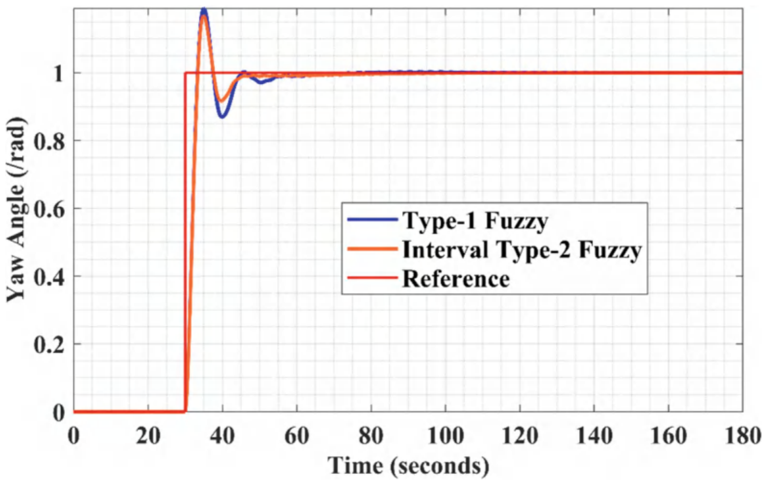

Fig. 10 Performance result in step response-Yaw

wave inputs for pitch and yaw angles. The resulting pitch and yaw angle responses, which is illustrated in Figs. 15, 16, 17 and 18 demonstrate that the IT2-FLC effectively suppresses oscillations compared to the T1-FLC.

In these simulations, a band-width white noise is included to the given variables. Both the controllers are performed in the noisy conditions for both pitch and yaw angles which is shown in Figs. 19 and 20. It is observed that, IT2-FLC gives less overshoot than T1-FLC for both pitch and yaw angles.

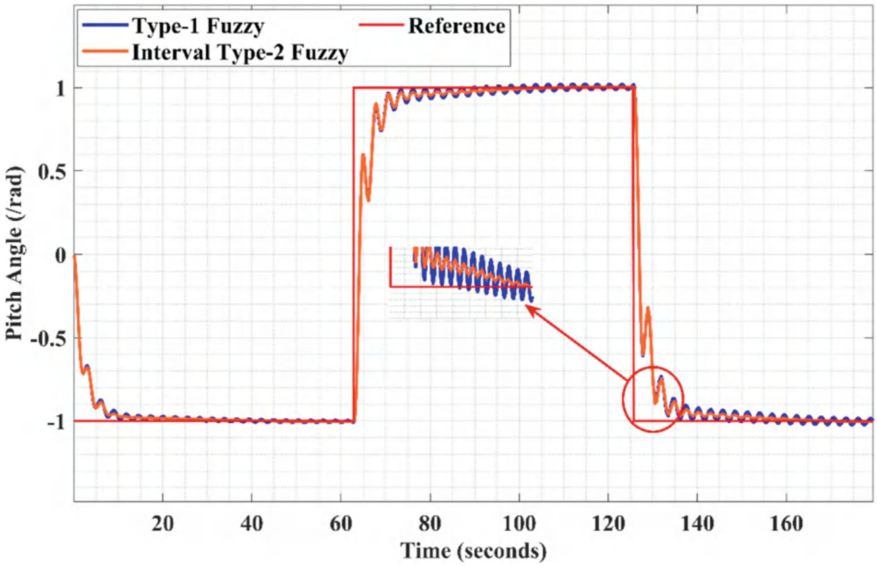

Fig. 11 Performance result in square wave response-Pitch (using TriMF)

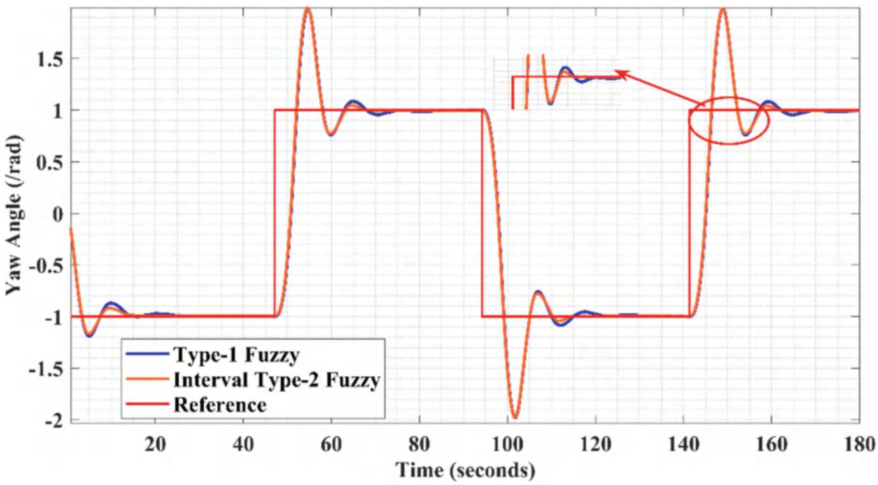

Fig. 12 Performance result in square wave response-Yaw (using TriMF)

The benchmarks of above performance evaluation of both controllers in step response are shown in Table 7, square wave response is in Table 8 and in the presence of noise are shown in Table 9.

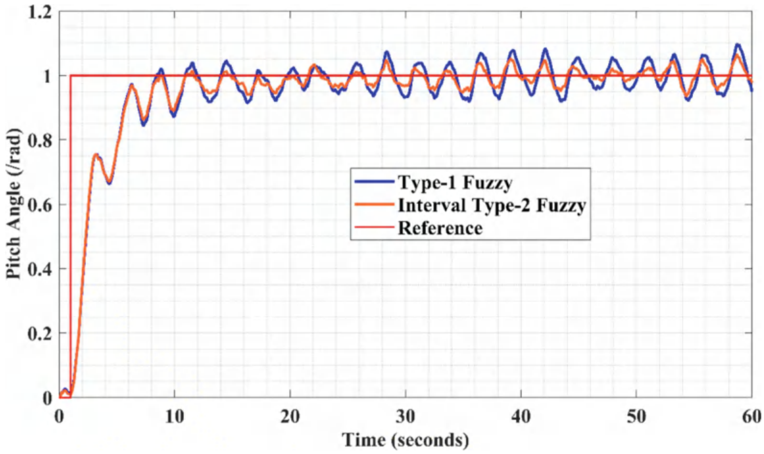

Fig. 13 Performance result in the presence of noise-Pitch (using TriMF)

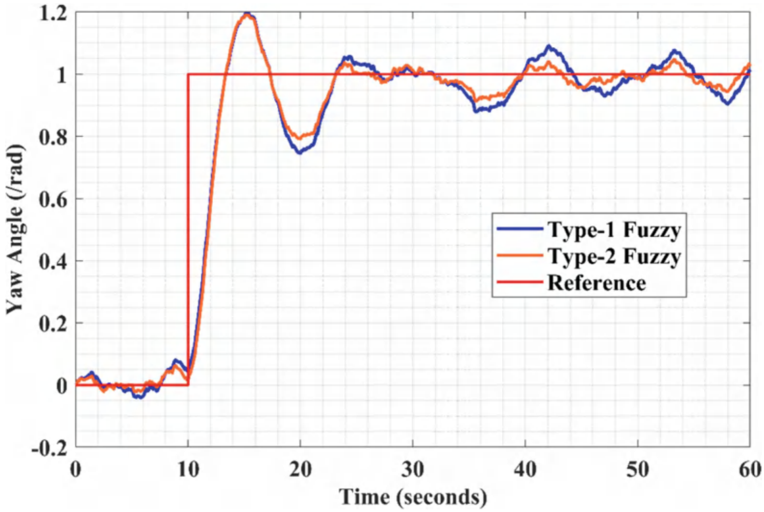

Fig. 14 Performance result in the presence of noise-Yaw (using TriMF)

Table 4 Benchmark result in step response (using TriMF)

Position	Controller	Rise time	Settling time	Overshoot	ITAE
Pitch angle	T1-FLC	4.119	29.3841	0.7854	45.25
	IT2-FLC	4.1527	18.2329	0.881	20.48
Yaw angle	T1-FLC	2.3243	53.2516	19.0379	116.5
	IT2-FLC	2.3637	43.9436	16.9349	113.9

Table 5 Benchmark result in square wave response (using TriMF)

Position	Controller	Rise time	Settling time	Overshoot	ITAE
Pitch angle	T1-FLC	4.3386	179.4649	0.3258	1174
	IT2-FLC	4.1899	159.5652	0.002	1095
Yaw angle	T1-FLC	1.00356	167.2015	98.7094	3559
	IT2-FLC	1.0033	160.8853	99.8059	3376

Table 6 Benchmark result in the presence of noise (using TriMF)

Position	Controller	Rise time	Overshoot	ITAE
Pitch angle	T1-FLC	4.0274	15.5009	74.43
	IT2-FLC	4.1436	8.9426	45.39
Yaw angle	T1-FLC	2.5263	18.5932	111
	IT2-FLC	2.4039	16.1914	81.23

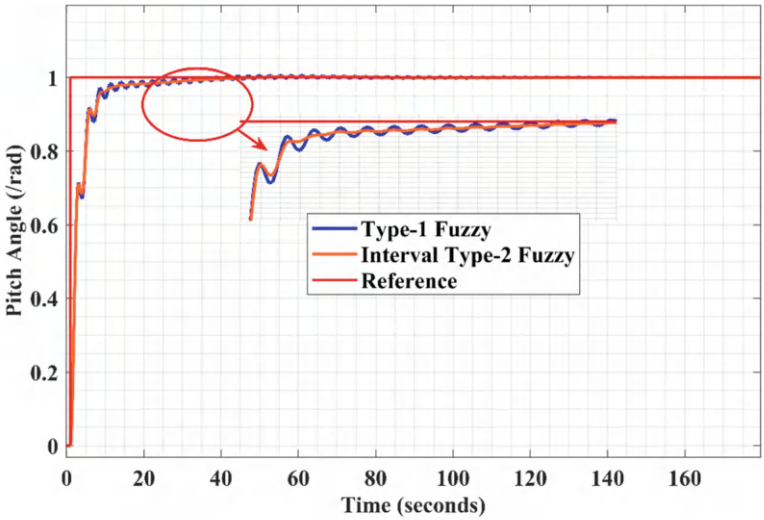

Fig. 15 Performance result in step response-Pitch (using TrapMF)

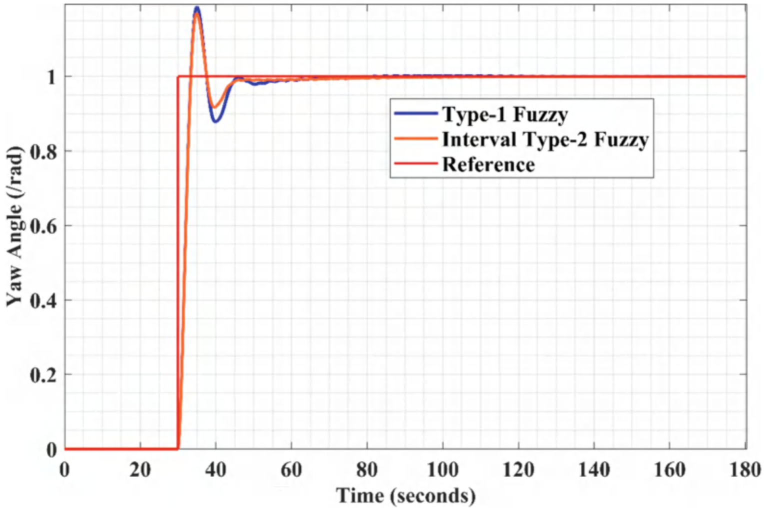

Fig. 16 Performance result in step response-Yaw (using TrapMF)

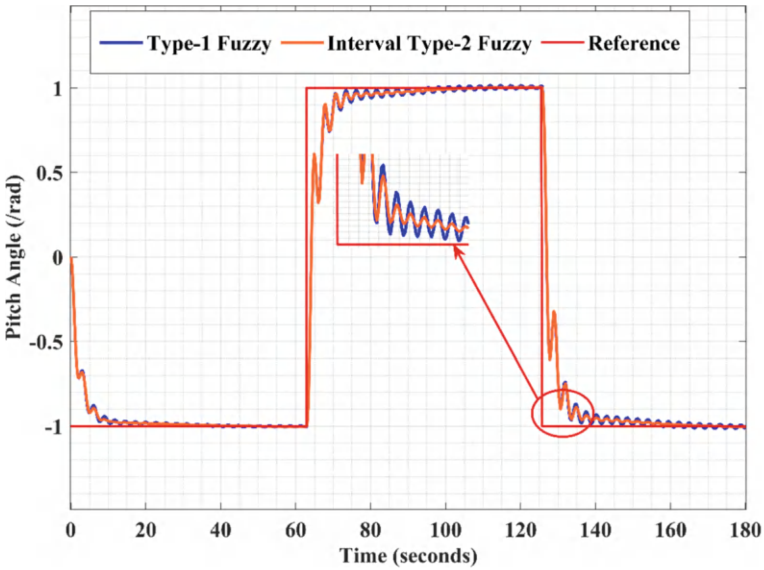

Fig. 17 Performance result in square wave response-Pitch (using TrapMF)

Table 7 Benchmark result in step response (using TrapMF)

Position	Controller	Rise time	Settling time	Overshoot	ITAE
Pitch angle	T1-FLC	4.1128	23.8952	0.5708	28.82
	IT2-FLC	4.214	15.8321	0.2194	20.49
Yaw angle	T1-FLC	2.3462	50.8336	18.7369	115.7
	IT2-FLC	2.3692	44.0723	17.0769	114.4

Table 8 Benchmark result in square wave response (using TrapMF)

Position	Controller	Rise time	Settling time	Overshoot	ITAE
Pitch angle	T1-FLC	4.2463	178.8794	0.0457	1130
	IT2-FLC	4.2028	157.1853	0.0014	1094
Yaw angle	T1-FLC	1.0311	165.7934	100	3543
	IT2-FLC	1.0343	160.8536	100	3398

Table 9 Benchmark result in the presence of noise (using TrapMF)

Position	Controller	Rise time	Overshoot	ITAE
Pitch angle	T1-FLC	4.0316	15.2002	69.22
	IT2-FLC	4.1814	7.6014	42.35
Yaw angle	T1-FLC	2.5238	17.6182	102.8
	IT2-FLC	2.4049	16.4121	81

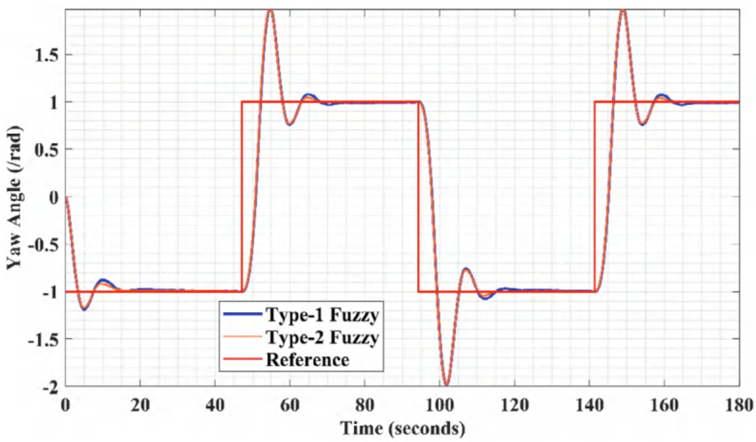

Fig. 18 Performance result in square wave response-Yaw (using TrapMF)

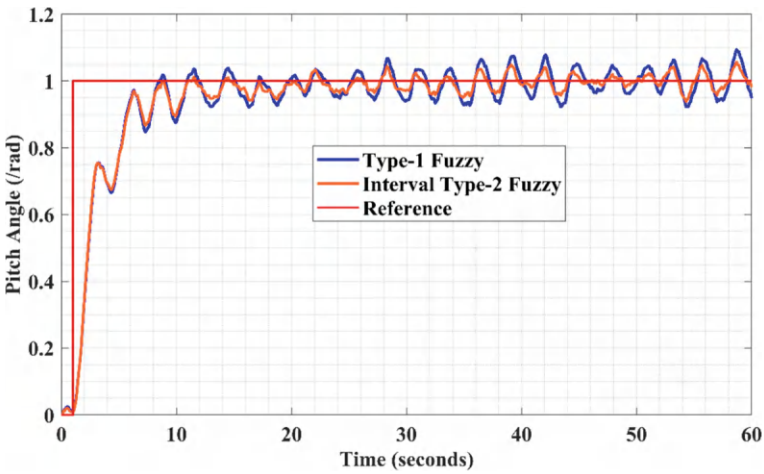

Fig. 19 Performance result in the presence of noise-Pitch (using TrapMF)

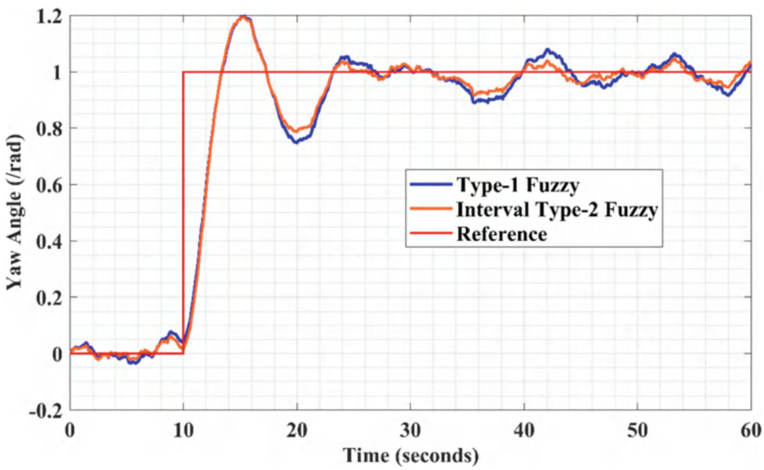

Fig. 20 Performance result in the presence of noise-Yaw (using TrapMF)

7 Conclusion

The TRMS is more difficult to control because of the model uncertainties and external disturbances. In order to overcome these difficulties, an IT2-FLPID controller is proposed for control system. The proposed controller is analyzed and compared to the

T1-FLC with certain benchmarks. For in depth analysis, we also compare the TriMF to the TrapMF. The simulation results shows that the proposed controller improves better performance than T1-FLC due to fast settling time in step and square wave response and less overshoot in the presence of noise and the proposed controller gradually eliminates the oscillations when compared to type-1 counterparts. For comparing TriMF and TrapMF, it is observed that the performance of IT2-FLC is similar but in case of T1-FLC, the input variable using TrapMF improves better performance than TriMF.

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Modeling a Fuzzy Transportation Problem for Securing Mission Critical Servers Using Attack Graph

Edithstine Rani Mathew and Lovelymol Sebastian

Abstract The fuzzy transportation problem has been researched extensively in many fields but till now no one has applied it in the field of cybersecurity. In this work, we have modeled a novel security metric system, to calculate the overall security risk of a given network by mapping it to a fuzzy transportation problem. Further, we are quantifying the overall security risk factor of a given network. As an initial step, we are identifying the mission-critical resources like Servers hosted in that network. Then we are using various vulnerability scanners like Nessus to scan the network and to identify the vulnerabilities and their risk factor in all the connected machines in the given network. The attacker's path from each source extracted out of the attack graph would be mapped to a fuzzy transportation problem and would be used for quantifying the overall vulnerability of a given network. We are trying to minimize the security risk of a given network.

Keywords Attack Graph · Fuzzy transportation problem · Trapezoidal fuzzy number

1 Introduction

Commodity transportation from sources to destinations is important in global economics. This increases the popularity of transportation problems. The transportation problem is applied in different phases like scheduling, manufacturing, investment, selection of location, HR management, etc. Chakraborty [1] explained how to solve a fully fuzzy transportation problem using triangular fuzzy numbers. In this paper, we presented new methods to study fuzzy transportation models to prevent cyber-attacks from intruders. Vulnerability detection and analysis of systems and

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networks in the cyber world is a complex task. It is not just used in the field of systems and networks but is also used in various other sectors like power grid [2], railway and airline transport systems [3, 4], for detecting the level of risk an organization is exposed to. As part of vulnerability analysis, it is mandatory to quantify the vulnerabilities an organization is exposed to. Q Liu conducted a comprehensive review of cyber-attacks and cyber security, focusing on emerging trends and recent developments [5].

Apart from the above security metrics, researchers have introduced Common Vulnerability Scoring System (CVSS) [6, 7]. There are three versions of the CVSS and various researchers have analyzed the efficiency and impact of features used in the CVSS [8]. In the CVSS, the impact of every discovered software vulnerability is quantified. CVSS contains three parts namely, base score, temporal score, and Environmental Score. The base score is fixed as soon as the vulnerability is discovered based on various factors like the Attack Vector, Attack Complexity, Privileges Required, User Interaction, Scope, Confidentiality, Integrity and Availability is assigned a CVE ID published in National Vulnerability Database (NVD). The temporal score and the environmental score depend totally on the network where the system and service are deployed. An attack graph is a concise depiction of all system routes that lead to a state where an intrusion has been successful. Various researchers have proposed the usage of the CVSS in attack graphs [9, 10]. In practice, many researchers have found that the CVSS is just used for prioritization of the vulnerabilities based on its score value [11] Our proposed system uses CVSS Scoring System by selecting a particular attack path from the attack graph and to formulate it as a transportation problem.

An attack path is created when one or more vulnerabilities are found that can be used by attackers to move between assets in a network and get access to particular ones, creating an exploitable path between the assets. Unlike the usual methods, where they quantify the security risk an organization is exposed to, we are proposing a method that quantifies the effort a hacker needs to take for exploiting a particular mission-critical vulnerability. Lower value shows that the system is secure and it is hard for a hacker to reach and exploit the mission-critical service.

1.1 Experimental Setup

As part of setting up a testbed, we have created a network of fifty systems, which is similar to a network used in a small-scale organization or lab environment of an educational institute. The systems are set up using virtual machines. Three servers in the network are considered to be of critical importance. This is since these servers are used for various mission-critical activities.

1. Web Server (D1): Ubuntu Server 12.04 with Apache 2.2.22 and PHP 5.3.10 as the server-side scripting language. The organization's website, intranet, and employee credentials are stored on this server.

2. Database Server (D2): Ubuntu server 14.04.6 with MySQL Server 5.7 as the database server. Financial transactions are stored here.
3. File and App Server (D3): Ubuntu server 18.04.4 with Apache 2.4, PHP 5.5.38 as a server-side scripting engine, and MySQL Server 5.7 as the database server. FTP Server and Moodle web application is also loaded in this server. Employee data that includes their certificates and recruitment-related evaluations are carried out using this server.

The virtualization software that is used to set up this testbed is Virtual Box, VMWare EXSi and Turnkey Open Source Hypervisor (XCP-ng). The threat modeling is done both as a skilled hacker (an outsider) (S1) and an insider (S2). We used different vulnerability scanners like Nessus and Nmap to assess the vulnerabilities of the systems connected to the network. We created a list of vulnerabilities after assessing the systems connected to the network.

Based on the above information, we have generated an attack graph though which a skilled hacker would be able to get access into the critical systems and could alter its data. We have also calculated the overall vulnerability metric of each of these systems with help of the base score obtained using the CVSS scoring system. Since in the real world, the vulnerability of the systems does not just depend upon the vulnerabilities of the software installed in these machines but also depends upon the user's behavior pattern. So in this work, we are also considering various behavioral patterns of the users, like installation of pirated software in their systems.

Result and Discussion. The intranet is connected to the internet via single ISPs. The connection from intranet and ISP link is secured using a packetfence [Open Source network access control (NAC)] Solution. On modeling attack graph, it is true that there exist multiple paths for the insider to reach the critical resources by exploiting vulnerabilities on the path. It is to be noted that one of the paths could be the same as the path that is taken by the outsider, excluding the initial node. Since this is already considered, it is not recommended to consider this path, and it may be also noted that the path considered has to also contain the path from the initial source to the current starting point.

We are mapping the attack graph to a fuzzy transportation problem [12] to understand the least cost (the minimum vulnerability) with which a hacker would be able to reach any one of the critical system D1/D2/D3, exploiting various vulnerabilities in one or more of the systems, which are connected in this network. So, we aim to minimize the overall value so to find the minimum amount of vulnerabilities, that a hacker needs to exploit so that he would be able to reach any of the critical systems. In this work, we are considering a scenario where a skilled hacker is trying to reach the critical infrastructure from outside an organization. We are assuming that he is not supported by any of the insiders who are working in the organization. This is since the overall security system gets weakened, once an insider comes into the picture.

For the fuzzy transportation problem, we are considering the values as trapezoidal fuzzy number [13].

The attack paths are given in Figs. 1, 2, 3, 4, 5 and 6 and the fuzzy transportation problem generated from the attack path is shown in Table 1.

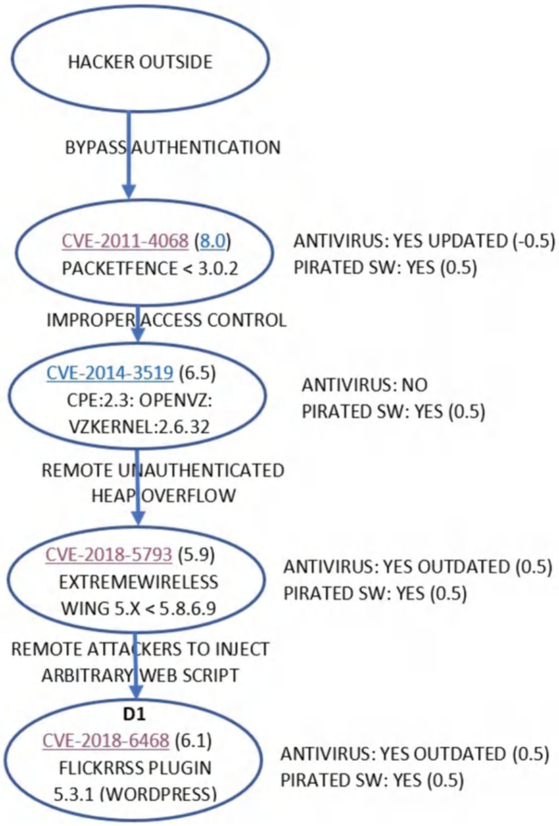


Fig. 1 Attack path from internet to D1

The circle in given attack paths indicates the software, its version and its CVE ID, along with its CVSS score. The name of the vulnerability of that version of software and its exploitation is listed in the inbound arrow.

In this scenario, supply or availability is taken as the maximum available vulnerability considering all paths that exist between the source and any of the destinations.

Similarly, the demand is considered as the minimum required vulnerability that a hacker needs to reach a particular server (destination) from any of the sources, irrespective of the number of hops in the path, and the path that doesn't contain any antivirus solution. Also the objective coefficients denotes the vulnerability of the minimum distance (hops) path that exists between the source to the destination.

Now we solve the fuzzy transportation problem given in Table 1 to find the minimum effort that a hacker needs to put in to reach any of the three mission-critical servers. In this problem vulnerability in the path is considered as trapezoidal fuzzy number. We are also proposing a new method for determining the difference between two trapezoidal fuzzy numbers, which we use to solve the problem.

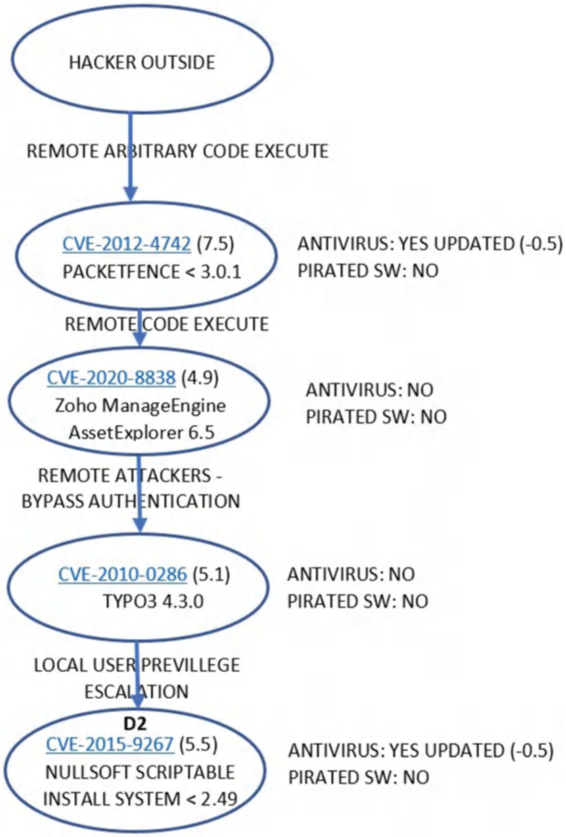


Fig. 2 Attack path from internet to D2

2 A New Method for Subtraction on Trapezoidal Fuzzy Number

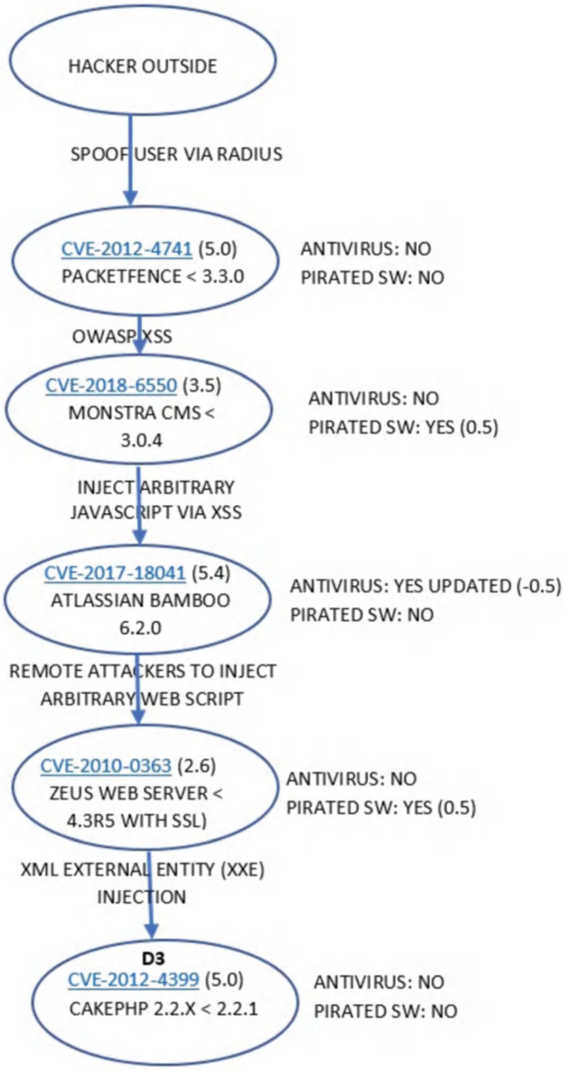
We have developed a new method to find the difference between two trapezoidal fuzzy numbers (TrFN). Suppose $\tilde{M} = (\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, \tilde{m}_4)$ and $\tilde{N} = (\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4)$ be two TrFN then $\tilde{M} \Theta_{gH} \tilde{N} = (\tilde{m}_1 - \tilde{n}_1, \tilde{m}_2 - \tilde{n}_2, \tilde{m}_3 - \tilde{n}_3, \tilde{m}_4 - \tilde{n}_4)$ where Θ_{gH} represents generalized Hukuhara difference. Suppose $\tilde{M} \Theta_{gH} \tilde{N} = \tilde{P}_\alpha$ where $\tilde{P}_\alpha = [\tilde{P}_\alpha^-, \tilde{P}_\alpha^+]$

Now $\tilde{M}_\alpha = \left[\tilde{M}_\alpha^-, \tilde{M}_\alpha^+\right] = [\tilde{m}_1 + \alpha(\tilde{m}_2 - \tilde{m}_1), \tilde{m}_4 - \alpha(\tilde{m}_4 - \tilde{m}_3)]$ and

$$\tilde{N}_\alpha = \left[\tilde{N}_\alpha^-, \tilde{N}_\alpha^+\right] = [\tilde{n}_1 + \alpha(\tilde{n}_2 - \tilde{n}_1), \tilde{n}_4 - \alpha(n_4 - n_3)], \forall \alpha \in [0,1].$$

$$\begin{aligned}
\text{Now}\ \tilde{P}_\alpha &= \left[\tilde{P}_\alpha^-, \tilde{P}_\alpha^+\right] \\
&= [\tilde{M}_\alpha^-, \tilde{M}_\alpha^+]\Theta_{gH}\left[\tilde{N}_\alpha^-, \tilde{N}_\alpha^+\right] \\
&= [\tilde{M}_\alpha^- - \tilde{N}_\alpha^-, \tilde{M}_\alpha^+ - \tilde{N}_\alpha^+]\ \text{since}\ \tilde{m}_4 - \tilde{m}_1 \leq \tilde{n}_4 - \tilde{n}_1. \\
&= [\tilde{m}_1 + \alpha(\tilde{m}_2 - \tilde{m}_1) - (\tilde{n}_1 + \alpha(\tilde{n}_2 - \tilde{n}_1)), \tilde{m}_4 - \alpha(\tilde{m}_4 - \tilde{m}_3) - (\tilde{n}_4 - \alpha(\tilde{n}_4 - \tilde{n}_3)) \\
&= [\tilde{m}_1 - \tilde{n}_1 + \alpha\{(\tilde{m}_2 - \tilde{n}_2) - (\tilde{m}_1 - \tilde{n}_1)\}, \tilde{m}_4 - \tilde{n}_4 - \alpha\{(\tilde{m}_4 - \tilde{n}_4) + (\tilde{m}_3 - \tilde{n}_3)\}]
\end{aligned}$$

Fig. 3 Attack path from internet to D3

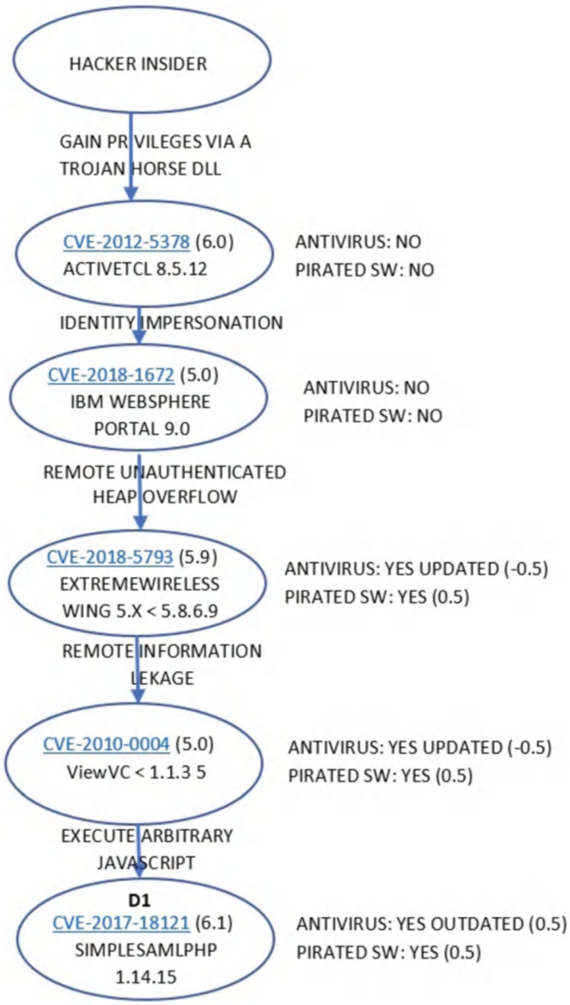


Assume $\tilde{m}_1 - \tilde{n}_1 + \alpha\{(\tilde{m}_2 - \tilde{n}_2) - (\tilde{m}_1 - \tilde{n}_1)\} \leq x \leq \tilde{m}_4 - \tilde{n}_4 - \alpha\{(\tilde{m}_4 - \tilde{n}_4) + (\tilde{m}_3 - \tilde{n}_3)\}$

$$\Rightarrow \frac{x - (\tilde{m}_1 - \tilde{n}_1)}{(\tilde{m}_2 - \tilde{n}_2) - (\tilde{m}_1 - \tilde{n}_1)} \geq \alpha$$

Thus

Fig. 4 Attack path from intranet to D1



$$\mu_p^-(x) = \frac{x - (\tilde{m}_1 - \tilde{n}_1)}{(\tilde{m}_2 - \tilde{n}_2) - (\tilde{m}_1 - \tilde{n}_1)}$$

$$\mu_p^-(x) = \frac{1}{(\tilde{m}_2 - \tilde{n}_2) - (\tilde{m}_1 - \tilde{n}_1)} > 0\, if\ (\tilde{m}_2 - \tilde{n}_2) > (\tilde{m}_1 - \tilde{n}_1)$$

Thus $\mu_p^-(x)$ is a non-decreasing function and

$$\mu_p^-((\tilde{m}_2 - \tilde{n}_2) = 1, \mu_p^-((\tilde{m}_2 - \tilde{n}_2) = 0.$$

Similarly we have

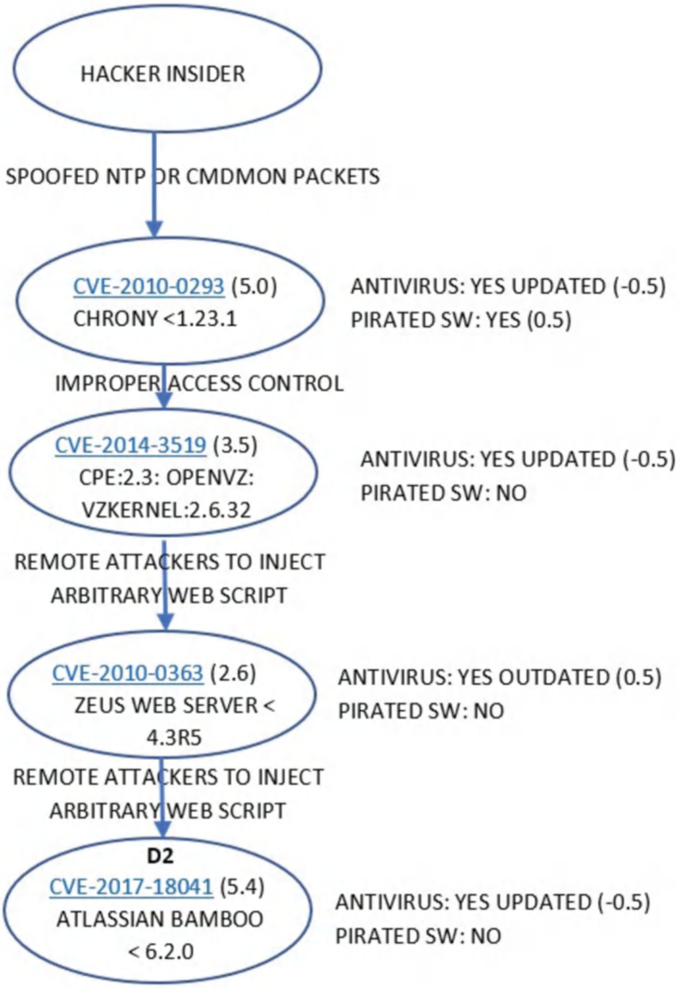


Fig. 5 Attack path from intranet to D2

$$x \leq \tilde{m}_4 - \tilde{n}_4 - \alpha\{(\tilde{m}_4 - \tilde{n}_4) + (\tilde{m}_3 - \tilde{n}_3)\}$$

$$\Rightarrow \frac{(\tilde{m}_4 - \tilde{n}_4) - x}{(\tilde{m}_4 - \tilde{n}_4) - (\tilde{m}_3 - \tilde{n}_3)} \geq \alpha$$

ie $\mu_p^+(x) = \frac{(\tilde{m}_4 - \tilde{n}_4) - x}{(\tilde{m}_4 - \tilde{n}_4) - (\tilde{m}_3 - \tilde{n}_3)}$

$$\mu_p^+(x) = \frac{-1}{(\tilde{m}_4 - \tilde{n}_4) - (\tilde{m}_3 - \tilde{n}_3)} < 0 if\, (\tilde{m}_4 - \tilde{n}_4) > (\tilde{m}_3 - \tilde{n}_3),$$

Fig. 6 Attack path from intranet to D3

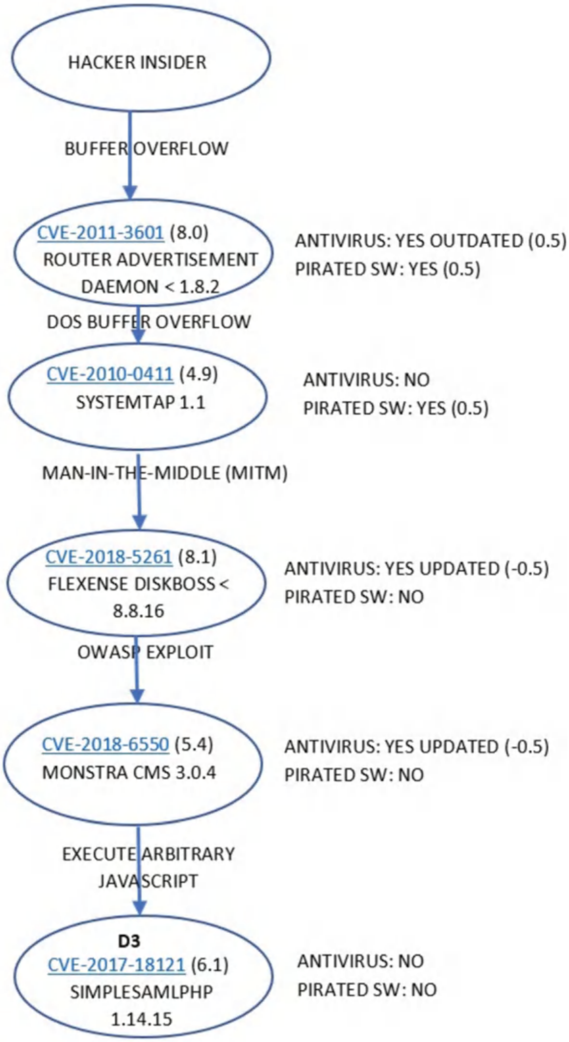


Table 1 Fuzzy Transportation Problem generated using Attack Graph

	Web server (D1)	Database server (D2)	File and App server (D3)	Availability (vulnerability
A skilled hacker (outsider) (S_1)	(6, 7, 7.5, 8)	(4, 5, 6.5, 7)	(3, 4, 4.5, 5)	(6, 8, 10, 11)
An inside hacker (S_2)	(5, 6, 8, 9)	(3, 4, 4.5, 5)	(6, 7, 8, 8.5)	(7, 9, 11, 12)
Demand (Vulnerability of the server)	(2, 3, 4, 4.5)	(5, 7, 9, 10)	(6, 7, 8, 8.5)	

Thus $\mu_p^+(x)$ is a non-increasing function and

$$\mu_p^-(\tilde{m}_4 - \tilde{n}_4) = 0, \mu_p^-(\tilde{m}_3 - \tilde{n}_3) = 1$$

Therefore the membership function can be written as

$$\mu_{\tilde{P}}(x) = \begin{cases} \frac{x-(\tilde{m}_1-\tilde{n}_1)}{(\tilde{m}_2-\tilde{n}_2)-(\tilde{m}_1-\tilde{n}_1)}, \tilde{m}_1 - \tilde{n}_1 \leq x \leq \tilde{m}_2 - \tilde{n}_2 \\ \frac{(\tilde{m}_4-\tilde{n}_4)-x}{(\tilde{m}_4-\tilde{n}_4)-(\tilde{m}_3-\tilde{n}_3)}, \tilde{m}_3 - \tilde{n}_3 \leq x \leq \tilde{m}_4 - \tilde{n}_4 \\ 0, x \leq \tilde{m}_1 - \tilde{n}_1, x \geq \tilde{m}_4 - \tilde{n}_4 \\ 1, \widetilde{m}_2 - \tilde{n}_2 \leq x \leq \tilde{m}_3 - \tilde{n}_3 \end{cases}$$

So $\tilde{M}\Theta_{gH}\tilde{N} = (\tilde{m}_1 - \tilde{n}_1, \tilde{m}_2 - \tilde{n}_2, \tilde{m}_3 - \tilde{n}_3, \tilde{m}_4 - \tilde{n}_4) if\, \tilde{m}_4 - \tilde{m}_1 \leq \tilde{n}_4 - \tilde{n}_1$.

3 Steps to Find the Initial Fuzzy Basic Feasible Solution (IFBFS) and Optimal Solution of FTP

1. Formulate the FT table from the problem (P1).
2. $i = 1$ check $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$. If yes goto 3. Otherwise goto case 1.

Case 1: If $\sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j$ then introduce a dummy column with zero fuzzy number for all of its costs. Assume fuzzy demand $(\widetilde{\eta})$ in such a way that it will satisfy $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \oplus \widetilde{\eta}$. Then goto (iii).

Case 2: If $\sum_{i=1}^{m} \tilde{a}_i < \sum_{j=1}^{n} \tilde{b}_j$ then introduce a dummy row with zero fuzzy number for all of its costs. Assume fuzzy source $(\widetilde{\theta})$ in a manner that will satisfy $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \oplus \widetilde{\theta}$. Then goto 3.

3. To obtain the initial solution, use the fuzzy vogel's approximation approach or the fuzzy north-west corner method.
4. Use the fuzzy modified distribution method to identify the best optimal result.

Using the fuzzy north-west corner rule, the initial fuzzy basic feasible solution (67, 103, 146.5, 171.25) is presented in Table 2.

Now, we apply the initial fuzzy basic feasible solution (IFBFS) generated by the fuzzy north-west corner (FNWC) rule and evaluate it using the fuzzy modified distribution method. Below are the steps for obtaining the fuzzy optimal solution.

1. Initially, either a_i or b_j is set to zero as a fuzzy number. Assigning a zero fuzzy number to either a_i or b_j is recommended when there are multiple allocations in a row or column, as this simplifies calculations significantly. Determine the values of a_i or b_j using the relationship $c_{ij} = a_j \oplus b_j$ for all occupied cells (i, j).

Table 2 Initial fuzzy basic feasible solution

	Web server (D1)	Database server (D2)	File and App server (D3)	Availability (vulnerability
A skilled hacker (outsider) (S_1)	[(2, 3, 4, 4.5)] (6, 7, 7.5, 8)	[(4, 5, 6, 6.5)] (4, 5, 6.5, 7)	(3, 4, 4.5, 5)	(6, 8, 10, 11)
An inside hacker (S_2)	(5, 6, 8, 9)	[(1, 2, 3, 3.5)] (3, 4, 4.5, 5)	[(6, 7, 8, 8.5)] (6, 7, 8, 8.5)	(7, 9, 11, 12)
Demand (Vulnerability of the server)	(2, 3, 4, 4.5)	(5, 7, 9, 10)	(6, 7, 8, 8.5)	

2. Calculate the opportunity cost for unoccupied cells (i, j) using the formula $d_{ij} = c_{ij}\Theta_{gH}(a_j \oplus b_j)$.
3. Analyze each d_{ij} sign
   - If d_{ij} is a positive fuzzy number, then the current FBFS is considered optimal.
   - An empty cell (i, j) can be added to the solution to improve it if one or more of the d_{ij} are negative. The input place for the solution is chosen to be the empty cell with the largest negative d_{ij} value.
4. Form a closed path starting from the unoccupied cell with the most negative FN d_{ij}. Begin the path by marking a plus sign on the first available empty cell. Then, traverse down the column (or row), alternating between plus and minus signs at each occupied cell, and continue along rows (or columns) until reaching another empty cell. Close the path and go back to the allocated unoccupied cell.
5. Among the cells marked with a minus sign along the edges of the closed loop, select the smallest value ζ. Assign this value to the designated empty cell and adjust the values of additional occupied cells marked with plus signs accordingly. To satisfy $x_{ij'} \oplus \zeta = x_{ij}$, calculate the value of the occupied cell $x_{ij'}$ with the negative sign.
6. Improve the solution by allocating units to the designated empty cell in step 5, and subsequently compute the updated transportation cost
7. Verify the optimality of the improved solution once more. The method is complete when all positive fuzzy number d_{ij} results for empty cells.

The fuzzy optimal solution obtained is presented in Table 3, as outlined in the proposed algorithm

Thus, the minimum effort that a hacker needs to put in the network to reach any of the three mission-critical servers is (0,1,2,2.5) ⊗ (6, 7, 7.5, 8) ⊕ (6,7,8,8.5) ⊗ (3,4,4.5,5) ⊕ (2,2,2,) ⊕ (5,6,8,9) ⊕ (5,7,9,10) ⊗ (3,4,4.5,5) = (43,75,107.5,130.5). Defuzzifying the optimal value we get the crisp value as 89.75. i.e., In this problem, the minimum effort that a hacker needs to put in to reach any of the three mission critical servers is 89.75. The security measures applied are adequate and the network is secure to the possible acceptable level, even though 100% security is a myth.

Table 3 Optimal solution

	Web server (D1)	Database server (D2)	File and App server (D3)	Availability (vulnerability
A skilled hacker (outsider) (S_1)	[(0, 1, 2, 2.5)] (6, 7, 7.5, 8)	(4, 5, 6.5, 7)	[(6, 7, 8, 8.5)] (3, 4, 4.5, 5)	(6, 8, 10, 11)
An inside hacker (S_2)	[(2, 2, 2, 2)] (5, 6, 8, 9)	[(5, 7, 9, 10)] (3, 4, 4.5, 5)	(6, 7, 8, 8.5)	(7, 9, 11, 12)
Demand (Vulnerability of the server)	(2, 3, 4, 4.5)	(5, 7, 9, 10)	(6, 7, 8, 8.5)	

4 Conclusion

In this paper, we have formulated the security of systems and networks as a fuzzy transportation problem. We have focused on minimizing the overall vulnerability, and maximize the effort a hacker needs to put in, to reach any of the three mission critical servers. Based on our experiment setup and analysis, we have considered two cases, where one hacker is from internet, and another hacker from intranet, trying to hack the critical servers to obtain financial gains. This forms the first study on the application of transportation problems for quantifying the security of the network based on the generated attack graph. The proposed methodology could be extended to an entire network and an automated dashboard could be used for displaying the security risk of the entire organization.

For citations of references, we prefer the use of square brackets and consecutive numbers. Citations using labels or the author/year convention are also acceptable. The following bibliography provides a sample reference list with entries for journal articles [1], an LNCS chapter [2], a book [3], proceedings without editors [4], as well as a URL [5].

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The Ranking Function-Based Defuzzification Technique and Application of Type-2 Fuzzy Logic to Study a Triple Goal-Based 4D-Transportation Problem with Carbon Emission Effect

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Abstract The number of vehicles to meet our needs in daily life is increasing. So, due to types of vehicles and road conditions, etc. greenhouse gas emissions, global warming, and carbon emissions are growing in nature. The effects of carbon emissions in our daily lives are pervasive and multifaceted, impacting various aspects of our environment, health, and well-being. The transportation system is considered as one of the major sources of carbon emissions. The goal of this article is to create a four-dimensional, multi-objective green transport plan that takes into account the carbon price, offset, and cap regulations. In real-life circumstances, all input parameters of the proposed model are not in crisp value form. To produce a realistic four-dimensional transportation system, a twofold uncertainty namely type-2 intuitionistic fuzzy is included in this study. A novel ranking defuzzification method is introduced in this model to transform it into a deterministic form. Lastly, based on nonfuzzy and fuzzy methods, determined the various optimal solutions of the proposed model. Two real-world case studies are analyzed to demonstrate the applicability of the proposed investigation. Managerial insights and conclusions along with future research scope of this study are finally presented.

Keywords Four-dimensional transportation problem · Carbon emission · Type-2 intuitionistic fuzzy set · Pareto optimal solution · Carbon tax · Cap and offset scheme

Abbreviations

L.P.P	Linear programming problem
TP	Transportation problem
2D-TP	Two-dimensional transportation problem

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STP	Solid transportation problem
3D-TP	Three-dimensional TP
4D-TP	Four-dimensional TP
MI4D-TP	Multi-item 4D-TP
MOTP	Multi-objective TP
MOSTP	Multi-objectives solid TP
DM	Decision-maker
MOMI4D-TP	Multi-objective multi-item 4D-TP
T2-FS	Type-2 fuzzy set
I-FS	Intuitionistic fuzzy set
I-FN	Intuitionistic fuzzy number
FS	Fuzzy set
T1-FS	Type-1 fuzzy set
MF	Membership function
MG	Membership grade
T2I-FN	Type-2 intuitionistic fuzzy number
FN	Fuzzy number
CV	Critical value
RF	Ranking function
CaE	Carbon emission
MOOP	Multi-objective optimization problem
FP	Fuzzy programming
GC	Global criterion
GP	Goal programming
WGP	Weighted goal programming
IFP	Intuitionistic fuzzy programming
NCP	Neutrosophic compromise programming
FGP	Fuzzy goal programming
NFS	No feasible solution
MO4D-TP	Multi-objective 4D-TP
IT2FL	Interval type-2 fuzzy logic
IF	Intuitionistic fuzzy

1 Introduction

Today's globe is very concerned about CaEs. It has a significant impact on the rise in greenhouse gas emissions in the atmosphere. In the actual world, the transport sector is primarily responsible for the majority of CaEs. First created by Hitchcock [15] in 1941, the 2D-TP or TP is a unique kind of decision-making issue where the objective is to minimize the cost of carrying the item from some origins to some destinations. There are supply and demand limitations in the TP system. STP or 3D-TP is the extension of traditional TP, first proposed by Haley [14]. Demand, supply, and conveyance capacity restrictions are the three categories of constraints that the STP model is concerned with. However, a single route option in the TP system is insufficient in our real-world scenario. Thus, under an STP framework, STP becomes a 4D-TP when many routes are taken into consideration for transportation. In real-life TPs, the DM may need to optimize multiple objectives simultaneously for various reasons. If 4D-TP deals with multiple objectives then it is called MO4D-TP. In real-life TPs, the DM may need to optimize multiple objectives simultaneously for various reasons. When multiple items are transported through a MO4D-TP structure from some sources to some destinations through some routes, it is known as a MOMI4D-TP model.

The definition of the parameters of TPs may not be possible due to a variety of uncontrollable events, including but not limited to unstable financial markets, a lack of input data, extreme weather, poor road conditions, etc. In 1965, Zadeh [51] introduced the concept of FST to overcome this ambiguity. FS can handle the satisfaction case of fuzzy uncertainty, but it is unable to handle the uncertainty displeasure situation. Consequently, handling these kinds of ambiguous data situations becomes an extremely difficult undertaking. Based on this concept, Atanassov [4] introduced the I-FS which is a generalization of FS. As a result, I-FS provides a more appropriate and comprehensive formulation to address the previously described uncertain scenarios. On the other hand, fuzzy integers that are triangular or trapezoidal are frequently used to handle ambiguous data in application difficulties. Considering the numerous studies conducted, we conclude that the trapezoidal form of a fuzzy number is typically used because of its ability to provide more accurate information due to its simplicity. But occasionally, the data's two-dimensional shape is unable to address all of the ambiguities. After that, Zadeh also suggested using a T2-FS to get out of a similar predicament. Additional information about the various T2-FS and I-FS connections may be found in [10, 26]. In order to overcome the uncertainties in a green 4D-TP system, a unique 4-dimensional Trapezoidal T2I-FN with a degree of reluctance is introduced here.

1.1 Motivation

CaEs refer to the release of CO_2 and other greenhouse gases into the atmosphere. These emissions primarily result from human activities such as burning fossil fuels for energy, industrial processes, transportation, deforestation, and agriculture. CO_2 is the most significant greenhouse gas emitted by human activities and is a major contributor to global warming and climate change. Efforts to reduce CaEs are crucial in mitigating the impacts of climate change and transitioning to a more sustainable future. Transportation systems rely on selecting the right kind of conveyance to move products from one location to another while reducing CaEs. But the market for green cars is growing every day as a result of rising greenhouse gas emissions, environmental deterioration, global warming, and other factors. Advanced technology is taken into consideration while creating green automobiles that run on alternative fuels. Examples of these include fuel cells, hydrogen, natural gas, and compressed air, hybrid electric, and battery-electric vehicles. These cars improve the environment

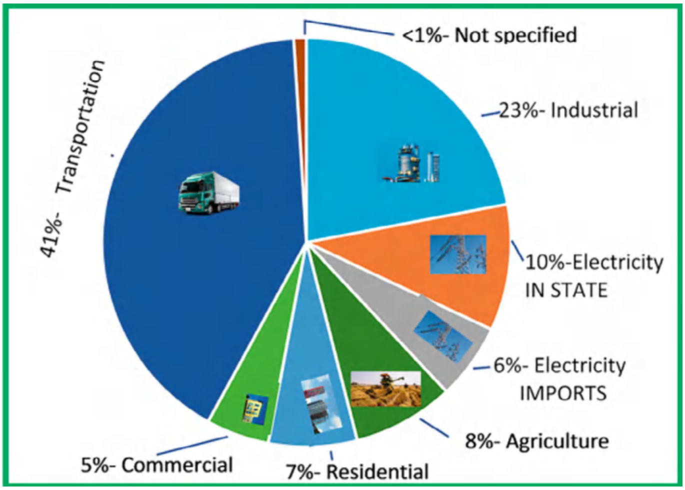

Fig. 1 Sources of carbon emission

by lowering CaEs and air pollution. A graphical representation of Worldwide CaEs by various economic sectors between 2016 and 2017 is mentioned in Fig. 1. Implementing a comprehensive strategy like carbon emission tax, cap and offset policies in the transportation sector can significantly contribute to reducing greenhouse gas emissions and combating climate change. CaE tax is one type of tax on the amount of carbon emitted by vehicles. The tax can be levied on various types of transportation fuels such as gasoline, diesel, and aviation fuel, based on their carbon content. Cap policy sets a cap on the total amount of CaEs allowed within a specific jurisdiction, often by issuing a limited number of permits or allowances. Companies in the transportation sector must hold enough permits to cover their emissions. If they emit more than their allocated permits, they can purchase additional permits from companies that have surplus allowances. In transportation systems, an offset policy refers to a strategy implemented to mitigate or balance the environmental impacts, typically CaEs, associated with transportation activities. The goal is to reduce the net carbon footprint of transportation operations by investing in projects that either reduce emissions elsewhere or remove CO_2 from the atmosphere. Inspired by this concept, we examined a smart, environmentally friendly and economical based four-dimensional transport system with considering carbon emission, cap, carbon tax, and offset policy.

2 Literature Review

Using GP approach, Jana et al. [16] investigated a fuzzy multi-objective profit TP model. Using Zimmermann's fuzzy programming, Rizk-Allah et al. [35] formulated a multi-objective TP model and they found its best compromise solution. To find the optimal solution of the intuitionistic fuzzy multi-objective TP model, Roy et al. [40] developed GP technique and IF programming. Malik and Gupta [28] described an IF multi-objective TP model via based on GP approach. Based on Pythagorean-hesitant fuzzy computational algorithm, Adhami and Ahmad [1] formulated and solved a multiobjective TP model. Mahajan and Gupta [27] analyzed multiple objective TP model under the fully intuitionistic fuzzy environment using different MFs. An IT2FL controller approach was created by Jana et al. [17] to evaluate the dangers that construction site workers face. Under the framework of uncertainty theory, Sahoo et al. [46] examined a supply chain model with visibility and risk factors.

The STP model is a generalization of the well-known TP model, where source and destination are replaced with three-dimensional qualities in the objective and constraint set, which was first proposed by mathematician Schell [48]. Under fuzzy environment, using possibility measures principle Ojha et al. [34] examined an entropy-based STP model. Under type-2 fuzzy environment and via credibility optimization approaches (namely expected value, pessimistic value, and optimistic value criterion), Liu et al. [25] solved an STP model. Giri et al. [11] developed a fully fuzzy multi-item STP model with fixed charge cost. Sahoo et al. investigated an uncertain supply chain model with visibility and risk criteria. Using interval approximation and reduction method Kundu et al. [22] solved a Multi-item STP model under type-2 fuzzy set. Under fuzzy environment, Kocken and Sivri [20] developed an STP model and they determined optimal solutions through a parametric method. In terms of lower approximations and upper approximations of the rough intervals, Das et al. [6] examined a profit STP model. Using gamma type-2 defuzzification technique, Sengupta et al. [49] studied a CaE-based STP model. Mardanya and Roy [29] solved a fuzzy multi-objective STP model using fuzzy programming and interval programming. Das et al. [7] developed a type-2 fuzzy green MOSTP model using IF programming approach. Using the IT2FL paradigm, Sahoo et al. [42] investigated the STP model.

In the case of fixed charge 4D-TP model, Halder et al. [13] focused on the significance of routes for damageable or breakable substitutable objects. Under fuzzy and rough interval environments, Bera et al. [5] proposed a budget constraint-based 4D-TP model. Under type-2 fuzzy-random environment, Akter et al. [2] developed a CaE-based 4D-TP model. Sahoo et al. [41] presented a MOMI4D-TP model using uncertainty theory and GP technique. Using neutrosophic programming method, weighted sum method and max min Zimmermann method Aktar et al. [3] studied a fixed charge-based green MOMI4D-TP model. A 4D-TP model with damageable and replaceable items was constructed and solved by Sahoo et al. [44] using the type-2 uncertain normal critical value based reduction approach. In the framework of uncertainty theory, an entropy-based 4D-TP model with discounted costs was analyzed by Sahoo et al. [45]. Giri and Roy [12] developed a neutrosophic green 4D-TP model with fixed charge cost. Table 1 presents and reviews some recent research on several transportation-related issues.

Table 1 Some remarkable research contributions on various TP models

Author(s) ↓	OF type		Env. ↓	Items		Dim. ↓	Delivery time	Dwell time	Cap and offset					Solution technique
↓	Single	Multiple	↓	Single	Multiple	↓	↓	↓	↓	LUT	CaE	TT	TC	↓
Jana et al. [16]	N	Y	T2F	Y	N	2	N	N	N	Y	N	Y	N	EV, CM
Roy et al. [40]	N	Y	IF	Y	N	2	Y	N	N	N	N	Y	Y	IFP
Rizk-Allah et al. [35]	N	Y	Neutrosophic	Y	N	2	N	N	N	N	N	Y	Y	NCPA
Malik and Gupta [28]	N	Y	IF	Y	N	2	N	N	N	N	N	N	Y	GP
Sahoo et al. [42]	Y	N	T2F	N	Y	3	N	N	N	N	N	N	Y	FL
Ojha et al. [34]	N	Y	Fuzzy	N	Y	3	N	N	N	N	N	Y	Y	PMFEZA
Liu et al. [25]	Y	N	T2F	Y	N	3	N	N	N	N	N	N	Y	PV, OV,& EV
Giri et al. [11]	Y	N	Fully fuzzy	N	Y	3	N	N	N	N	N	N	Y	FCCP
Sengupta et al. [49]	N	Y	Gamma T2F	Y	N	3	N	N	N	N	Y	N	Y	NIAGC
Mohammed and Wang [33]	N	Y	Fuzzy	Y	N	3	N	N	N	N	Y	Y	Y	GP
Midya et al. [31]	N	Y	IF	Y	N	3	N	N	N	N	Y	Y	Y	Min-max GP
Mogale et al. [32]	N	Y	Crisp	Y	N	3	N	Y	N	N	N	Y	Y	NCRO
Konur and Schaefer [21]	Y	N	Crisp	Y	N	–	N	N	Y	N	Y	N	Y	PA
Aktar et al. [2]	Y	N	T2FR	N	Y	4	N	N	N	N	Y	N	Y	GRG
Samanta et al. [47]	N	Y	LR-type IF	N	Y	4	N	N	N	N	N	Y	N	CCM
Kar et al. [18]	Y	N	Fully neutrosophic	N	Y	4	N	N	N	N	N	N	Y	SAF, WVF
Giri and Roy [12]	N	Y	Neutrosophic	Y	N	4	N	N	N	N	Y	Y	Y	PHFP, NP
Shivani and Rani [50]	N	Y	IF	N	Y	4	N	N	N	N	Y	N	Y	EV
Present study	N	Y	**T2IFS**	N	Y	4	Y	Y	Y	Y	Y	Y	Y	**T2IFS**

• Env.: Environment, Dim.: Dimension, OF: Objective function, LUT: Loading & unloading time, T2F: Type-2 fuzzy, TT: Transportation time, TC: Transportation cost, CaE: Carbon emission, EV: Expected value, CM: Credibility measure, IT2FL: Interval type-2 fuzzy logic, NCPA: Neutrosophic compromise Programming approach, IF: Intuitionistic fuzzy, FL: Fuzzy logic, IFP: Intuitionistic fuzzy programming, PMFEZA: Possibility measure of fuzzy equality & Zimmermann's approach, PV: Pessimistic value, OV: Optimistic value, EV: Expected value, FCCP: Fuzzy chance constraint programming, NIAGC: Nearest interval approximation and generalized credibility, T2FR: Type-2 fuzzy random, GRG: Generalized reduced gradient, CCM: Convex combination method, SAF: Score and accuracy function, WVF: Weighted value function, T2IFS: Type-2 intuitionistic fuzzy set, FR: Fuzzy rough, NCRO: Non-dominated sorting chemical reaction optimization, PHFP: Pythagorean hesitant fuzzy programming, NP: Neutrosophic programming, Y: Yes, N: No, PA: Proposed algorithm

2.1 Research Gaps

Table 1 shows a summary of the literature review of the different TP models. Through this analysis, the following topics are identified as research gaps:

- A thorough review of the prior literature reveals that there have only been very few studies conducted in the field of green transport using multi-objective 4D-TP models under uncertain environment (cf. [2, 12, 50]). Also, under the 3D-TP framework, only a few Scientists examined about carbon emission effect (cf. [31, 33, 49]). Under type-2 intuitionistic fuzzy environment, but till now no Scientist has analyzed the green 4D-TP model.
- Under different types of uncertainties such as T2FR [2], IF [50], LR-type IF [47], Neutrosophic [12], and fully neutrosophic [18], many researchers have investigated about 4D-TP models. Under type-2 intuitionistic fuzzy, yet, nobody examined the 4D-TP model.
- Many scientists focus mainly on transportation time and cost (cf. [32, 34, 35, 40]) but very few examine the carbon emission impact.
- Huge amounts of CaE are released into the atmosphere via transportation along the whole supply chain, and this is thought to be a major contributing cause to an artificial climate change. The governments support a few policies, including the CaE tax, cap, and offset scheme, to limit carbon discharge. This had a major impact on the transport network and also very few researchers have concentrated on this issue [21].
- Few academics have included delivery time, dwell time, loading & unloading time, and vehicle time etc. as goals in addition to other objectives related to the transportation system [16, 32].
- Many authors have typically regarded the parameters that make up the goals and constraints of TP models in a precise and deterministic manner. However, due to inadequate knowledge of item demand and supply constraints on available transportation options, etc., the parameters of real-world optimization issues are imprecise. As a result, uncertainty is a major factor in many decision-making issues.

To fill these gaps, a type-2 intuitionistic fuzzy environment is used to propose a mathematical model based on real-world situations, called green 4D-TP model. Within the context of the 4D-TP model, the suggested model is built with three objective functions-environmental (carbon emission), economical (transport cost), and satisfactory (transport time). These functions help DMs to stay credible in the cutthroat global market by increasing customer satisfaction, profit, and the environment. In the proposed model introduces some practical and realistic features such as delivery time and dwell time, CaE tax, and cap and offset scheme which have been rarely used by researchers. In our mentioned model, we want to increase order satisfaction by reducing both total cost, time and carbon emissions. Thus, the main contributions of the present study are described in the next subsection.

2.2 Some Contributions of the Present Study

- Under type-2 intuitionistic fuzzy, a real-life based multi-objective green 4D-TP model with a CaE tax, cap, and offset scheme is introduced.
- The fixed charge cost, total transportation cost, maintenance cost, dwell time, transportation time, loading and unloading time, and CaE cost by different modes of transportation are also taken into account for formulating the proposed model.
- To manage the uncertainties, a novel type-2 trapezoidal fuzzy number is presented and defuzzified using a suggested ranking function.
- To find the best pareto-optimal solution of the suggested green 4D-TP model, two methods namely non-fuzzy method and fuzzy method are appointed.
- Two real-world case studies are analyzed to demonstrate the applicability of the proposed investigation.

2.3 Structure of this Paper

For the benefit of its readers, the current study is set up as follows. Sect. 2 contains the literature review and Sect. 3 presents a few theoretical ideas that will be used throughout the work. The proposed model construction is described in Sect. 4 and its deterministic transformation is mention in Sect. 5. Secion 6 gives solution techniques. Numerical examples with result analysis have been outlined in Sects. 7 and 8 shows managerial insights of the proposed model. Finally, in Sect. 9, a conclusion regarding the paper's future research directions is drawn. In the last part of this paper, some references are listed.

3 Some Theoretical Concepts

A few preliminary concepts regarding T2-FS, IFS, and IFN are mentioned in this section.

• **T1-FS and T2-FS**: We employ T1-FSs when we are unable to identify a member's MF as either 0 or 1. Like this, we employ T2-FSs when the conditions are so hazy that it is difficult for us to determine the MG even as a precise number in the interval [0, 1]. The notion of a T2-FS was first presented by Zadeh in 1975 [52]. In contrast to a T1-FS, where the MG is a crisp value in [0, 1], a T2-FS has a fuzzy MF, meaning that the MG for each element of this set is a FS in [0, 1]. These sets can be applied in scenarios when the MF's structure or some of its parameters are unclear. We recommend some research papers [19, 30] for further information regarding T2-FS.

• **Definition 1** (I-FS [4]) We let U be a universal set and $u \in U$. An I-FS $\widetilde{I}$ in the set U is stated by a set of ordered triplet form:

$$\widetilde{I} = \left\{ \langle u, \phi_{\tilde{I}}(u), \psi_{\tilde{I}}(u) \rangle \mid u \in U \right\}$$

where $\phi_{\tilde{I}}(u)$ and $\psi_{\tilde{I}}(u)$ are respectively known as MG and non-MG such that $\phi_{\tilde{I}}(u) : U \to [0, 1]$, $\psi_{\tilde{I}}(u) : U \to [0, 1]$ and $0 \le \phi_{\tilde{I}}(u) + \psi_{\tilde{I}}(u) \le 1 \;\; \forall u \in U$.

• **Definition 2** (Trapezoidal I-FN [23]) A Trapezoidal I-FN or type-1 Trapezoidal I-FN is denoted by $\widetilde{T}$ and defined as $\widetilde{T} = \langle (\theta_1, \theta_2, \theta_3, \theta_4); \phi_{\tilde{T}}, \psi_{\tilde{T}} \rangle$ with the MF and non-MF are can be expressed as

$$\phi_{\tilde{T}}(u) = \begin{cases} \beta_{\tilde{T}} \left(\frac{u-\theta_1}{\theta_2-\theta_1} \right), & \text{if } \theta_1 \le u \le \theta_2, \\ \beta_{\tilde{T}}, & \text{if } \theta_2 \le u \le \theta_3, \\ \beta_{\tilde{T}} \left(\frac{\theta_4-u}{\theta_4-\theta_3} \right), & \text{if } \theta_3 \le u \le \theta_4, \\ 0, & \text{if } u < \theta_1 \textbf{ or } u > \theta_4, \end{cases}$$

$$\psi_{\tilde{T}}(u) = \begin{cases} \frac{\theta_2 - u + \gamma_{\tilde{T}}(u-\theta_1)}{\theta_2-\theta_1}, & \text{if } \theta_1 \le u \le \theta_2, \\ \gamma_{\tilde{T}}, & \text{if } \theta_2 \le u \le \theta_3, \\ \frac{u-\theta_3 + \gamma_{\tilde{T}}(\theta_4-u)}{\theta_4-\theta_3}, & \text{if } \theta_3 \le u \le \theta_4, \\ 1, & \text{if } u < \theta_1 \textbf{ or } u > \theta_4. \end{cases}$$

In this case, $\beta_{\tilde{T}}$ and $\gamma_{\tilde{T}}$ are the MD and non-MD such that $\beta_{\tilde{T}}, \gamma_{\tilde{T}} \in [0, 1]$ and $0 \le \beta_{\tilde{T}} + \gamma_{\tilde{T}} \le 1$.

• **Definition 3** (Trapezoidal T2I-FN) Here, we introduce a two-fold uncertainty environment that draws its foundation from the ideas of T2-FS and Trapezoidal I-FN. The definition of it and arithmetic operations are then covered. The characteristics of trapezoidal FNs make the selection of two-fold uncertainty crucial in the situation of the suggested model. Since the given FN can manage both symmetric and asymmetric uncertainties, posing a real-world problem makes more sense.

A Trapezoidal T2I-FN is denoted by $\widetilde{\widetilde{T}}$ in U, can be defined as

$$\widetilde{\widetilde{T}} = \langle (\widetilde{T}_1, \widetilde{T}_2, \widetilde{T}_3, \widetilde{T}_4); \Delta_{1\tilde{\tilde{T}}}, \Delta_{2\tilde{\tilde{T}}} \rangle, \tag{1}$$

where $\widetilde{T}_1$, $\widetilde{T}_2$, $\widetilde{T}_3$, $\widetilde{T}_4$ are all in Trapezoidal I-FN in nature and $\Delta_{1\tilde{\tilde{T}}}$, $\Delta_{2\tilde{\tilde{T}}}$ are respectively known as MG and non-MG of $\widetilde{T}$. As a result, equation (1) may be written as: $\widetilde{\widetilde{T}} = \langle (\langle (t_{11}, t_{12}, t_{13}, t_{14}); \phi_{\tilde{T}_1}, \psi_{\tilde{T}_1} \rangle, \langle (t_{21}, t_{22}, t_{23}, t_{24}); \phi_{\tilde{T}_2}, \psi_{\tilde{T}_2} \rangle, \langle (t_{31}, t_{32}, t_{33}, t_{34}); \phi_{\tilde{T}_3}, \psi_{\tilde{T}_3} \rangle, \langle (t_{41}, t_{42}, t_{43}, t_{44}); \phi_{\tilde{T}_4}, \psi_{\tilde{T}_4} \rangle); \Delta_{1\tilde{\tilde{T}}}, \Delta_{2\tilde{\tilde{T}}} \rangle$, where $\Delta_{1\tilde{\tilde{T}}} = \mathbf{min}\{\phi_{\tilde{T}_1}, \phi_{\tilde{T}_2}, \phi_{\tilde{T}_3}, \phi_{\tilde{T}_4}\}$ and $\Delta_{2\tilde{\tilde{T}}} = \mathbf{max}\{\psi_{\tilde{T}_1}, \psi_{\tilde{T}_2}, \psi_{\tilde{T}_3}, \psi_{\tilde{T}_4}\}$.

• **Arithmetic Operations of Trapezoidal T2I-FNs**:

Let, $\widetilde{\widetilde{T}} = \langle (\langle (t_{11}, t_{12}, t_{13}, t_{14}); \phi_{\tilde{T}_1}, \psi_{\tilde{T}_1} \rangle, \langle (t_{21}, t_{22}, t_{23}, t_{24}); \phi_{\tilde{T}_2}, \psi_{\tilde{T}_2} \rangle, \langle (t_{31}, t_{32}, t_{33}, t_{34}); \phi_{\tilde{T}_3}, \psi_{\tilde{T}_3} \rangle, \langle (t_{41}, t_{42}, t_{43}, t_{44}); \phi_{\tilde{T}_4}, \psi_{\tilde{T}_4} \rangle); \Delta_{1\tilde{\tilde{T}}}, \Delta_{2\tilde{\tilde{T}}} \rangle$ and $\widetilde{\widetilde{V}} = \langle (\langle (v_{11}, v_{12}, v_{13}, v_{14}); \phi_{\tilde{V}_1}, \psi_{\tilde{V}_1} \rangle, \langle (v_{21}, v_{22}, v_{23}, v_{24}); \phi_{\tilde{V}_2}, \psi_{\tilde{V}_2} \rangle, \langle (v_{31}, v_{32}, v_{33}, v_{34}); \phi_{\tilde{V}_3}, \psi_{\tilde{V}_3} \rangle, \langle (v_{41},$

$v_{42}, v_{43}, t_{44}); \phi_{\tilde{V}_4}, \psi_{\tilde{V}_4}\rangle); \Delta_{1\tilde{\tilde{V}}}, \Delta_{2\tilde{\tilde{V}}}\rangle$ be two Trapezoidal T2I-FNs. Hence, the addition, subtraction and scalar multiplication of these two Trapezoidal T2I-FNs can be written as:

(i) Addition: $\tilde{\tilde{T}} + \tilde{\tilde{V}} = \langle(\langle\langle(t_{11}+v_{11}, t_{12}+v_{12}, t_{13}+v_{13}, t_{14}+v_{14}); \phi_{\tilde{T}_1} \wedge \phi_{\tilde{V}_1}, \psi_{\tilde{T}_1} \vee \psi_{\tilde{V}_1}\rangle, \langle(t_{21}+v_{21}, t_{22}+v_{22}, t_{23}+v_{23}, t_{24}+v_{24}); \phi_{\tilde{T}_2} \wedge \phi_{\tilde{V}_2}, \psi_{\tilde{T}_2} \vee \psi_{\tilde{V}_2}\rangle, \langle(t_{31}+v_{31}, t_{32}+v_{32}, t_{33}+v_{33}, t_{34}+v_{34}); \phi_{\tilde{T}_3} \wedge \phi_{\tilde{V}_3}, \psi_{\tilde{T}_3} \vee \psi_{\tilde{V}_3}\rangle, \langle(t_{41}+v_{41}, t_{42}+v_{42}, t_{43}+v_{43}, t_{44}+v_{44}); \phi_{\tilde{T}_4} \wedge \phi_{\tilde{V}_4}, \psi_{\tilde{T}_4} \vee \psi_{\tilde{V}_4}\rangle); \Delta_{(1\tilde{\tilde{T}}+1\tilde{\tilde{V}})}, \Delta_{(2\tilde{\tilde{T}}+2\tilde{\tilde{V}})}\rangle$, where $\Delta_{(1\tilde{\tilde{T}}+1\tilde{\tilde{V}})} = \mathbf{min}\{\phi_{\tilde{T}_1} \wedge \phi_{\tilde{V}_1}, \phi_{\tilde{T}_2} \wedge \phi_{\tilde{V}_2}, \phi_{\tilde{T}_3} \wedge \phi_{\tilde{V}_3}, \phi_{\tilde{T}_4} \wedge \phi_{\tilde{V}_4}\}$ and $\Delta_{(2\tilde{\tilde{T}}+2\tilde{\tilde{V}})} = \mathbf{max}\{\psi_{\tilde{T}_1} \vee \psi_{\tilde{V}_1}, \psi_{\tilde{T}_2} \vee \psi_{\tilde{V}_2}, \psi_{\tilde{T}_3} \vee \psi_{\tilde{V}_3}, \psi_{\tilde{T}_4} \vee \psi_{\tilde{V}_4}\}$.

(ii) Subtraction: $\tilde{\tilde{T}} - \tilde{\tilde{V}} = \langle(\langle\langle(t_{11}-v_{44}, t_{12}-v_{43}, t_{13}-v_{42}, t_{14}-v_{41}); \phi_{\tilde{T}_1} \wedge \phi_{\tilde{V}_4}, \psi_{\tilde{T}_1} \vee \psi_{\tilde{V}_4}\rangle, \langle(t_{21}-v_{34}, t_{22}-v_{33}, t_{23}-v_{32}, t_{24}-v_{31}); \phi_{\tilde{T}_2} \wedge \phi_{\tilde{V}_3}, \psi_{\tilde{T}_2} \vee \psi_{\tilde{V}_3}\rangle, \langle(t_{31}-v_{24}, t_{32}-v_{23}, t_{33}-v_{22}, t_{34}-v_{21}); \phi_{\tilde{T}_3} \wedge \phi_{\tilde{V}_2}, \psi_{\tilde{T}_3} \vee \psi_{\tilde{V}_2}\rangle, \langle(t_{41}-v_{41}, t_{42}-v_{13}, t_{43}-v_{12}, t_{44}-v_{11}); \phi_{\tilde{T}_4} \wedge \phi_{\tilde{V}_1}, \psi_{\tilde{T}_4} \vee \psi_{\tilde{V}_1}\rangle); \Delta_{(1\tilde{\tilde{T}}-1\tilde{\tilde{V}})}, \Delta_{(2\tilde{\tilde{T}}-2\tilde{\tilde{V}})}\rangle$, where $\Delta_{(1\tilde{\tilde{T}}-1\tilde{\tilde{V}})} = \mathbf{min}\{\phi_{\tilde{T}_1} \wedge \phi_{\tilde{V}_4}, \phi_{\tilde{T}_2} \wedge \phi_{\tilde{V}_3}, \phi_{\tilde{T}_3} \wedge \phi_{\tilde{V}_2}, \phi_{\tilde{T}_4} \wedge \phi_{\tilde{V}_1}\}$ and $\Delta_{(2\tilde{\tilde{T}}-2\tilde{\tilde{V}})} = \mathbf{max}\{\psi_{\tilde{T}_1} \vee \psi_{\tilde{V}_4}, \psi_{\tilde{T}_2} \vee \psi_{\tilde{V}_3}, \psi_{\tilde{T}_3} \vee \psi_{\tilde{V}_2}, \psi_{\tilde{T}_4} \vee \psi_{\tilde{V}_1}\}$.

(iii) Multiplication: For any real value of c,

$$c \cdot \tilde{\tilde{T}} = \begin{cases} \langle(\langle\langle(ct_{11}, ct_{12}, ct_{13}, ct_{14}); \phi_{\tilde{T}_1}, \psi_{\tilde{T}_1}\rangle, \langle(ct_{21}, ct_{22}, ct_{23}, ct_{24}); \phi_{\tilde{T}_2}, \psi_{\tilde{T}_2}\rangle, \\ \langle(ct_{31}, ct_{32}, ct_{33}, ct_{34}); \phi_{\tilde{T}_3}, \psi_{\tilde{T}_3}\rangle, \langle(ct_{41}, ct_{42}, ct_{43}, ct_{44}); \phi_{\tilde{T}_4}, \psi_{\tilde{T}_4}\rangle); \\ \Delta_{1\tilde{\tilde{T}}}, \Delta_{2\tilde{\tilde{T}}}\rangle \text{ if } c \geq 0; \\ \langle(\langle\langle(ct_{41}, ct_{42}, ct_{43}, ct_{44}); \phi_{\tilde{T}_4}, \psi_{\tilde{T}_4}\rangle, \langle(ct_{31}, ct_{32}, ct_{33}, ct_{34}); \phi_{\tilde{T}_3}, \psi_{\tilde{T}_3}\rangle, \\ \langle(ct_{21}, ct_{22}, ct_{23}, ct_{24}); \phi_{\tilde{T}_2}, \psi_{\tilde{T}_2}\rangle, \langle(ct_{11}, ct_{12}, ct_{13}, ct_{14}); \phi_{\tilde{T}_1}, \psi_{\tilde{T}_1}\rangle); \\ \Delta_{1\tilde{\tilde{T}}}, \Delta_{2\tilde{\tilde{T}}}\rangle, \text{ if } c < 0. \end{cases}$$

3.1 *Suggested Defuzzification Method*

In real-world applications, the defuzzification process of FN is crucial for overcoming uncertainty. Actually, several defuzzification approaches are available in the literature, including CV-based reduction, linguistic approach, α-cut, and integration method, etc. Roy and Bhaumik [36] provide an efficient defuzzification strategy to address a RF and extract a triangular FN in order to solve a water management problem. Inspired by this method, we present a novel RF for Trapezoidal T2I-FN conversion into crisp forms. The RF tranforms each Trapezoidal T2I-FN into real line, mathematically it can be defined as: $\mathcal{R} : F(\tilde{\tilde{T}}) \to R$, where $F(\tilde{\tilde{T}})$ is a set of Trapezoidal T2I-FN. So, the RF for Trapezoidal T2I-FN can be expressed as:

$$\mathcal{R}(\tilde{\tilde{T}}) = \frac{1}{4}\left(\frac{\Delta_{1\tilde{\tilde{T}}} + \Delta_{2\tilde{\tilde{T}}}}{2}\right)\left\{\frac{t_{12}+t_{22}+t_{32}+t_{42}}{4} + \frac{t_{11}+t_{21}+t_{31}+t_{41}}{4} + \frac{t_{13}+t_{23}+t_{33}+t_{43}}{4} + \frac{t_{14}+t_{24}+t_{34}+t_{44}}{4}\right\}$$

where $\tilde{\tilde{\tilde{T}}}$ is mentioned in equation (1). Let $\tilde{\tilde{\tilde{T}}}$ and $\tilde{\tilde{\tilde{V}}}$ are two trapezoidal T2I-FNs then we can write:

i. If $\mathcal{R}(\tilde{\tilde{\tilde{T}}}) = \mathcal{R}(\tilde{\tilde{\tilde{V}}}) \Longrightarrow \tilde{\tilde{\tilde{T}}} =_{\mathcal{R}} \tilde{\tilde{\tilde{V}}}$, i.e., $\min\{\tilde{\tilde{\tilde{T}}}, \tilde{\tilde{\tilde{V}}}\} = \tilde{\tilde{\tilde{T}}}$ or, $\tilde{\tilde{\tilde{V}}}$,
ii. If $\mathcal{R}(\tilde{\tilde{\tilde{T}}}) < \mathcal{R}(\tilde{\tilde{\tilde{V}}}) \Longrightarrow \tilde{\tilde{\tilde{T}}} <_{\mathcal{R}} \tilde{\tilde{\tilde{V}}}$, i.e., $\min\{\tilde{\tilde{\tilde{T}}}, \tilde{\tilde{\tilde{V}}}\} = \tilde{\tilde{\tilde{T}}}$,
iii. If $\mathcal{R}(\tilde{\tilde{\tilde{T}}}) > \mathcal{R}(\tilde{\tilde{\tilde{V}}}) \Longrightarrow \tilde{\tilde{\tilde{T}}} >_{\mathcal{R}} \tilde{\tilde{\tilde{V}}}$, i.e., $\min\{\tilde{\tilde{\tilde{T}}}, \tilde{\tilde{\tilde{V}}}\} = \tilde{\tilde{\tilde{V}}}$.

3.2 Some Solution Concept

• **Pareto-optimal solution (POS)**: Let, Ω be the feasible set of solution and P^3 be a MOOP. A solution $X = \{x^{pos}_{isdvp}\} \in \Omega$ is said to be a POS of P^3 iff there is no other solution $X = \{x_{isdvp}\} \in \Omega$ such that $F^j(x_{isdvp}) < F^j(x^{pos}_{isdvp})$ for at least one j and $F^j(x_{isdvp}) \leq F^j(x^{pos}_{isdvp})$ for $j = 1, 2, 3$.

• **Anti-ideal solution (AIS)**: A solution $X = \{x^{ais}_{isdvp}\} \in \Omega$ is said to be a AIS of P^3 if it satisfies the followig condition: $F^j(x^{ais}_{isdvp}) = \mathbf{max}_{\{x_{isdvp}\}\in\Omega} F^j(x_{isdvp})$ for j = 1,2,3.

• **Ideal solution (IS)**: A solution $X = \{x^{is}_{isdvp}\} \in \Omega$ is said to be a AIS of P^3 if it satisfies the followig condition: $F^j(x^{is}_{isdvp}) = \mathbf{min}_{\{x_{isdvp}\}\in\Omega} F^j(x_{isdvp})$ for j= 1,2,3.

4 Mathematical Modelling

In this section, the proposed problem is described under two-fold uncertainty, namely the green multi-objective-based 4D-TP model with dwell time. This article examines an unusual strategic formulation from the perspectives of the environment, the economy, and customer happiness. This study examines a robust framework for green logistics, which transport various items via some types of vehicles from some sources to some destinations through some routes. The suggested model aims to concurrently identify the optimal solutions and budgets for possible sites while lowering the total conveyance cost, time, and CaE cost under a carbon reduction policy. In addition to the goals indicated above, the following factors are also taken into account: **(i)** Expenses such as processing fees, tolls, packing costs, safety costs, and so forth are classified as fixed-charge costs. **(ii)** The fuel consumption of the vehicles is taken into account when calculating the costs associated with logistics and CaEs. **(iii)** There may be a few obstacles in the route, such as a bridge crossing, broken down objects on the path, and so forth, that impact the travel time, also known as dwell time. **(iv)** Vehicle maintenance expenses are based on how far the path is. **(v)** The amount of time needed for goods to be loaded and unloaded, which raises the delivery time accuracy. **(vi)** Reduce the carbon footprint by adhering to the Kyoto Protocol (2007) for the purpose of minimizing transportation-related CaEs. **(vii)** The location and the products being transported determine the total budget of a possible site; these factors are considered decision variables and are therefore addressed as variable budgets. This research's primary goal is to develop and solve a green 4DTP model that accounts for great weather occurrences in order to control CaEs, a crucial aspect of daily life.

4.1 List of Notations and Assumptions

To explain the said model, the following notations and presumptions must be made:

Indexes:

(i) S & D: Number of sources and destinations, where $s \in \{1, 2, 3, \ldots, S\}$ and $d \in \{1, 2, 3, \ldots, D\}$.
(ii) V & I: Number of vehicles and different items, where $v \in \{1, 2, 3, \ldots, V\}$ and $i \in \{1, 2, 3, \ldots, I\}$.
(iii) P: Number of available routes or paths from each origin to each destination where $p \in \{1, 2, 3, \ldots, P\}$.

Parameters:

(iii) $\widetilde{\widetilde{A}}_{is}$ = Total available quantity of ith item at sth origin (in gm), which is trapezoidal T2I-FN in nature.
(iv) $\widetilde{\widetilde{R}}_{id}$ = Total required quantity of ith item at dth destination (in gm), which is trapezoidal T2I-FN in nature.
(v) $\widetilde{\widetilde{F}}_{vp}$ = Total capacity of vth conveyance via pth route (in gm), which is trapezoidal T2I-FN in nature.
(vi) E_{sdp} = Distance from sth origin to dth destination via pth route (in Km).
(vii) C_{isdvp} = Cost for transporting one unit of ith item using vth conveyance via pth route from sth origin to dth destination (in \$).
(viii) T_{isdvp} = Travel time for transporting one unit of ith item using vth conveyance via pth route from sth origin to dth destination (in hour).
(ix) Γ_{isdvp} = Amount of CaE for transporting one unit of ith item using vth conveyance via pth route from sth origin to dth destination (in gm).
(x) τ_{isdvp} = Dwell time for transporting one unit of ith item using vth conveyance via pth route from sth origin to dth destination (in hour).
(xi) μ_v = Maintenance cost of vth conveyance for per unit distance (in \$).
(xii) L_{isvp} = Loading time for transporting one unit of ith item using vth conveyance via pth route at sth origin (in hour).
(xiii) j = Number of objective functions and κ = CaE cap i.e., limited capacity of CaE permit.
(xiv) $\widehat{L}_{idvp}$ = Unloading time for transporting one unit of ith item using vth conveyance via pth route at dth destination (in hour).

(xv) Φ_{isdvp} = Fixed-charge cost (processing fees, safety costs, loading and unloading fees, tolls, packing fees, and so forth are a few examples) for transporting one unit of ith item using vth conveyance via pth route from sth origin to dth destination (in \$).

(xvi) Θ = Tax for each unit of CaE and G = a large quantity.

(xvii) L^j & U^j = Lower and upper bound of the jth objective function.

(xviii) Bu_{dp} = Total budget at dth destination via pth route and Λ = Cost of penalties per unit of emissions over the cap.

Decision variable:

– x_{isdvp} = the transported amount of ith item from sth source point to dth destination using vth conveyance through pth route (in \$).

Binary auxiliary variables (BAV):

- y_{isdvp} is a BAV whose value is 1 if ith item is transported from sth source point to dth destination point using vth conveyance through pth route and 0 elsewhere.
  i.e. $y_{isdvp} = \begin{cases} 1 & \textbf{if } x_{isdvp} \geq 0 \\ 0 & \textbf{elsewhere}. \end{cases}$

There are following **assumptions**:

(i) The amount of CaEs depends on the distance travelled by vehicles and their fuel consumption.

(ii) Modes of transport are heterogeneous in nature and delivered items are homogeneous in nature. Transportation costs are directly proportional to the units of shipped items.

(iii) The routes are excellent and seamless. No deterioration during transportation of goods.

(iv) The conveyance ($\widetilde{\widetilde{F_{vp}}}$), demand ($\widetilde{\widetilde{R_{id}}}$) and supply ($\widetilde{\widetilde{A_{is}}}$) parameters are in trapezoidal T2I-FN nature. Also, $\widetilde{\widetilde{F_{vp}}}, \widetilde{\widetilde{R_{id}}}, \widetilde{\widetilde{A_{is}}} > 0, \forall\ i, s, d, v, p$.

4.2 Mathematical Model Formulation

In this article, we develop a mathematical model with considering dwell time, offset policy and carbon tax namely a multi-objective-based green 4D-TP model which developed both clean environment and an economic condition. The first objective function (F^1) in this model refers to transportation cost; the second objective function (F^2) is transportation time; the third objective function (F^3) is CaE; and there are three constraints in this model: source constraints, demand constraints, and vehicle capacity constraints. Let, ith item transported from sth source point to dth destination using vth conveyance through pth route. The graphical form of the 4D-TP model is shown in Fig. 2. The challenge is determining how much amount is moved by various vehicles along various routes from sources to destinations so as to minimize the three-goal functions. Hence the mathematical form of the proposed green 4D-TP model can be represented as:

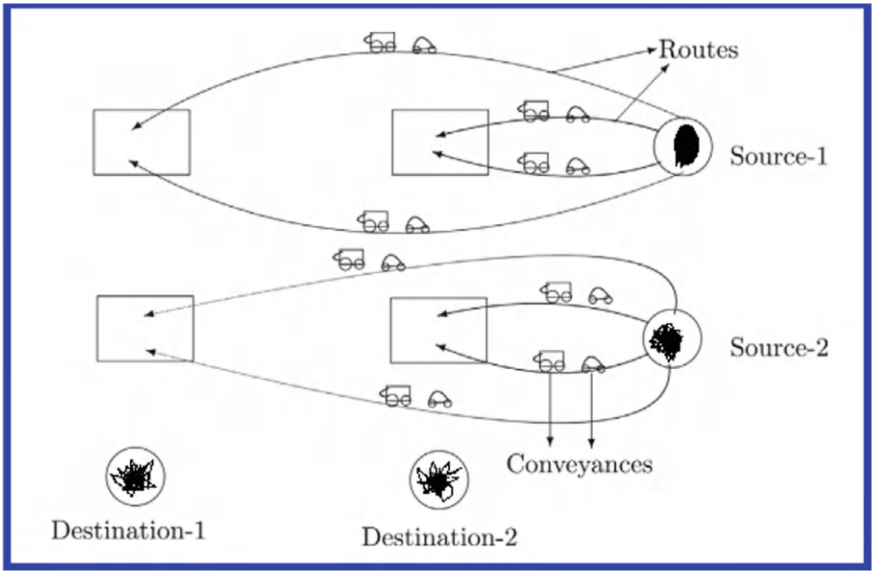

Fig. 2 Graphical form of 4D-TP model

<u>**Model-I**</u>:

$$\begin{cases}
\textbf{Min} \quad \underbrace{F^1}_{\textbf{Total cost}} = \sum\limits_{i,s,d,v,p} \left\{ C_{isdvp} \cdot x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp}\right) y_{isdvp} \right\} \\
\textbf{Min} \quad \underbrace{F^2}_{\textbf{Total time}} = \sum\limits_{i,s,d,v,p} \left\{ \left(L_{isvp} + \widehat{L}_{idvp}\right) x_{isdvp} + \left(T_{isdvp} + \tau_{isdvp}\right) y_{isdvp} \right\} \\
\textbf{Min} \quad \underbrace{F^3}_{\textbf{Total CaE}} = \Theta \sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} + \Lambda \left(\sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} - \kappa \right)^{+} \\
\textbf{subject to}: \\
\sum\limits_{d,v,p} x_{isdvp} \le \widetilde{\widetilde{A_{is}}}, \ \forall \ i, s. \quad (Ia) \\
\sum\limits_{s,v,p} x_{isdvp} \ge \widetilde{\widetilde{R_{id}}}, \ \forall \ i, d. \quad (Ib) \\
\sum\limits_{i,s,d} x_{isdvp} \le \widetilde{\widetilde{F_{vp}}}, \ \forall \ v, p. \quad (Ic) \\
\sum\limits_{i,s} \widetilde{\widetilde{A_{is}}} \ge \sum\limits_{i,d} \widetilde{\widetilde{R_{id}}} \ \ \& \ \sum\limits_{v,p} \widetilde{\widetilde{F_{vp}}} \ge \sum\limits_{i,d} \widetilde{\widetilde{R_{id}}} \quad (Id) \\
x_{isdvp} \ge 0, \quad y_{isdvp} = \begin{cases} 1 & \textbf{if} \quad x_{isdvp} > 0 \\ 0, & \textbf{elsewhere} \end{cases} \ \forall \ i, s, d, v, p. \quad (Ie) \\
x_{isdvp} \le G \cdot y_{isdvp} \quad \forall \ i, s, d, v, p. \quad (If) \\
\sum\limits_{i,s,v} \left\{ \left(\Theta \cdot \Gamma_{isdvp} + C_{isdvp}\right) x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp}\right) y_{isdvp} \right\} \\
\quad + \Lambda \left(\sum\limits_{i,s,v} \Gamma_{isdvp} \cdot x_{isdvp} - \kappa \right)^{+} \le Bu_{dp}, \ \forall \ d, p. \quad (Ig)
\end{cases} \tag{2}$$

The first objective function implies the economic aspect and this function determines the overall transportation cost (minimum). The 2nd objective function implies the customers' satisfaction and this function determines overall transportation time (minimum). The 3rd objective function implies the environmental aspects and this function optimizes the overall CaE cost under offset and tax policy. The constraints (Ia), (Ib), and (Ic) respectively imply that supply, demand, and conveyance capacity restriction. Constraint (Id) represent the feasibility criterion of the proposed model. The binary restrictions and non-negativity criteria make up constraints (Ie). The relationship between the decision variables and BAV is shown by the constraint (If). Finally, constraint (Ig) implies that the total cost at dth destination via pth route under the offset regulation and carbon tax does not exceed the optimal budget. Note that, $\left(\sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} - \kappa \right)^{+} = \textbf{max} \left(\sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} - \kappa , \ 0 \right)$ and $\sum\limits_{i,s,d,v,p} = \sum\limits_{i=1}^{I} \sum\limits_{s=1}^{S} \sum\limits_{i=d}^{D} \sum\limits_{v=1}^{V} \sum\limits_{p=1}^{P}$.

5 Deterministic Transformation

Since in model-I, the conveyance, demand, and supply parameters are in trapezoidal T2I-FN nature so we cannot solve easily. So, in this case, to translate trapezoidal T2I-FN into a deterministic form, a ranking defuzzification method is presented.

Model-II:

$$\begin{cases} \textbf{Min} \quad \underbrace{F^1}_{\textbf{Total cost}} = \sum\limits_{i,s,d,v,p} \left\{ C_{isdvp} \cdot x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp} \right) y_{isdvp} \right\} \\ \textbf{Min} \quad \underbrace{F^2}_{\textbf{Total time}} = \sum\limits_{i,s,d,v,p} \left\{ \left(L_{isvp} + \widehat{L}_{idvp} \right) x_{isdvp} + \left(T_{isdvp} + \tau_{isdvp} \right) y_{isdvp} \right\} \\ \textbf{Min} \quad \underbrace{F^3}_{\textbf{Total CaE}} = \Theta \sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} + \Lambda \left(\sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} - \kappa \right)^+ \\ \textbf{subject to}: \\ \sum\limits_{d,v,p} x_{isdvp} \leq \mathcal{R}(\widetilde{\widetilde{A}}_{is}), \ \forall \ i, s. \quad (IIh) \\ \sum\limits_{s,v,p} x_{isdvp} \geq \mathcal{R}(\widetilde{\widetilde{R}}_{id}), \ \forall \ i, d. \quad (IIi) \\ \sum\limits_{i,s,d} x_{isdvp} \leq \mathcal{R}(\widetilde{\widetilde{F}}_{vp}), \ \forall \ v, p. \quad (IIj) \\ \sum\limits_{i,s} \mathcal{R}(\widetilde{\widetilde{A}}_{is}) \geq \sum\limits_{i,d} \mathcal{R}(\widetilde{\widetilde{R}}_{id}) \ \ \& \ \sum\limits_{v,p} \mathcal{R}(\widetilde{\widetilde{F}}_{vp}) \geq \sum\limits_{i,d} \mathcal{R}(\widetilde{\widetilde{R}}_{id}) \quad (IIk) \\ \textbf{the constraints} \ (Ie) - (Ig). \end{cases} \tag{3}$$

From the 3rd objective function we classify two possible zones based on the carbon cap and when $\left\{ \sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} \right\} \leq \kappa$, the first one occurs. When $\left\{ \sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} \right\} \geq \kappa$, the 2nd one occurs. Based on the first criteria, the new sub model (i.e., Model-IIA) is formulated as follows:

Model-IIA:

$$\begin{cases} \textbf{Min} \quad \underbrace{F^1}_{\textbf{Total cost}} = \sum\limits_{i,s,d,v,p} \left\{ C_{isdvp} \cdot x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp} \right) y_{isdvp} \right\} \\ \textbf{Min} \quad \underbrace{F^2}_{\textbf{Total time}} = \sum\limits_{i,s,d,v,p} \left\{ \left(L_{isvp} + \widehat{L}_{idvp} \right) x_{isdvp} + \left(T_{isdvp} + \tau_{isdvp} \right) y_{isdvp} \right\} \\ \textbf{Min} \quad \underbrace{F^3}_{\textbf{Total CaE}} = \Theta \sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} \quad (IIAl) \\ \textbf{subject to the constraints (Ie)-(If) and (IIh)–(IIk),} \\ \left\{ \sum\limits_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} \right\} \leq \kappa, \quad (IIAm) \\ \sum\limits_{i,s,v} \left\{ \left(\Theta \cdot \Gamma_{isdvp} + C_{isdvp} \right) x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp} \right) y_{isdvp} \right\} \leq Bu_{dp}, \ \forall \ d, p. \\ \qquad (IIAn) \end{cases} \tag{4}$$

Based on the second criteria, the new sub model (i.e., Model-IIB) is formulated as follows:

Model-IIB:

$$\left\{\begin{array}{l}
\textbf{Min} \quad \underbrace{F^1}_{\textbf{Total cost}} = \sum_{i,s,d,v,p} \left\{ C_{isdvp} \cdot x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp}\right) y_{isdvp} \right\} \\
\textbf{Min} \quad \underbrace{F^2}_{\textbf{Total time}} = \sum_{i,s,d,v,p} \left\{ \left(L_{isvp} + \widehat{L}_{idvp}\right) x_{isdvp} + \left(T_{isdvp} + \tau_{isdvp}\right) y_{isdvp} \right\} \\
\textbf{Min} \quad \underbrace{F^3}_{\textbf{Total CaE}} = \Theta \sum_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} + \Lambda \left(\sum_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} - \kappa \right) \quad (IIBp) \\
\textbf{subject to the constraints (Ie)-(If) and (IIh)-(IIk),} \\
\left\{ \sum_{i,s,d,v,p} \Gamma_{isdvp} \cdot x_{isdvp} \right\} \geq \kappa, \qquad (IIBq) \\
\sum_{i,s,v} \left\{ C_{isdvp} \cdot x_{isdvp} + \left(\mu_v \cdot E_{sdp} + \Phi_{isdvp}\right) y_{isdvp} \right\} \\
\quad + \left(\Lambda + \Theta\right) \sum_{i,s,v} \Gamma_{isdvp} \cdot x_{isdvp} - \left(\Lambda \cdot \kappa\right) \leq Bu_{dp}, \ \forall \ d, p. \ (IIBr)
\end{array}\right. \tag{5}$$

6 Solution Procedures

Under the T2I-FN environment, the proposed model i.e., multi-objective-based green 4D-TP model is analysed. Using RF principle with trapezoidal T2I-FN, Model-I is transformed into a deterministic model (i.e., Model-II). The DM in a MOOP must simultaneously optimize the opposing objective functions. Selecting an ideal point at which all of the objective functions reach their maximum values is so challenging. As a result, we need to look at POSs. Many researchers have solved MOOP using various non-fuzzy and fuzzy methods such as FGP [9], NCP [35], IFP [37], WGP [38], GP [43], GC [39], and FP [24] methods. To solve the MOOP, none of the aforementioned approaches-IFP, GC, FP, and NCP-need to know the DM's objectives (weights or goals) in advance. Of the four approaches mentioned above, GC method and FP method offer a straightforward mathematical framework that facilitates comprehension and application. Furthermore, in comparison to the other approaches, the two methods consistently produce a POS in a comparatively short amount of memory and processing time. To discover a Pareto optimal front, GC and FP method use the concept of the shortest distance from the ideal point; the techniques do not require any prior knowledge about objective functions from the DM. To solve Model-II, we utilize both a fuzzy approach i.e., FP method, and a non-fuzzy approach i.e., the GC method. Based on the carbon cap criterion, Model-II is split into two sub models namely Model-IIA and Model-IIB. Next, these two models are solved separately in order to derive POS. The best solution for Model-II is then determined by comparing the solutions. However, the POS of the corresponding model is the optimal solution

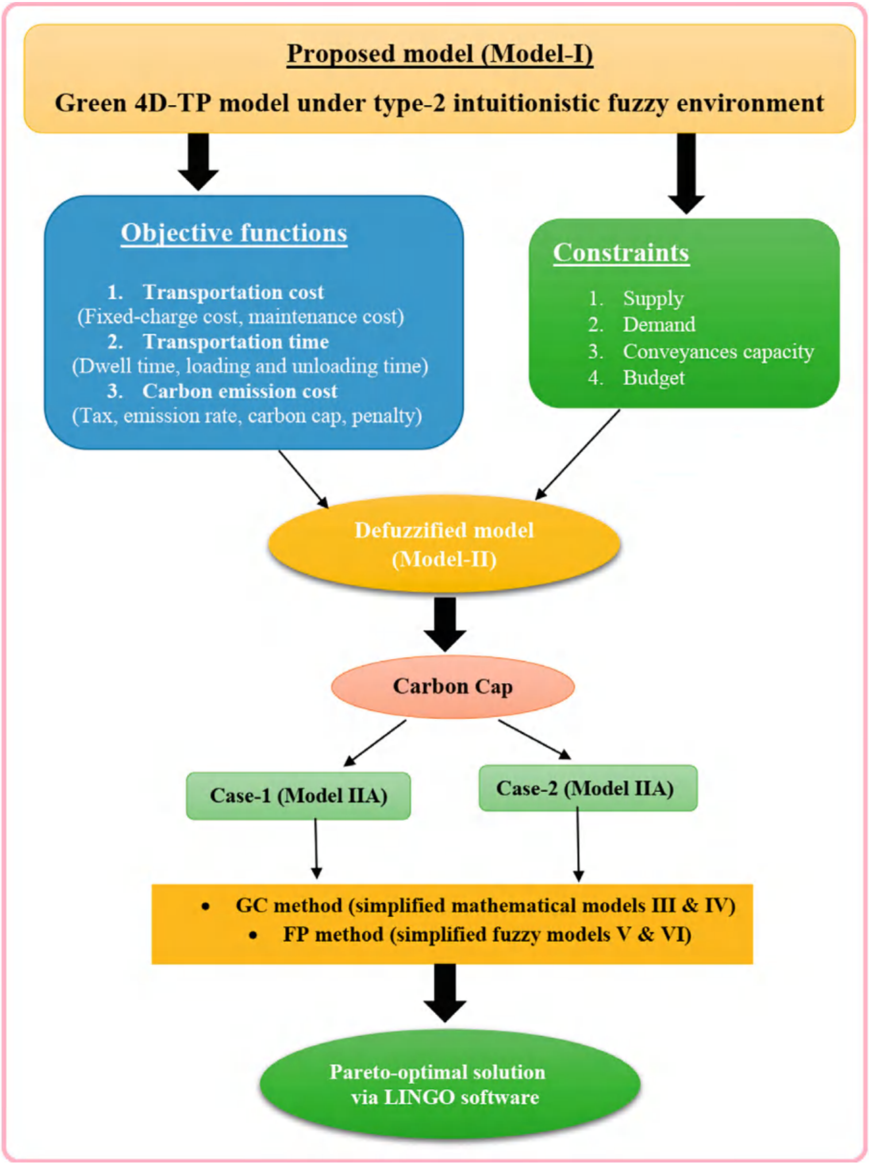

Fig. 3 Schematic diagram of the proposed model

of Model-II if one of the two models (i.e., Models-IIA and IIB) has a POS and the other has no feasible solution. Schematic diagram of the proposed model (model formulation to solution techniques) is represented in Fig. 3.

6.1 GC Method

To determine the POS, using GC method [39] Model-II can be reformulated as:

Model-III: (Based on Model-IIA)

$$\begin{cases} \textbf{Min} \left(\sqrt{\sum_{j=1}^{3} \left\{ \frac{\mathbf{F}^j(x_{isdvp}) - F^j_{\mathbf{min}}}{F^j_{\mathbf{max}} - F^j_{\mathbf{min}}} \right\}^2} \right) \\ \textbf{s. t.} \\ \textbf{the constraints (Ie)-(If) and (IIh)-(IIk),} \\ \textbf{the constraints (IIAm)-(IIAn).} \end{cases} \tag{6}$$

Model-IV: (Based on Model-IIB)

$$\begin{cases} \textbf{Min} \left(\sqrt{\sum_{j=1}^{3} \left\{ \frac{\mathbf{F}^j(x_{isdvp}) - F^j_{\mathbf{min}}}{F^j_{\mathbf{max}} - F^j_{\mathbf{min}}} \right\}^2} \right) \\ \textbf{s. t.} \\ \textbf{the constraints (Ie)-(If) and (IIh)-(IIk),} \\ \textbf{the constraints (IIBq)-(IIBr).} \end{cases} \tag{7}$$

• **Theorem 1** Let $X = \{x_{isdvp}\}$ be an optimal solution of Model-III (or Model-IV), then it must also be a POS $X = \{x_{isdvp}\}$ of Model-IIA (or Model-IIB).

***P*roof** By contradiction, this Theorem can be demonstrated. Let $X = \{x_{isdvp}\}$ be an optimal solution of Model-III (or Model-IV) which is not a POS of Model-IIA (or Model-IIB). Then, there exists a solution $X = \{x^*_{isdvp}\}$ such that $X = \{x^*_{isdvp}\}$ dominates $X = \{x_{isdvp}\}$.

$$\Rightarrow \left(\sqrt{\sum_{j=1}^{3} \left\{ \frac{\mathbf{F}^j(x^*_{isdvp}) - F^j_{\mathbf{min}}}{F^j_{\mathbf{max}} - F^j_{\mathbf{min}}} \right\}^2} \right) < \left(\sqrt{\sum_{j=1}^{3} \left\{ \frac{\mathbf{F}^j(x_{isdvp}) - F^j_{\mathbf{min}}}{F^j_{\mathbf{max}} - F^j_{\mathbf{min}}} \right\}^2} \right)$$

which stands in stark contrast to the idea that $X = \{x_{isdvp}\}$ is an optimal solution of Model-III (or Model-IV).

6.2 FP Method

To determine the POS, using FP method [24] Model-II can be reformulated as:

Model-V: (Based on Model-IIA)

$$\begin{cases} \textbf{Max} \quad \eta \\ \textbf{s. t.} \\ F^j(x_{isdvp}) + \eta\,(U^j - L^j) \leq U^j \quad ; \; j = 1, 2, 3. \\ \textbf{the constraints (Ie)-(If) and (IIh)-(IIk),} \\ \textbf{the constraints (IIAm)-(IIAn),} \\ \eta \geq 0. \end{cases} \tag{8}$$

<u>Model-VI</u>: (Based on Model-IIB)

$$\begin{cases} \textbf{Max} \quad \eta \\ \textbf{s. t.} \\ F^j(x_{isdvp}) + \eta\,(U^j - L^j) \leq U^j \quad ; \; j = 1, 2, 3. \\ \textbf{the constraints (Ie)-(If) and (IIh)-(IIk),} \\ \textbf{the constraints (IIBq)-(IIBr),} \\ \eta \geq 0. \end{cases} \tag{9}$$

In this case, η represents the level of satisfaction of a solution and $\eta = \textbf{min}\{\varpi(F^j(x_{isdvp})) : j = 1, 2, 3\}$, where $\varpi(F^j(x_{isdvp}))$ is a MF similar to each j^{th} objective function and it is defined below:

$$\varpi(F^j(x_{isdvp})) = \begin{cases} 1, & \text{if } F^j(x_{isdvp}) \leq L^j, \\ \frac{U^j - F^j(x_{isdvp})}{U^j - L^j}, & \text{if } L^j \leq F^j(x_{isdvp}) \leq U^j, \\ 0, & \text{if } F^j(x_{isdvp}) \geq U^j. \end{cases}$$

where $L^j = F^{jj}, F^{kj} = F^j(\,(x_{isdvp})^k\,)\,; j = 1, 2, 3.$ and $U^j = \textbf{max}\{\,F^{1k}, F^{2k}, F^{3k}\,\}$. To gain further insight into this methodology, please consult the paper [24].

• **Theorem 2** Let $X = \{x_{isdvp}\}$ be an optimal solution of Model-V (or Model-VI), then it must also be a POS $X = \{x_{isdvp}\}$ of Model-IIA (or Model-IIB).
Proof See [8, 39].

7 Numerical Examples

Here, two real-life-based cases are provided to support the suggested paradigm and resolution methods. Here, two real-life-based cases are provided to support the suggested paradigm and resolution methods. The parameters are unknown because of incomplete information, the state of the market, and the climate. Hence, DMs must face certain challenging issues. We consider a real-world issue that has to do with this kind of uncertainty.

7.1 Case Study-1

"Raja Paribahan" is a reputed and trusted transport company in Uluberia, Howrah, West Bengal, India. Various types of transport such as trucks, tempos, goods, boats, launches, buses, motorcycles, totos, rickshaws, autos etc. are available in this company. Their aim is to find the two types of items (LED TV and Fridge) to be transported from the two Whole Seller shops (at Diamond Harbour and Alipore) to the two local shops (at Amta and Udaynarayanpur) via two different routes (Uluberia and Santragachi) using two different vehicles (trucks and boats) in such way that at the minimum possible transportation cost and transportation time including the dwell time, delivery time and a minimum amount of CaE under the offset regulation, cap and tax. It is mentioned that the company is given a CaE cap under the cap, tax, and offset policy. The company may only be required to pay the standard tax per unit emission if its discharges fall short of the cap. However, in addition to the standard carbon tax, the company must pay an offset as a penalty if its emissions exceed the cap. All supporting hypothetical data for this phenomenon are designed in the next section.

7.2 Input Data

Table 2 represents the fixed-charge, transportation cost, and dwell time. Transportation time and CaE cost are described in Table 3. The loading and unloading times and distances are reported in Table 4. Availabilities, demands, and conveyance capacities are given in Table 5. We considered these three constraints are type-2 trapezoidal T2I-FN in nature and their crisp values are in Table 5. The maintenance cost: $\mu_1 = 0.75$ (in \$) and $\mu_2 = 0.40$ (in \$), CaE cap: $\kappa = 1653$, CaE tax $(\Theta) = 9$.

7.3 Case Study-2

The other parameters are the same as in the previous case study-1, and here we consider the CaE cap $\kappa = 742$.

7.4 Results Analysis

Here, based on the non-fuzzy and fuzzy methods we determined the POS of Model-II. In order to provide the DM with an option to acquire desired results, the application examples are conducted using two distinct solution strategies with similar input datas. To determine the optimal results of these models via an optimization tool namely

Table 2 Fixed-charge (Φ_{isdvp}), Transportation cost (C_{isdvp}) and dwell time (τ_{isdvp})

i	s	d	v	Φ_{isdv1}	Φ_{isdv2}	C_{isdv1}	C_{isdv2}	τ_{isdv1}	τ_{isdv2}
1	1	1	1	235	47	66	127	81	63
			2	137	293	170	290	78	124
		2	1	285	216	313	219	248	89
			2	203	355	210	89	50	67
	2	1	1	134	120	234	207	178	216
			2	291	427	129	242	63	91
		2	1	143	329	143	257	78	55
			2	225	241	127	303	69	82
2	1	1	1	55	138	61	108	81	107
			2	300	233	130	246	169	88
		2	1	224	97	120	159	62	80
			2	232	137	150	100	92	75
	2	1	1	230	290	370	407	133	248
			2	275	394	416	381	175	79
		2	1	178	138	180	252	85	66
			2	119	149	234	127	150	67

Table 3 Transportation time (T_{isdvp}) and CaE cost (Γ_{isdvp})

i	s	d	v	T_{isdv1}	T_{isdv2}	Γ_{isdv1}	Γ_{isdv2}
1	1	1	1	189	80	101	127
			2	145	276	70	98
		2	1	167	322	214	178
			2	203	209	155	90
	2	1	1	88	136	73	142
			2	291	127	125	240
		2	1	143	245	143	259
			2	221	135	120	233
2	1	1	1	76	130	69	109
			2	245	178	77	245
		2	1	223	77	121	150
			2	232	137	153	102
	2	1	1	237	291	372	344
			2	270	222	315	104
		2	1	176	139	183	255
			2	111	140	230	121

Table 4 Loading time (L_{isvp}), unloading time ($\widehat{L}_{idvp}$) and distance (E_{sdp})

i	s	v	p	L_{isvp}	i	d	v	p	$\widehat{L}_{idvp}$	s	d	p	E_{sdp}
1	1	1	1	75	1	1	1	1	87	1	1	1	96
			2	139				2	175				
		2	1	245			2	1	102			2	132
			2	105				2	78				
	2	1	1	92		2	1	1	170		2	1	80
			2	116				2	68				
		2	1	214			2	1	66			2	207
			2	80				2	132				
2	1	1	1	84	2	1	1	1	184	2	1	1	55
			2	258				2	170				
		2	1	97			2	1	111			2	128
			2	140				2	136				
	2	1	1	123		2	1	1	83		2	1	67
			2	158				2	95				
		2	1	168			2	1	160			2	246
			2	250				2	51				

LINGO 18.0 software on HP laptop with configuration 2.10 GHz Intel Core i5, 8 GB RAM is considered. The optimal solutions of both case studies are described in Tables 6 and 7. Based on case study-1, the optimal solution is derived by GC method and its graphical representation is given in Fig. 4. Based on case study-2, the optimal solution is derived by FP method and its graphical representation is given in Fig. 5. Table 6 leads us to the conclusion that, in terms of decreased computing complexity and sustained accuracy, the solution produced by the suggested GC method is better than the solutions acquired by the other solution approaches. But, Table 7 shows that it occurs in the exact opposite way. Consequently, depending on the nature of the issue, the DM is free to select any one of the solution approaches. As a result, the conclusions reached are verified in light of the solution procedures. Furthermore, the analytical findings show that the financial and customer satisfaction and environmental goals have all been optimized and that the best outcomes have also been identified. We investigate the possibility that a company will choose the less expensive vehicles that produce more carbon if the cap is higher than the threshold. Due to this, the company just has to pay the standard CaE tax and does not need to offset any excess emissions, which lowers both the overall transportation cost and the CaE cost. As a result, the business will profit more. Once more, the company selects the expensive conveyances with lower emissions if the margin is less than the total emission. As a result, the overall cost of transport and CaEs rise since they must provide an offset as a penalty, which lowers their profit. Because of this, the company will constantly be concerned about CaEs brought on by the movement of goods. To raise its carbon cap, the corporation can also invest in carbon offset projects. Subsequently, the carbon

Table 5 Availabilities ($\widetilde{\widetilde{A}}_{is}$), demands ($\widetilde{\widetilde{R}}_{id}$) and conveyances capacities ($\widetilde{\widetilde{F}}_{vp}$)

$\widetilde{\widetilde{A}}_{11}$: $\langle\langle\langle(30, 45, 80, 125); 0.55, 0.2\rangle, \langle(64, 98, 120, 145); 0.75, 0.3\rangle, \langle(105, 130, 160, 185)$ $; 0.60, 0.15\rangle, \langle(110, 160, 180, 200); 0.75, 0.3\rangle$, $\mathcal{R}(\widetilde{\widetilde{A}}_{11}) = 102.90$
$\widetilde{\widetilde{A}}_{12}$: $\langle\langle\langle(75, 80, 110, 130); 0.50, 0.2\rangle, \langle(100, 140, 170, 190); 0.45, 0.1\rangle, \langle(105, 130, 160, 190)$ $; 0.70, 0.1\rangle, \langle(180, 215, 220, 245); 0.25, 0.15\rangle$, $\mathcal{R}(\widetilde{\widetilde{A}}_{12}) = 82.47$
$\widetilde{\widetilde{A}}_{21}$: $\langle\langle\langle(60, 75, 80, 145); 0.30, 0.25\rangle, \langle(64, 95, 120, 140); 0.35, 0.30\rangle, \langle(115, 130, 160, 180)$ $; 0.30, 0.10\rangle, \langle(130, 140, 150, 165); 0.40, 0.20\rangle$, $\mathcal{R}(\widetilde{\widetilde{A}}_{21}) = 77.91$
$\widetilde{\widetilde{A}}_{22}$: $\langle\langle\langle(65, 75, 80, 135); 0.55, 0.2\rangle, \langle(165, 195, 220, 245); 0.85, 0.3\rangle, \langle(195, 230, 260, 285)$ $; 0.50, 0.25\rangle, \langle(200, 210, 220, 245); 0.27, 0.13\rangle$, $\mathcal{R}(\widetilde{\widetilde{A}}_{22}) = 68.39$
$\widetilde{\widetilde{R}}_{11}$: $\langle\langle\langle(50, 55, 80, 120); 0.45, 0.2\rangle, \langle(65, 90, 120, 140); 0.60, 0.3\rangle, \langle(105, 130, 160, 180)$ $; 0.30, 0.2\rangle, \langle(110, 125, 130, 145); 0.25, 0.13\rangle$, $\mathcal{R}(\widetilde{\widetilde{R}}_{11}) = 81.30$
$\widetilde{\widetilde{R}}_{12}$: $\langle\langle\langle(20, 45, 80, 110); 0.15, 0.12\rangle, \langle(50, 98, 120, 155); 0.55, 0.13\rangle, \langle(100, 130, 160, 180)$ $; 0.30, 0.11\rangle, \langle(164, 198, 220, 249); 0.70, 0.3\rangle$, $\mathcal{R}(\widetilde{\widetilde{R}}_{12}) = 172.48$
$\widetilde{\widetilde{R}}_{21}$: $\langle\langle\langle(78, 145, 180, 225); 0.15, 0.1\rangle, \langle(80, 90, 120, 140); 0.60, 0.3\rangle, \langle(95, 130, 160, 180)$ $; 0.60, 0.15\rangle, \langle(99, 110, 120, 155); 0.25, 0.23\rangle$, $\mathcal{R}(\widetilde{\widetilde{R}}_{21}) = 136.55$
$\widetilde{\widetilde{R}}_{22}$: $\langle\langle\langle(80, 85, 90, 125); 0.22, 0.1\rangle, \langle(94, 125, 140, 155); 0.42, 0.13\rangle, \langle(105, 130, 150, 195)$ $; 0.26, 0.15\rangle, \langle(120, 135, 140, 175); 0.33, 0.10\rangle$, $\mathcal{R}(\widetilde{\widetilde{R}}_{22}) = 117.43$
$\widetilde{\widetilde{F}}_{11}$: $\langle\langle\langle(75, 85, 90, 125); 0.55, 0.2\rangle, \langle(61, 95, 126, 158); 0.35, 0.2\rangle, \langle(105, 130, 160, 180)$ $; 0.60, 0.15\rangle, \langle(124, 175, 220, 245); 0.35, 0.23\rangle$, $\mathcal{R}(\widetilde{\widetilde{F}}_{11}) = 95.38$
$\widetilde{\widetilde{F}}_{12}$: $\langle\langle\langle(49, 55, 80, 121); 0.35, 0.12\rangle, \langle(60, 91, 120, 149); 0.75, 0.3\rangle, \langle(110, 130, 160, 185)$ $; 0.30, 0.21\rangle, \langle(126, 198, 220, 245); 0.35, 0.20\rangle$, $\mathcal{R}(\widetilde{\widetilde{F}}_{12}) = 88.57$
$\widetilde{\widetilde{F}}_{21}$: $\langle\langle\langle(45, 65, 80, 95); 0.25, 0.20\rangle, \langle(64, 98, 100, 145); 0.30, 0.20\rangle, \langle(75, 80, 90, 95)$ $; 0.46, 0.10\rangle, \langle(90, 98, 120, 145); 0.55, 0.23\rangle$, $\mathcal{R}(\widetilde{\widetilde{F}}_{21}) = 159.31$
$\widetilde{\widetilde{F}}_{22}$: $\langle\langle\langle(20, 45, 75, 120); 0.45, 0.12\rangle, \langle(56, 98, 120, 140); 0.75, 0.3\rangle, \langle(100, 130, 150, 185)$ $; 0.60, 0.15\rangle, \langle(140, 198, 220, 245); 0.70, 0.31\rangle$, $\mathcal{R}(\widetilde{\widetilde{F}}_{22}) = 140.54$

policy assists the policymaker in cutting CaEs as well as the organisation in making the best judgements possible to enhance economic performance (Table 8).

7.5 Sensitivity Analysis

The effects of the input parameters on the stability of the proposed model will be examined in order to analyse the effect of the parameters on the experimental results, since the major research parameters described above are generated from a similar set of test data. Here, we vary the values of the parameters to test the stability of the suggested model and the robustness of POSs. The complexity in the suggested model arises when the ranges are determined after the object has undergone parametric modifications yet the POSs that are obtained stay the same. In fact, the more limits and choice factors there are, the more complex it becomes. Das and Roy [8] presented an easy method to examine the stability and sensitivity of an optimization model. Here,

Table 6 The POS of Model-II based on case study-1

Solution methods	Memory (K)	CPU time(s)	Solution of Model-IIA	Solution of Model-IIB
GC method	75	0.46	(Let, $Bu_{11} = 3572$, $Bu_{12} = 1962$, $Bu_{21} = 4239$, $Bu_{22} = 2875$, $\eta = 0.692$.) $x_{11111} = 28.53$, $x_{21121} = 35.40$, $x_{12211} = 19.81$, $x_{21122} = 46.02$, $x_{21112} = 11.75$, $x_{12211} = 07.80$, $x_{21111} = 51.05$, $x_{12221} = 25.32$, $x_{12222} = 34.21$. $\mathbf{F^1 = 23502}$, $\mathbf{F^2 = 14572}$, $\mathbf{F^3 = 37152.40}$.	NFS
FP method	80	0.42	(Let, $Bu_{11} = 3794$, $Bu_{12} = 2362$, $Bu_{21} = 4981$, $Bu_{22} = 3068$, $\eta = 0.692$.) $x_{12221} = 23.98$, $x_{21221} = 41.57$, $x_{22211} = 19.43$, $x_{21122} = 89.01$, $x_{21112} = 60.41$, $x_{12211} = 9.55$, $x_{21112} = 27.48$, $x_{12221} = 10.88$, $x_{12222} = 67.04$. $F^1 = 32480$, $F^2 = 14970$, $F^3 = 40256$.	NFS
NCP method [35]	124	0.81	(Let, $Bu_{11} = 6172$, $Bu_{12} = 2562$, $Bu_{21} = 4739$, $Bu_{22} = 3175$, $\eta = 0.543$.) $x_{11211} = 65.99$, $x_{11121} = 41.76$, $x_{12211} = 08.35$, $x_{21122} = 52.68$, $x_{21112} = 45.27$, $x_{12211} = 20.55$, $x_{21111} = 24.69$, $x_{12221} = 28.77$, $x_{12222} = 43.23$. $F^1 = 31576$, $F^2 = 15031$, $F^3 = 42170$. (Take $\varpi = 0.386$)	NFS
IFP method [37]	96	0.74	(Let, $Bu_{11} = 5491$, $Bu_{12} = 3970$, $Bu_{21} = 4299$, $Bu_{22} = 3170$, $\eta = 0.386$.) $x_{11111} = 28.53$, $x_{21121} = 35.40$, $x_{12211} = 19.81$, $x_{21122} = 46.02$, $x_{21112} = 11.75$, $x_{12211} = 07.80$, $x_{21111} = 51.05$, $x_{12221} = 25.32$, $x_{12222} = 34.21$. $F^1 = 24732$, $F^2 = 14487.25$, $F^3 = 37469$. (Take $\varpi = 0.386$)	NFS

Table 7 The POS of Model-II based on case study-2

Solution methods	Memory (K)	CPU time(s)	Solution of Model-IIB	Solution of Model-IIA
GC method	77	2.39	(Let, $Bu_{11} = 3588$, $Bu_{12} = 1967$, $Bu_{21} = 4259$, $Bu_{22} = 2888$, $\eta = 0.689$.) $x_{11111} = 78.04$, $x_{21121} = 43.75$, $x_{12211} = 31.40$, $x_{21112} = 42.67$, $x_{21122} = 37.68$, $x_{12211} = 55.04$, $x_{21111} = 27.85$, $x_{12221} = 29.45$, $x_{12222} = 28.60$. $F^1 = 35467$, $F^2 = 13225$, $F^3 = 37967.75$.	NFS
FP method	84	0.97	(Let, $Bu_{11} = 4372$, $Bu_{12} = 2269$, $Bu_{21} = 4231$, $Bu_{22} = 3405$, $\eta = 0.838$.) $x_{11211} = 18.70$, $x_{11121} = 22.98$, $x_{12211} = 33.78$, $x_{22122} = 54.97$, $x_{21112} = 59.31$, $x_{12211} = 14.70$, $x_{21111} = 68.44$, $x_{12221} = 25.90$, $x_{12222} = 36.68$. $\mathbf{F^1 = 33061}$, $\mathbf{F^2 = 13965}$, $\mathbf{F^3 = 36380}$.	NFS
NCP method [35]	113	2.10	(Let, $Bu_{11} = 3841$, $Bu_{12} = 2430$, $Bu_{21} = 3758$, $Bu_{22} = 2973$, $\eta = 0.672$.) $x_{21111} = 35.71$, $x_{21121} = 44.97$, $x_{12211} = 32.45$, $x_{21122} = 78.50$, $x_{21112} = 56.36$, $x_{22211} = 19.55$, $x_{21111} = 46.39$, $x_{12221} = 80.31$, $x_{12222} = 09.78$. $F^1 = 23502$, $F^2 = 14572$, $F^3 = 3752.40$.	NFS
IFP method [37]	87	1.88	(Let, $Bu_{11} = 3772$, $Bu_{12} = 1939$, $Bu_{21} = 4235$, $Bu_{22} = 2870$, $\eta = 0.758$.) $x_{11112} = 67.84$, $x_{21121} = 45.79$, $x_{12211} = 28.30$, $x_{21122} = 36.78$, $x_{21112} = 59.23$, $x_{22211} = 28.65$, $x_{21111} = 61.27$, $x_{12221} = 25.49$, $x_{12222} = 37.50$. $F^1 = 34680$, $F^2 = 11972$, $F^3 = 39784$. (Take $\varpi = 0.386$) (Take $\varpi = 0.673$)	NFS

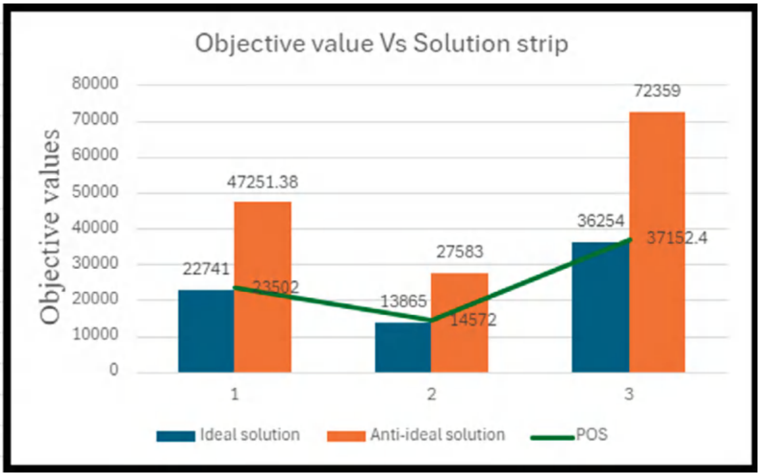

Fig. 4 Graphical form of the GC method for case study-1

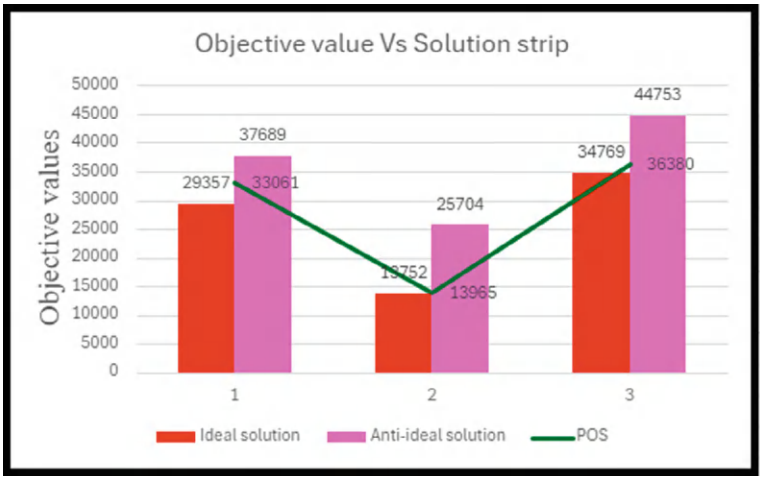

Fig. 5 Graphical form of the FP method for case study-2

we used this concept. Let, $\mathcal{R}(\widetilde{\widetilde{A}}_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}_{id})$, and $\mathcal{R}(\widetilde{\widetilde{F}}_{vp})$ are respectively changes to $\mathcal{R}(\widetilde{\widetilde{A}}^{*}_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}^{*}_{id})$, and $\mathcal{R}(\widetilde{\widetilde{F}}^{*}_{vp})$; for $i, s, d, p, v = 1, 2$. The ranges of $\mathcal{R}(\widetilde{\widetilde{A}}^{*}_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}^{*}_{id})$, and $\mathcal{R}(\widetilde{\widetilde{F}}^{*}_{vp})$ in which the suggested models are stable and the solutions continue to be POSs for these input parameters are readily determined using the previously described procedures. As a result, the proposed model's parameter validity ranges are taken into consideration. To find the validity ranges of the parameters in the proposed model, perform steps 1-4 in this instance. The obtained ranges of essential

Table 8 Three types of solutions of Model-II based on both case studies

(For case study-1)		Via GC method		(For case study-2)		Via FP method
	F^1	F^2	F^3	F^1	F^2	F^3
Ideal solution	22741	13865	36254	29357	13752	34769
Anti-ideal solution	47251.38	27583	72359	37689	25704	44753
POS	23502	14572	37152.40	33061	13965	36380

parameters are displayed in Table 9. In fact, the suggested model's range of possible parameters is comparatively reached in a similar manner.

8 Managerial Insights

This research yields important and profitable managerial insights that will benefit the various private and government organizations involved in the logistics sector. When Model-II is divided into two sub-models, organizations can choose the best POS based on the results. Actually, in order to distribute the goods with the stated goals, organizations may decide which routes would go to the ideal destination. The effects of CaEs under policies involving a carbon offset, cap, and carbon tax are briefly discussed. Organizations can determine when their profit will be lower (or higher) by using that analysis. As a result, they are able to modify their benefits and environmental consciousness, which could improve their standing in the global market. In order to help businesses choose a more suitable delivery time that improves their product service, the dwell time for the path impediments is also factored into the conveyance time. To determine which input ranges are appropriate for the organizations, sensitivity and stability analyses are performed.

9 Conclusion and Final Remarks

This study presents a strategic dilemma of green transportation systems under twofold uncertainty by taking into account the financial, environmental goals and customer happiness. An unprecedented multi-objective 4D-TP model with the three competing purposes mentioned above under a carbon policy has been developed to support the decision. Simultaneously, the best outputs and the quantities of dispersed com-

Table 9 The range of capacity, demand and supply parametersof both case studies

For case study-1		For case study-2	
Crisp values of	Ranges of	Crisp values of	Ranges of
$\mathcal{R}(\widetilde{\widetilde{A}}_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}_{id})$, **and** $\mathcal{R}(\widetilde{\widetilde{F}}_{vp})$	$\mathcal{R}(\widetilde{\widetilde{A}}^*_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}^*_{id})$, **and** $\mathcal{R}(\widetilde{\widetilde{F}}^*_{vp})$	$\mathcal{R}(\widetilde{\widetilde{A}}_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}_{id})$, **and** $\mathcal{R}(\widetilde{\widetilde{F}}_{vp})$	$\mathcal{R}(\widetilde{\widetilde{A}}^*_{is})$, $\mathcal{R}(\widetilde{\widetilde{R}}^*_{id})$, **and** $\mathcal{R}(\widetilde{\widetilde{F}}^*_{vp})$
$\mathcal{R}(\widetilde{\widetilde{A}}_{11}) = 102.90$	$102.90 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{11}) \leq 200$	$\mathcal{R}(\widetilde{\widetilde{A}}_{11}) = 102.90$	$63.28 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{11}) < \infty$
$\mathcal{R}(\widetilde{\widetilde{A}}_{12}) = 82.47$	$75.69 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{12}) \leq 87.25$	$\mathcal{R}(\widetilde{\widetilde{A}}_{12}) = 82.47$	$74.20 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{11}) \leq 93.11$
$\mathcal{R}(\widetilde{\widetilde{A}}_{21}) = 77.91$	$46.17 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{21}) \leq 63.49$	$\mathcal{R}(\widetilde{\widetilde{A}}_{21}) = 77.91$	$62.80 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{21}) \leq 67.00$
$\mathcal{R}(\widetilde{\widetilde{A}}_{22}) = 68.39$	$68.39 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{22}) \leq 59.77$	$\mathcal{R}(\widetilde{\widetilde{A}}_{22}) = 68.39$	$68.39 \leq \mathcal{R}(\widetilde{\widetilde{A}}^*_{22}) \leq 51.42$
$\mathcal{R}(\widetilde{\widetilde{R}}_{11}) = 81.30$	$81.30 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{11}) \leq 101.32$	$\mathcal{R}(\widetilde{\widetilde{R}}_{11}) = 81.30$	$81.30 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{11}) \leq 95.71$
$\mathcal{R}(\widetilde{\widetilde{R}}_{12}) = 172.48$	$172.48 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{12}) \leq 182$	$\mathcal{R}(\widetilde{\widetilde{R}}_{12}) = 172.48$	$172.48 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{12}) \leq 190.55$
$\mathcal{R}(\widetilde{\widetilde{R}}_{21}) = 136.55$	$136.55 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{21}) \leq 151.04$	$\mathcal{R}(\widetilde{\widetilde{R}}_{21}) = 136.55$	$121 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{21}) \leq 130.46$
$\mathcal{R}(\widetilde{\widetilde{R}}_{22}) = 117.43$	$117.43 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{22}) \leq 128$	$\mathcal{R}(\widetilde{\widetilde{R}}_{22}) = 117.43$	$102 \leq \mathcal{R}(\widetilde{\widetilde{R}}^*_{22}) \leq 116.25$
$\mathcal{R}(\widetilde{\widetilde{F}}_{11}) = 95.38$	$95.38 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{11}) \leq 106.23$	$\mathcal{R}(\widetilde{\widetilde{F}}_{11}) = 95.38$	$95.38 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{11}) \leq 109.77$
$\mathcal{R}(\widetilde{\widetilde{F}}_{12}) = 88.57$	$88.57 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{12}) \leq 94.83$	$\mathcal{R}(\widetilde{\widetilde{F}}_{12}) = 88.57$	$88.57 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{12}) < 99.50$
$\mathcal{R}(\widetilde{\widetilde{F}}_{21}) = 159.31$	$159.31 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{21}) \leq 168.45$	$\mathcal{R}(\widetilde{\widetilde{F}}_{21}) = 159.31$	$159.31 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{21}) \leq 175.84$
$\mathcal{R}(\widetilde{\widetilde{F}}_{22}) = 88.57$	$86.35 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{22}) < \infty$	$\mathcal{R}(\widetilde{\widetilde{F}}_{22}) = 88.57$	$86.35 \leq \mathcal{R}(\widetilde{\widetilde{F}}^*_{22}) < \infty$

modities via certain routes by various transportation modes at the same time have been questioned. Budgetary restrictions, dwell time, maintenance costs, fixed-charge costs, and loading and unloading times are some of the other significant contributions contributed to this study. To manage the uncertainties, a novel version of the trapezoidal type-2 fuzzy number has been introduced. The previously indicated uncertainty was considerably reduced with the introduction of a straightforward linear ranking function and required less computing power. Both a fuzzy and a non-fuzzy technique are applied to successfully solve the proposed formulation. After that, two examples have been provided to validate the previously discussed model and solution methodologies. Lastly, choices on lowering CaEs from transport systems have also been considered. We have also concluded that the logistics system can regulate the amount of CaEs as a result of our model formulation and solution.

The suggested methods can also be used to other decision-making issues, like logistic networks, supply chain management, inventory control, multi-criteria decision-making issues, etc. Furthermore, our approach may be extended too many contexts, like rough sets, urban transportation planning, biofuel supply chain systems, risk management, solid waste management, robust environment, etc. Furthermore, to handle the green 4D-TP model, interested scientists might design the non-membership and membership functions as hyperbolic, exponential, etc. instead of linear functions.

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Exploring Machine Learning Techniques for Enhanced Chronic Kidney Disease Diagnosis: A Comprehensive Survey

V. Chandra Kumar and R. Kalpana

Abstract The growing significance of machine learning techniques in medical diagnosis has prompted numerous studies focusing on Chronic Kidney Disease (CKD) detection. This survey paper presents a comprehensive overview of the recent advancements in applying machine learning algorithms for CKD diagnosis. By analyzing a wide array of research articles and studies, this paper aims to provide a holistic understanding of the diverse methodologies employed in CKD detection using machine learning. The survey delves into various machine learning algorithms, such as Logistic Regression, Decision Trees, Support Vector Machines, and K-Nearest Neighbors, highlighting their strengths, limitations, and suitability for CKD detection. The paper also explores the pivotal role of feature selection in diagnosis models. Furthermore, examines the challenges associated with imbalanced datasets, noise, and missing values, and explores how researchers have addressed these issues in their approaches. Through an in-depth analysis of existing literature, this survey paper synthesizes the current state of CKD detection using machine learning and identifies trends, gaps, and possible areas for future research. The ultimate aim is to facilitate a comprehensive understanding of the field, aiding researchers and medical professionals in making informed decisions when developing and applying machine learning-based CKD diagnosis systems.

Keywords Machine learning · CKD · Pre-processing · Feature selection · ML models · Evaluation parameters

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1 Introduction

Chronic Kidney Disease (CKD) is a large global health issued [1], affecting millions of people worldwide. CKD is categorized by the gradual loss of kidney working over interval [2], leading to various complications and a reduced quality of life. Early detection and timely intervention are crucial to slow down disease progression, prevent complications [3], and improve patient outcomes [4]. However, CKD detection can be challenging [4], as symptoms may not be apparent in the early stages, and traditional diagnostic methods may not be sensitive enough for early detection [5]. In recent years, the field of healthcare has witnessed remarkable advancements in machine learning, a subfield of artificial intelligence [6]. Machine Learning (ML) models have demonstrated their potential to revolutionize medical diagnostics and decision-making processes [7]. Their ability to process large and complex datasets, recognize deep-patterns, and make predictions based on available data has opened up new avenues for enhancing CKD detection and management. The importance of CKD detection and the pivotal role that machine learning plays in revolutionizing CKD diagnosis, risk assessment, and personalized treatment strategies. By harnessing the power of machine learning, healthcare professionals can better combat the challenges posed by CKD and improve patient outcomes [8]. The detection and classification of CKD using ML is of significant importance in healthcare for several reasons [9, 10]:

- *Early Detection and Intervention*: ML models can analyze a wide range of long-suffering data of patients including medical records, and imaging data etc. By identifying patterns and associations, these models can detect CKD at an early stage when symptoms may not be evident. Early detection allows for timely intervention and appropriate treatment, preventing disease progression and improving patient outcomes.
- *Personalized Risk Assessment*: ML models can provide custom-made assessments for individuals depend on their unique medical times gone by, lifestyle, and biomarker levels.
- *Efficient Resource Allocation*: CKD is a predominant and costly health condition that places a major problem on healthcare. Machine learning models along with optimization can lead to cost savings and better resource management.
- *Handling Large and Complex Datasets*: ML algorithms outrival at handing out enormous amounts of data, which is common in healthcare. CKD detection involves analyzing various clinical variables, imaging data, and genetic information. Machine learning can handle these complex datasets and uncover hidden relationships between different factors.
- *Predictive Modeling*: Machine learning models can go beyond simple CKD diagnosis and predict disease progression and patient outcomes. By analyzing data, these models can forecast the trajectory of CKD in individual patients, allowing for proactive management and preventive measures

- *Improving Diagnosis Accuracy*: Machine learning models can integrate multiple data sources and variables, leading to more precise and consistent CKD diagnosis. By involving a extensive range of factors, these models can reduce the risk of misdiagnosis and improve diagnostic accuracy.
- *Real-time Monitoring*: Machine learning models can be integrated into healthcare systems to provide real-time monitoring of patients at risk of CKD or those undergoing treatment. This continuous monitoring allows for early detection of changes in patients' health status, enabling prompt medical intervention.
- *Supporting Clinical Decision-Making*: Machine learning algorithms have the potential to function as valuable aids for healthcare providers in making informed decisions. By presenting relevant patient information and risk scores, these models can assist clinicians in making well-informed and evidence- based decisions.
- *Research and Discovery*: Machine learning can aid researchers in identifying new biomarkers and risk factors associated with CKD. By analyzing large-scale datasets, these models can reveal novel insights into disease mechanisms and potential therapeutic targets.

In its entirety, integration of machine learning models into CKD detection holds the assure of a transformative impact on healthcare, promoting advancements in early diagnosis, personalized treatment strategies, efficient resource allocation, and ultimately, enhanced patient outcomes. It is imperative, however, to navigate challenges including safeguarding data privacy, ensuring model interpretability, and upholding ethical considerations. These measures are indispensable to ensure the responsible and efficacious implementation of machine learning within healthcare domains. Collaboration among data scientists, healthcare practitioners, and policymakers stands as a central factor in harnessing the full potential that machine learning offers in CKD detection and broader healthcare contexts [11]. The subsequent sections of the paper unfold systematically: Sect. 2 delves into the Search Method, Sect. 3 explores CKD datasets, Sect. 4 examines pre-processing techniques, Sect. 5 encompasses feature selection methods, and Sect. 6 elaborates on the models employed for CKD detection. Sect. 7 encompasses the evaluation parameters, and the paper concludes with Sect. 8, summarizing the insights gleaned from the preceding sections.

2 Machine Learning Techniques

2.1 Classification Algorithms

Machine learning models significantly enhance the detection of CKD by leveraging data analysis, pattern recognition, classification, handling imbalanced data, supporting early detection, enabling personalized medicine, managing missing data and noise, aiding clinical decisions, and contributing to research insights. Their role extends beyond traditional diagnostic methods, offering a data-driven and innovative approach to improve CKD detection accuracy and patient care. Several Machine Learning techniques were employed on CKD detection depicted (Fig. 1).

A. **Logistic Regression**

Logistic Regression (LR) [9] a highly employed algorithm for tackling binary classification endeavors, such as the detection of CKD. The central mechanism involves constructing a model that gauges the probability of a given case being associated with a specific category, employing the logistic function. In the specific context of CKD, notable attributes like age, blood pressure, and blood creatinine levels are leveraged as inputs to this model. Through careful analysis of these distinct input elements, the LR model calculates the probability of an individual harboring the disease or remaining disease-free, enabling accurate predictions related to CKD presence or absence based on the available dataset. As a cornerstone tool, Logistic Regression within the CKD

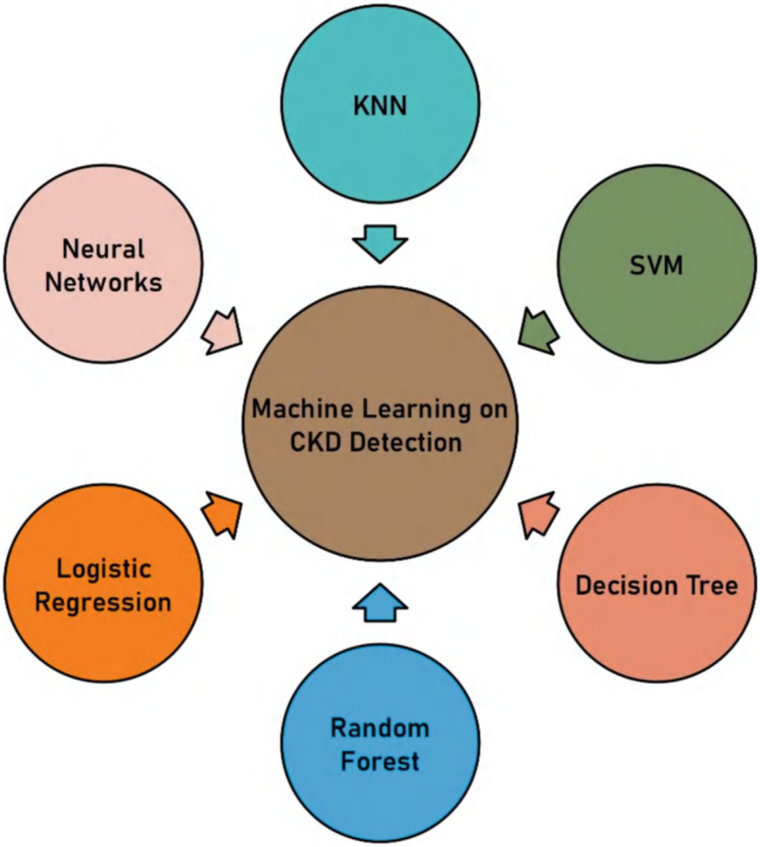

Fig. 1 Techniques of machine learning on CKD detection

context harnesses meticulously computed probabilities of class membership, effectively categorizing patients and furnishing valuable insights into their potential health condition.

Advantages:

- *Interpretability*: Logistic Regression provides straightforward interpretability, allowing healthcare professionals to understand the impact of individual features on CKD detection.
- *Efficiency*: *C*omputationally efficient and can handle large datasets with relatively low resource requirements.
- *Good for Simple Cases*: Logistic Regression performs well when the relationship between features and CKD presence is approximately linear.

Limitations:

- *Linear Assumption*: LR assumes linear relationships between features and the log-odds of the target, limiting its ability to capture non-linear relationships in complex CKD data.
- *Feature Independence Assumption*: The algorithm assumes feature independence, which might not hold in cases where features are correlated.
- *Imbalanced Data*: If the CKD dataset has an imbalanced class distribution, with a significantly higher count of CKD-negative cases compared to CKD-positive cases, Logistic Regression might be biased towards the majority class. This can lead to reduced sensitivity in detecting CKD cases, which are comparatively rarer.
- *Limited Complexity*: Logistic Regression constructs linear decision boundaries, which might not capture intricate decision boundaries present in complex CKD datasets.
- *Sensitive to Outliers and Lack of Robustness to Irrelevant Features*: LR can be sensitive to outliers where patient data might contain outliers, the resulting model might be skewed and not accurately represent the underlying CKD patterns.
- *Binary Classification*: It is design for binary classification tasks, which might not be suitable for more nuanced CKD diagnosis scenarios.

B. **Decision Tree**

Decision Tree [10] models can capture complex relationships between features and CKD, allowing for easy interpretation. Decision trees can be especially useful in diagnosing CKD by identifying critical thresholds for key biomarkers. Decision tree offer a visual framework for illustrating decision-making sequences that hinge on distinct feature values. By constructing these models, intricate associations between features and CKD can be encapsulated, granting a user-friendly means of comprehension. In the realm of CKD diagnosis, decision trees hold particular significance, as they are adept at pinpointing pivotal thresholds related to crucial biomarkers. This prowess aids in accurately discerning the presence of CKD and contributes to an enhanced understanding of the disease's diagnostic cues.

Advantages:

- *Easy understandability and Interpretability*: Decision Trees provide intuitive visualizations of decision-making processes, aiding healthcare practitioners in understanding the classification process.
- *Non-linear Relationships*: These models have the capability to capture complex non-linear relationships between features.
- Handling Missing Values: Decision Trees can handle missing data without significant preprocessing.

Limitations:

- *Over-fitting*: Decision Trees may encounter challenges of over fitting training data, potentially capturing irrelevant noise and resulting in reduced generalization performance when applied to unseen cases of Chronic Kidney Disease (CKD).
- *Instability*: Small changes in the data can result in significantly different tree structures, potentially affecting reliability.
- *Inability to Capture Complex Patterns*: In more intricate CKD cases, Decision Trees might struggle to capture complex relationships present in the data.
- *Bias Towards Dominant Classes*: In the presence of imbalanced class distributions, where one class (e.g., CKD-negative cases) significantly outweighs the other (e.g., CKD-positive cases), Decision Trees can exhibit bias towards the dominant class. This can lead to reduced accuracy in identifying the rarer CKD cases.
- Feature Scaling: Decision Trees are not sensitive to feature scaling, meaning they can handle features with different scales without adjustment and might not fully exploit the information present in these features.
- Unstable Structure: Minor alterations in the training data can result in substantial changes in the structure of the resulting Decision Tree. This instability can affect the model's reliability and interpretability, making it challenging to rely on consistent diagnostic outcomes.
- Limited Interpretable Depth: Shallow Decision Trees might not capture the intricacies of CKD relationships, while deep trees could lead to overfitting. Finding the right balance in terms of tree depth and interpretability can be a challenge, especially for complex CKD datasets.

C. **Support Vector Machines (SVM)**:

SVM constitute a robust approach in the realm CKD detection. In the context of CKD, SVM [11] are utilized to differentiate between patients with and without the disease based on relevant features. SVMs aim to find a decision boundary that maximizes the margin between the two classes while accounting for potential misclassifications, leading to a reliable classification mechanism for CKD diagnosis. The power of SVMs lies in their capability to handle complex datasets and non-linear relationships, making them suitable for CKD detection where intricate interactions between patient attributes and disease presence might exist. SVMs employ a kernel trick where optimal class separation is more feasible. By identifying the most informative features and constructing a decision boundary with the largest margin, SVMs contribute significantly to accurate CKD diagnosis. This model's versatility and ability to handle diverse data distributions empower healthcare practitioners and researchers to effectively distinguish between CKD cases and non-cases, enhancing the overall diagnostic process.

Advantages:

- *Non-linear Classification*: SVMs can capture non-linear relationships between features and CKD presence using various kernel functions.
- *Global Optimal Solution*: SVMs aim to find a global optimal solution, leading to effective decision boundaries.
- *Robust to Noise*: SVMs can handle noisy data and generalize well with appropriate regularization.

Limitations:

- *Computational Intensity*: SVMs can be computationally intensive, especially for large CKD datasets.
- *Limited Handling of Noisy Data*: SVMs aim to find a decision boundary with the maximum margin, which can be sensitive to noisy data points. Outliers or mislabeled instances in the CKD dataset might adversely affect the optimal decision boundary, leading to reduced accuracy ψ
- *Imbalanced Data*: If the CKD dataset has an imbalanced class distribution, SVMs might prioritize the majority class during model training. This can result in a bias towards the more prevalent class and lower sensitivity in detecting the rarer CKD cases.
- *Interpretability*: The resulting model might be less interpretable, especially with non-linear kernels.
- *Scalability*: Handling very large CKD datasets might pose scalability challenges.

D. **K-Nearest Neighbors (KNN)**

The K-Nearest Neighbors (KNN) [12] algorithm emerges as a valuable tool in the landscape of Chronic Kidney Disease (CKD) detection. Operating under the principle of similarity, KNN classifies data points by identifying the K nearest neighbors to a given instance and determining the prevalent class among them. In the context of CKD, KNN leverages patient attributes like age, blood pressure, and creatinine levels to assess the likelihood of an individual having or not having the disease. By measuring the proximity of a new patient's features to those of existing cases in the dataset, KNN aids in effectively categorizing patients as either CKD-positive or CKD-negative, contributing to accurate diagnostic outcomes. KNN's adaptability and straightforwardness make it particularly suitable for CKD detection, where it doesn't impose assumptions about data distribution and can handle various types of features. This algorithm's reliance on local neighborhood information allows it to capture intricate patterns in the data that might be indicative of CKD presence. KNN's performance can be influenced by the choice of distance metric and the value of K, the number of neighbors considered. By selecting an appropriate distance measure and tuning K to match the dataset's characteristics, KNN can effectively contribute to the medical field's diagnostic capabilities in the realm of CKD.

Advantages:

- *Intuitive Concept*: KNN is easy to understand, as it classifies based on proximity to existing instances.
- *Non-linear Relationships*: KNN can capture non-linear associations between features and CKD presence.

  Simple Implementation: KNN's simplicity makes it easy to implement and use.

Limitations of KNN

- ***Sensitivity to Feature Scaling***: KNN relies on distance metrics to determine the similarity between data points. If the features are not properly scaled, those with larger ranges can disproportionately influence the distance calculation, leading to skewed results.
- *Computational Intensity*: KNN can be computationally expensive, particularly with large datasets typical in medical diagnostics, since the algorithm requires the calculation of distances between the query point and every point in the training dataset.
- *Determination of Optimal K Value*: The performance of KNN is highly dependent on the choice of the parameter 'K' (the number of neighbors). Selecting an inappropriate value can lead to either overfitting or underfitting, affecting CKD detection accuracy.

E. **Random Forests (RF)**

RF is an ensemble technique [13] that integrated multiple decision trees. They can handle higher-dimensional data and capture patterns that are more robust. For CKD detection, random forests can aggregate the predictions of individual trees to improve accuracy and reduce over fitting.

Advantages:

- *Improved Accuracy*: Random Forests aggregate predictions from multiple decision trees, resulting in enhanced overall accuracy compared to individual trees. In CKD detection, this translates to more reliable identification of patients with or without the disease.
- *Reduced Overfitting*:*M*itigate the risk of overfitting that single decision trees might face. This can be particularly beneficial when dealing with smaller CKD datasets, preventing the model from capturing noise.
- *Feature Importance*: Random Forests provide feature importance scores, indicating the contribution of each attribute to the model's predictions. In CKD diagnosis, this insight can help identify which patient attributes play a pivotal role in determining disease presence.

- *Robustness to Noise*: Random Forests' ensemble nature allows them to be more robust to noisy data or outliers in the CKD dataset, as the impact of individual trees' errors is mitigated by the ensemble average.

Limitations:

- *Complexity*: The ensemble of decision trees can lead to a complex model, potentially making interpretation challenging. In the medical context of CKD detection, where interpretability is crucial, the complexity might limit its utility.
- *Computational Intensity*: Constructing and training numerous decision trees can be computationally intensive, especially for large CKD datasets. This might hinder real-time or resource-constrained applications.
- *Bias towards Dominant Classes*: CKD dataset has imbalanced class distribution, where one class predominates might lean towards that majority class, affecting its ability to detect rarer CKD cases.
- *Model Size*: The ensemble nature algorithms can lead to larger model sizes compared to individual decision trees, which could impact storage and deployment, particularly in resource-constrained environments.
- *Loss of Interpretability*: The ensemble approach might obscure the interpretability that individual decision trees offer, making it harder to explain the reasoning behind specific CKD predictions.
- *Hyper parameter Tuning*: Random Forests involve tuning hyper parameters like the number of trees, which can be time-consuming and require careful consideration to achieve optimal performance.

2.2 Feature Selection Techniques

The implication of feature selection that affects the performance of classifiers for CKD detection cannot be overstated. Two primary methods for feature selection, namely filter- and wrapper-based approaches, are pivotal in this context. Wrapper-based techniques leverage classifiers to iteratively construct ML models using various predictor variables [15]. By systematically evaluating subsets of features, these methods determine the combination that yields optimal model performance. Through this process, wrapper-based methods enhance the accuracy and efficiency of CKD classifiers. Conversely, filter-based methods operate independently of specific learning algorithms. They assess the correlation between predictor and independent variables, employing statistical criteria to identify relevant features for classification tasks. Filter-based methods contribute to the identification of informative variables by selecting features based on statistical criteria, thereby improving the performance of ML classifiers for CKD detection. In summary, the accurate detection of CKD using machine learning classifiers relies heavily on the strategic selection of features [16]. Both wrapper- and filter-based methods are essential components in this process, playing critical roles in optimizing classifier performance and ultimately advancing

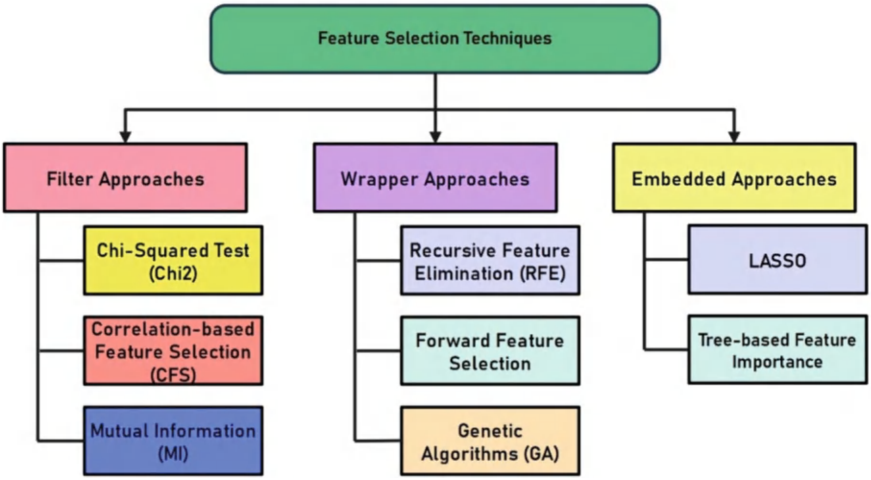

Fig. 2 Feature selection techniques

patient outcomes in CKD diagnosis and management. Few popular feature selections were shown in Fig. 2.

2.2.1 Filter Approaches

A. ***Chi-squared Test (Chi2)***: The chi-square (χ^2) distribution is a fundamental concept in statistics, particularly useful for analyzing categorical data and testing hypotheses. It's characterized by a parameter called degrees of freedom (df), which varies depending on the context of its application. The probability density function (PDF) of the chi-square distribution expresses the likelihood of obtaining a particular value. It's defined as a function of the random variable x and the degrees of freedom k represented as Eq. 1.

$$\chi^2 = \sum (\mathbf{O_i} - \mathbf{E_i})^2 / \mathbf{E_i} \tag{1}$$

$\chi 2$ is the chi-square statistic, O is the observed frequency for each category and E is the expected frequency under the assumption of independence between the variables.

B. ***Correlation-based Feature Selection (CFS)***: Correlation-based Feature Selection (CFS) is a methodical approach for selecting a subset of features

i. *Compute Feature-Target Correlation*:

   a. Correlation coefficient between each Xi feature and Y target variable.
   b. Correlation can be computed using techniques like Pearson correlation coefficient or other correlation measures.

ii. *Compute Feature-Feature Correlation*:

   a. Calculate the pairwise correlation coefficients between all pairs of features Xi and Xj.
   b. This provides insight into how features are correlated with each other.
   c. Denote *corr* (Xi,Xj) as the correlation between features Xi and Xj.

iii. *Evaluate Subset Correlation*:

   a. Develop a criterion to measure the correlation of features within a subset with the target variable while penalizing intercorrelations among features in the subset.
   b. Commonly, a weighted sum of individual feature-target correlations adjusted by the pairwise feature-feature correlations within the subset is used.

iv. *Subset Selection*:

   a. pick the subset of features that maximizes the criterion developed in the previous step.
   b. This subset comprises features highly correlated with the target variable while maintaining minimal correlation among themselves.

v. *Optional Optimization*:

   a. Depending on computational resources and dataset complexity, optimization techniques such as forward selection or genetic algorithms.

C. **Mutual Information (MI)**: MI measures the association between features and the target variable, aiding in the selection of informative features for CKD detection. The Mutual Information measures how much knowing the value of a feature reduces the uncertainty about the target variable. Features with higher Mutual Information scores with the target variable are considered more informative and are often selected for inclusion in the model depicted in Eq. 2.

$$\mathbf{I}(\mathbf{X}, \mathbf{Y}) = \sum\nolimits_{x \in X} \sum\nolimits_{y \in Y} \mathbf{p}(\mathbf{x}, \mathbf{y}) \log\left(\frac{\mathbf{p}(\mathbf{x}, \mathbf{y})}{\mathrm{p(x)p(y)}}\right) \tag{2}$$

- $I(X;Y)$ is the Mutual Information between feature X and the target variable Y.
- $p(x,y)$ is the joint probability mass function (PMF) of X and Y, representing the probability that $X = x$ and $Y = y$.
- $p(x)$ and $p(y)$ are the marginal probability mass functions of X and Y respectively.
- log denotes the natural logarithm.

2.2.2 Wrapper Approaches

A. ***Recursive Feature Elimination (RFE)***: RFE is a popular method in machine learning for selecting the most relevant features from a dataset. The process begins by training a model on the complete set of features. Based on model-specific importance criteria, the least significant features are identified and removed. This cycle of training, evaluating, and eliminating continues until the number of features reaches a predefined limit or another stopping condition is satisfied. RFE aids in identifying a smaller, more impactful set of features, which helps reduce dimensionality, enhance the model's interpretability, and improve its predictive accuracy.

B. ***Forward Feature Selection (FFS)***: FFS process begins with an empty set of features, and then iterates through each feature not currently in the feature set. For each feature, a machine learning model is trained using the current feature set plus the selected feature, and its performance is evaluated using a validation set or cross-validation. The feature that most improves the model's performance is added to the feature set. This process continues until a stopping criterion is met, such as reaching a desired number of features or no further improvement in model performance. Finally, the model is evaluated using a separate test set to assess its performance on unseen data

C. ***Genetic Algorithms (GA)***: GA evolves candidate feature subsets, evaluating their performance using a fitness function derived from the classifier's performance to select the best-performing feature subset. Genetic Algorithms (GAs) can be applied to the domain of CKD to address various optimization tasks, such as feature selection or parameter tuning for predictive models.

2.2.3 Embedded Approaches

A. ***LASSO (Least Absolute Shrinkage and Selection Operator)***: LASSO penalizes coefficients during model training, encouraging sparsity in the coefficient vector and performing feature selection by shrinking some coefficients to zero.

B. ***Tree-based feature importance***: This method employed by algorithms such as RF and Gradient Boosting Machines to evaluate the relevance of each feature to the overall model performance. This built-in feature importance measure aids in feature selection, enabling the identification of features that have the most significant impact on the predictive ability of the model.

3 Related Work

Huseyin Polat et al. introduced [17] Best First and Greedy stepwise search strategies to evaluate feature selection techniques and classification algorithms. These methods systematically assess feature subsets to identify informative features, enhancing overall efficiency. Sarah A et al. presented [18] a novel approach integrating an feature selection technique by information-gain with a cost-sensitive adaptive boosting (AdaBoost) classifier for CKD detection. This method prioritizes relevant features and adjusts for the imbalanced nature of CKD data to improve accuracy and sensitivity. Nusrat Tazin et al. demonstrated [19] that a ranking algorithm significantly enhances classification accuracy by selecting an appropriate number of attributes. They found that using 15 attributes led to the highest percentage improvement in accuracy for the given dataset. An ensemble feature selection method was proposed by another group Manonmani et al., [20], initially utilizing Density-based Feature Selection (DFS) to rank features then applying the Improved Teacher-Learner Based Optimization (ITLBO) algorithm to identify optimal feature subsets for CKD prediction. Shankar et al. proposed [21] a method for feature selection using ALO (Ant Lion Optimization) to choose optimal features for CKD classification, followed by sorting CKD data based on these features using a Deep Neural Network (DNN). Charleonnan et al. [22], conducted a comprehensive analysis on predictive models, including KNN, SVM, LR, and DT. The investigation focused on the Indians CKD dataset, with the primary aim of determining the optimal classifier for accurate CKD prediction. The findings revealed that SVM exhibited the most promising. Al-Momani, Rama, et al. Proposed [23] to achieve early CKD detection using machine-learning techniques, specifically Artificial Neural Network (ANN), SVM, and KNN. The significance of artificial intelligence is underscored by the critical need to identify these potentially life-threatening conditions at an early stage. The research focuses on analyzing a dataset comprising 400 samples and encompassing 13 distinct features. By subjecting the data to these three classification methods, their effectiveness was assessed. Notably, the outcomes indicate that the ANN classifier achieved the highest accuracy, boasting an impressive 99.2% accuracy rate. Dissanayakeet al. [24] formulated a CKD diagnostic system capable of identifying chronic kidney diseases across all stages, leveraging efficient feature selection approaches and machine learning methods. The data utilized in this study were collected from the Anuradhapura district in Sri Lanka. To enhance the quality of the dataset, comprehensive data preprocessing was conducted. Particularly, the imputation method KNN was employed to address absent values, incorporating various K values to optimize the imputation process. For robust feature selection, several techniques were employed, including the examination of the correlation matrix,

Jiongming et al. proposed [25] methodology for CKD diagnosis demonstrates viability both in terms of imputation of data and sample diagnosis. Following the unsupervised imputation of missing values utilizing the K-nearest neighbors (KNN) technique, the integrated model exhibited a commendable level of accuracy. This success leads us to anticipate that the application of this methodology within practical CKD diagnosis scenarios would yield favorable outcomes. Moreover, the potential extends beyond CKD, as this approach could be adapted for clinical data analysis in other medical conditions. The dataset utilized comprised a relatively modest 400 samples, potentially constraining the integrated model's (LOG and RF) generalization performance with an accuracy of 99.83. Furthermore, the dataset's binary nature, categorizing samples as either "ckd" or "notckd," precludes the model from diagnosing the severity of CKD. Wang et al. [26] Investigated the construction of a regression model, aimed at predicting creatinine values using 23 distinct features. The subsequent step integrates the anticipated creatinine values with the original set of

23 features to assess CKD risk. Through simulation, demonstrated that this method yields superior predictive outcomes in contrast to direct predictions solely from the 23 features. Regarding model selection, study engaged three machine learning models: RF, XGBoost, and ResNet. To enhance the creatinine predictor's efficacy, embraced ensemble learning, amalgamating predictions from eight predictors. The outcomes were optimized through the R-Squared (R2) metric, which led to the identification of an optimal under sampling strategy and regression model. Notably, the ensemble model excelled, achieving a peak R2 performance of 0.5590. Sarah et al. [18] In this study introduced a novel strategy that merges feature selection based on information gain (IG) with a cost-sensitive AdaBoost classifier, aimed at enhancing the accuracy of CKD detection. As benchmarks for performance evaluation, six alternative machine learning classifiers were implemented, comprising LR, DT, RF, SVM, XGBoost, and the conventional AdaBoost. The approach unfolds in two steps: Firstly, the IG technique gauges the significance of diverse attributes and ranks them. Subsequently, the models were trained using both the comprehensive feature set and the reduced selected features. The experimental findings underscore that the selected features indeed amplify the classifiers' performance. Moreover, the proposed approach, employing the cost- sensitive AdaBoost, outperforms both other classifiers and methodologies from recent literature. This substantiates the efficacy of merging IG-based feature selection and cost-sensitive AdaBoost as a potent method for CKD detection, with potential applications in early CKD detection via computer-aided diagnosis. To further advance this field, future research will be directed towards amassing substantial data volumes for machine learning model training. This includes datasets conducive to predicting disease severity, disease duration, and age of onset—parameters that hold significance in the comprehensive understanding and management of CKD. Polat et al. [17] applied both wrapper and filter methods to a Chronic Kidney Disease (CKD) dataset, employing distinct evaluators for each approach. The primary objective focused centers on assessing the accuracy rate of an SVM classifier trained on the full dataset against its accuracy on four reduced datasets generated through feature selection methods. The findings reveal that across all four methods, the dimension reduction of the CKD dataset consistently leads to improved diagnostic accuracy. Among the methods employed, the SVM classifier's accuracy on the dataset reduced by FilterSubsetEval using the Best First search engine stands out, showcasing the highest accuracy rate at 98.5%. Moreover, this method exhibits superior performance in key parameters including true positive (TP) rate, correctly classified instances, and the lowest values for metrics such as incorrectly classified instances and false positive (FP) rate. Notably, the pursuit of the smallest dataset dimension through feature selection methods doesn't necessarily guarantee the highest classification accuracy. For instance, SVM's accuracy on the minimal CKD dataset with seven attributes using ClassifierSubsetEval and the Greedy stepwise search engine reaches 98%. However, the SVM classifier's performance on the larger 13-attribute CKD dataset, achieved by applying FilterSubsetEval with the Best First feature selection method, achieves the highest accuracy rate at 98.5% in CKD diagnosis. Tekale et al. [27] a studied utilization of machine learning algorithms for anticipate CKD. In their research, they employed

a dataset containing 400 instances and encompassing 14 distinct features. Within their investigation, the authors applied both the decision tree and support vector machine techniques. Through preprocessing procedures, they effectively condensed the initial feature count from 25 down to 14. Among the algorithms evaluated, the support vector machine exhibited superior performance, yielding an impressive accuracy level of 96.75%. Xiao et al. [28] introduced a novel approach for forecasting the progression of chronic kidney disease. Their methodology involved the utilization of various predictive models. To assess the efficacy of these models, they conducted a performance-based comparison. For their investigation, they employed historical data from 551 patients afflicted with proteinuria, incorporating a comprehensive set of 18 distinctive features. The outcomes were categorized into mild, moderate, and severe classes. Upon analyzing the results, the researchers concluded that logistic regression outperformed the other models, achieving an impressive area under the curve (AUC) value of 0.873. Additionally, the logistic regression model exhibited a sensitivity of 0.83 and a specificity of 0.82. Yashf et al. [29] proposed an innovative approach was introduced to anticipate the likelihood of CKD utilizing machine learning algorithms. This was accomplished through a thorough analysis of data derived from patients diagnosed with CKD. The research employed the Random Forest and Artificial Neural Network techniques for predictive modeling. To streamline the dataset, the researchers extracted a subset of 20 features from the original 25. Subsequently, both the Random Forest and Artificial Neural Network models were deployed for evaluation. The outcome of the study highlighted the remarkableperformance of the Random Forest model, achieving an impressive accuracy rate of 97.12%. Dibaba Adeba Debal et al [30] have been conducted binary and multi-classification for stage predictionThe experimental results revealed that Random Forest, based on recursive feature elimination with cross-validation outperformed SVM and DT in terms of performance. Comparison of techniques available for CKD shown in Table 1.

4 Results and Discussion

The power of utilizing machine learning models based on size of the data set and quality of the data set. The dataset pertaining to Indians with Chronic Kidney Disease (CKD) comprises 400 instances and encompasses 24 attributes. These data were gathered from the UCI machine learning repository and similar type of datasets from other repositories [31–34]. Notably, the attributes within this dataset are classified into two distinct types: numeric attributes and nominal attributes. Existed surveys performed analysis on standard datasets and comparison of model accuracies of existed works shown in below Fig. 3 and proved that ensemble models and models with feature selection techniques improve the accuracy.

From the survey identified that the assessment of CKD prediction model performance involves in the following directions [35–46].

Table 1 Existed survey

Author	Model	Accuracy	Classification type
Charleonnan et al. [22]	SVM	98.3	Binary
	ANN-KNN Imputation for Missing values	99.2	Binary
D. V. Dissanayake [24]	Random Forest classification + Wrapper Feature Selection Methods	96	Binary
Qin et al. [25]	Integrated LR and Random Forest by using perceptron + KNN imputation	99.83	Binary
Wang et al. [26]	Regression Models	R2 0.5590	Binary
Ebiaredoh-Mienye [18]	AdaBoost + Information Gain	99.8	Binary
Polat et al. [17]	SVM + Best First Search engine	98.5%	Binary
Xiao et al. [28]	Logistic regression with 18 features	Aea under the curve (AUC) value of 0.873	Multi stage classification
Debal and Sitote [30]	SVM and RF with RFECV exhibited the highest accuracy, while XGBoost achieved a notable 82.56% accuracy for datasets with five classes	99.8% for binary classification and 82.56% accuracy for five-class datasets which is the highest	Multi stage classification

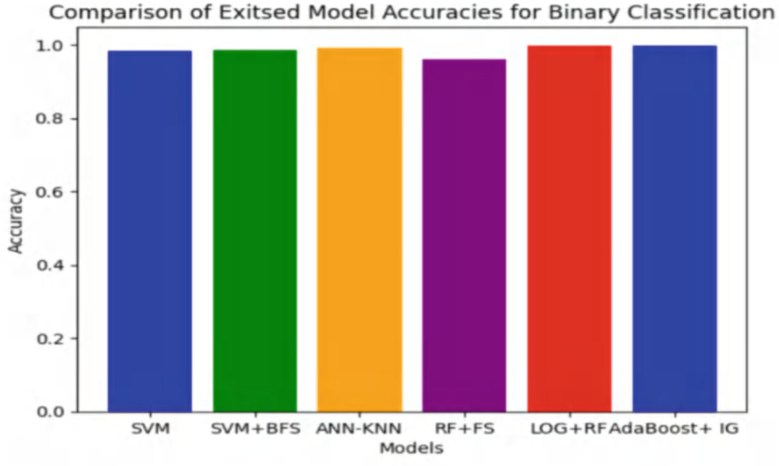

Fig. 3 Existed models accuracy performance

- *Integration of Deep Learning Techniques*: While the survey highlights a range of traditional ML algorithms, it falls short of delving deeply into the potential of deep learning techniques. Exploring the efficacy of these advanced techniques in CKD diagnosis, particularly in handling complex medical imaging data like kidney scans, could be a valuable avenue for future research.
- *Ensemble Methods for Improved Performance*: The survey briefly touches upon individual algorithms but overlooks the potential benefits of ensemble methods. Investigating the combination of multiple algorithms or models, such as Random Forests or Gradient Boosting, could enhance predictive accuracy and robustness, thereby warranting further exploration.
- *Explainable AI in CKD Diagnosis*: With increasing emphasis on the interpretability of machine learning models in medical applications, the survey misses the opportunity to discuss the integration of explainable artificial intelligence (XAI) techniques. Future research could explore methods to make complex CKD diagnosis models more transparent and interpretable for healthcare practitioners.
- *Real-time Monitoring and Early Detection*: The paper focuses primarily on diagnostic approaches but overlooks the potential for machine learning in continuous monitoring and early detection of CKD. Investigating real-time data streams and wearable sensor data to predict the onset or progression of CKD could be a promising research direction.
- *Personalized Medicine in CKD*: The survey mainly addresses generalized CKD diagnosis models. Future research could delve into developing personalized prediction models that consider an individual's medical history, genetics, lifestyle, and other personalized factors for more accurate and tailored predictions.
- *Integration of Multi-Modal Data*: Many CKD cases benefit from considering multiple data modalities, such as lab results, medical images, and patient demographics. The survey does not extensively explore the fusion of these diverse data sources using multimodal machine learning approaches, which could be a fruitful avenue for research.
- *Integration of Multi-Class Classification*: The integration of multi-stage classification in Chronic Kidney Disease (CKD) diagnosis involves a hierarchical approach that aims to detect level of problem. This methodology breaks down the classification task into multiple stages, each targeting a specific aspect of CKD diagnosis.
- *Validation and Clinical Adoption*: While the survey provides insights into model performance, it lacks in-depth discussions on the validation of models in real-world clinical settings. Future research could focus on conducting rigorous clinical validation studies to assess the practical applicability and impact of machine learning-based CKD diagnosis systems.
- *Benchmark Datasets and Reproducibility*: The survey could benefit from discussing standardized benchmark datasets commonly used in CKD diagnosis research. Additionally, emphasizing the importance of reproducibility and open sharing of code and data could encourage transparency and collaboration in the field.

5 Conclusion

The incorporation of machine learning techniques into medical diagnosis has sparked a surge of interest in detecting Chronic Kidney Disease (CKD). This survey paper has provided a wide-ranging overview of recent advancements in the application of machine learning algorithms for CKD diagnosis. Through an extensive review of diverse research articles and studies, this survey has aimed to offer a holistic understanding of the methodologies employed in utilizing machine learning for CKD detection. The exploration of various algorithms, including Logistic Regression, Decision Trees, Support Vector Machines, and K-Nearest Neighbors, has shed light on their respective strengths, limitations, and suitability within the context of CKD diagnosis. The essential role of feature selection and engineering in enhancing the accuracy of CKD diagnosis models has been highlighted. Moreover, this paper has delved into the challenges posed by imbalanced datasets, noise, and missing values, and has examined how researchers have effectively addressed these hurdles in their approaches. By conducting a comprehensive analysis of existing literature, this survey has synthesized the current landscape of CKD detection using machine learning, discerning prevailing trends, identifying gaps in research, and pinpointing promising avenues for future exploration. Ultimately, the overarching goal of this survey is to foster a robust comprehension of the field, empowering researchers and medical professionals to make well-informed decisions as they develop and implement machine learning-based CKD diagnosis systems. Looking ahead, the field of machine learning-enabled CKD diagnosis exhibits vast potential for growth. Future research could delve deeper into hybrid models that combine multiple algorithms to leverage their individual strengths and mitigate weaknesses. Furthermore, the exploration of explainable AI techniques could enhance the transparency and interpretability of CKD diagnosis models, fostering trust among medical practitioners. Addressing challenges such as privacy concerns, ethical considerations, and real-world deployment issues remains pivotal in ensuring the practical applicability of machine learning-based CKD diagnosis systems. As technology continues to evolve, collaboration between machine learning experts and medical professionals will play a pivotal role in advancing the accuracy, reliability, and clinical utility of CKD detection methodologies.

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Enhancing Fuzzy Multi Criteria Decision Making Technique in Engineering Design Problem

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Abstract Fuzzy logic provides a powerful tool for addressing the complexities and uncertainties in multicriteria decision-making (MCDM) problems. Allowing for degrees of truth effectively handles the vagueness inherent in human judgments and subjective evaluations. In MCDM, fuzzy logic employs fuzzy sets and membership functions to represent linguistic variables like "high," "medium," and "low." Key methodologies include fuzzy aggregation operators and fuzzy multi-attribute utility theory, which integrate individual criterion evaluations into a comprehensive decision metric. Applications span diverse fields such as engineering, economics, environmental management, and healthcare, where it aids in evaluating alternatives under uncertain conditions. Fuzzy logic's ability to capture and process imprecise information enhances the robustness and flexibility of MCDM processes, offering a valuable approach for decision-makers facing complex, multifaceted problems.

Keywords Fuzzy logic · Multicriteria decision making · Uncertainty · Fuzzy sets · Membership functions

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1 Introduction

Multicriteria decision-making (MCDM) problems are ubiquitous in both academic research and practical decision-making contexts across various domains, ranging from engineering and finance to environmental management and public policy. These decision problems involve evaluating and selecting the best alternative from a set of options based on multiple, often conflicting, criteria or objectives. For instance, in engineering, decision-makers may need to select the optimal product design considering cost, performance, reliability, and environmental impact. Similarly, in healthcare, clinicians may face the challenge of choosing the most suitable treatment option for a patient based on considerations such as efficacy, side effects, cost-effectiveness, and patient preferences [1].

The complexity of MCDM problems arises from several factors, including the presence of multiple decision criteria, the inherent trade-offs between competing objectives, and the uncertainty and subjectivity inherent in decision-making processes. Traditional decision-making approaches, such as simple additive weighting or pairwise comparison methods, often struggle to effectively address these complexities, leading to suboptimal or inconsistent decisions [2].

One of the key challenges in MCDM is the presence of uncertainty, imprecision, and subjectivity in the information available to decision-makers. Uncertainty arises from incomplete or imperfect knowledge about the value of decision criteria or the outcomes associated with different alternatives. Imprecision manifests as vagueness or ambiguity in the definition of decision criteria or decision-maker preferences. Subjectivity reflects the inherent differences in preferences and perspectives among stakeholders involved in decision-making [3].

Addressing these challenges requires the development of decision-making methodologies that can accommodate uncertainty, imprecision, and subjectivity systematically and transparently. Fuzzy logic provides a robust framework for tackling these challenges by allowing for representing and manipulating vague or ambiguous information through fuzzy sets, fuzzy numbers, and linguistic variables. Fuzzy logic offers a natural way to model human reasoning processes, enabling decision-makers to express their preferences in linguistic terms and reason with imprecise or uncertain data [4, 5].

2 Foundations of Multicriteria Decision Making

Multicriteria decision-making (MCDM) is a systematic approach to decision-making that involves evaluating and selecting the best alternative from a set of options based on multiple, often conflicting, criteria or objectives. At its core, MCDM seeks to address the inherent complexity of decision problems where decisions cannot be made solely based on a single criterion or dimension [2] (Fig. 1).

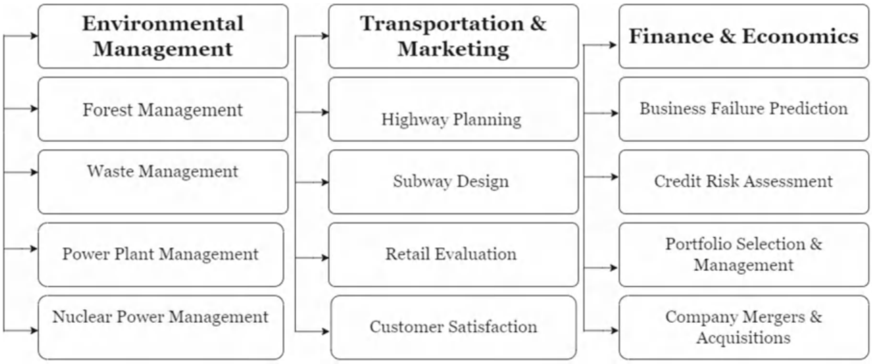

Fig. 1 Multicriteria decision making

2.1 Components of MCDM

MCDM problems typically consist of three main components: decision criteria, alternatives, and decision-makers' preferences.

- *Decision Criteria*: Decision criteria represent the factors or attributes that decision-makers consider when evaluating alternatives. These criteria can be quantitative (e.g., cost, performance metrics) or qualitative (e.g., reliability, environmental impact). The selection of appropriate decision criteria is crucial as it directly influences the outcome of the decision-making process [6].
- *Alternatives*: refer to the possible courses of action or solutions available to decision-makers. Each alternative represents a different combination of attributes or performance levels with respect to the decision criteria. The set of alternatives may range from a few well-defined options to a large and diverse set of possibilities [3].
- *Decision-makers' Preferences*: Decision-makers' preferences reflect their subjective judgments and priorities regarding the importance of different decision criteria and their preferences among the available alternatives. Preferences may vary among decision-makers based on their goals, values, beliefs, and risk attitudes. Incorporating decision-makers' preferences into the decision-making process is essential for ensuring the relevance and acceptability of the chosen alternative [7].

2.2 Complexity of MCDM Problems

MCDM problems are characterized by their inherent complexity, stemming from several factors (Fig. 2):

- *Multiple Criteria*: MCDM involves considering multiple decision criteria simultaneously, each with its own importance and relevance to the decision problem. The

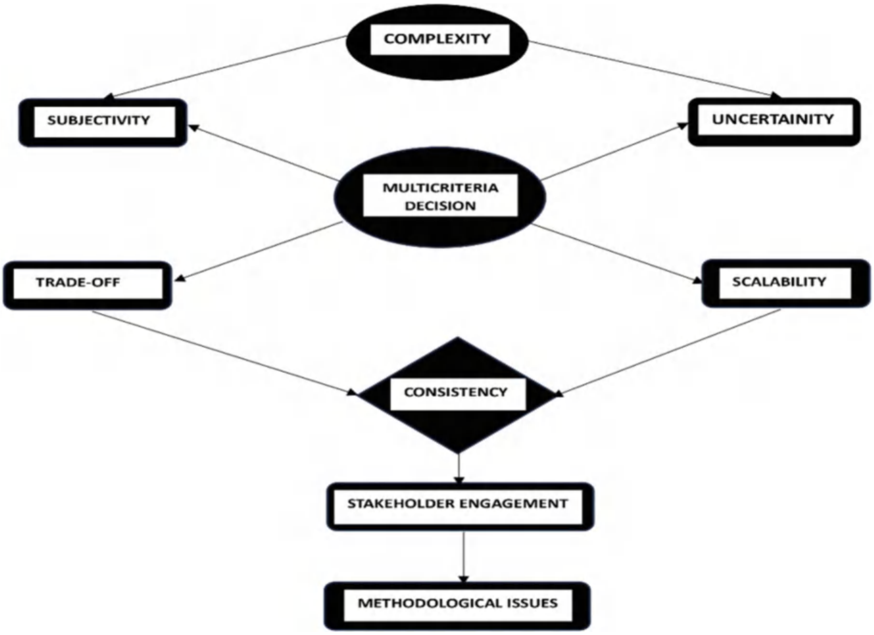

Fig. 2 Complexity of MCDM problems

interplay between these criteria often introduces trade-offs and conflicts, making the decision process challenging [8].

- *Trade-offs and Conflicts*: Decision criteria may compete with each other, leading to trade-offs where improving performance on one criterion may come at the expense of another. Resolving these trade-offs requires careful consideration of the relative importance of each criterion and the preferences of decision-makers.
- *Uncertainty and Risk*: MCDM problems are frequently subject to uncertainty and risk due to incomplete information, variability in outcomes, and external factors beyond the control of decision-makers. Uncertainty adds an additional layer of complexity to the decision-making process, requiring decision-makers to account for potential risks and their implications.
- *Subjectivity and Human Judgment*: The evaluation of decision criteria and the interpretation of outcomes often involve subjective judgments and human expertise. Decision-makers may have different perspectives, preferences, and levels of confidence in their assessments, introducing subjectivity into the decision process.
- *Traditional Decision-Making Approaches*:Traditionally, decision-makers have relied on simple decision-making approaches such as cost–benefit analysis, scoring models, or pairwise comparison methods (e.g., Analytic Hierarchy Process) to address MCDM problems. While these approaches provide structured frameworks for decision-making, they often oversimplify the complexities

of real-world decision problems and may fail to capture important nuances and uncertainties.

The foundations of multicriteria decision-making involve understanding the key components of decision problems, recognizing the inherent complexity and challenges involved, and acknowledging the limitations of traditional decision-making approaches in addressing these challenges. Effective MCDM requires developing and applying systematic decision-making methodologies that can accommodate multiple criteria, handle uncertainty and risk, and integrate decision-makers' preferences transparently and consistently [4].

2.3 Challenges in Multicriteria Decision Making

Multicriteria decision making (MCDM) is a complex process that involves assessing and comparing multiple criteria or objectives to arrive at a satisfactory solution. While MCDM techniques offer valuable tools for decision-makers, they also present several challenges that need to be addressed:

1. *Subjectivity and Uncertainty*:

Subjective Judgments: Decision criteria and their relative importance often vary among stakeholders based on individual perspectives, experiences, and priorities. This subjectivity can introduce bias and inconsistency into the decision-making process, complicating the evaluation of alternatives.

2. *Data Uncertainty*:

Multicriteria decision-making often involves dealing with incomplete, imprecise, or unreliable data. Uncertainty in data quality and reliability can undermine the credibility of decision outcomes and lead to suboptimal choices.

3. *Trade-Offs and Conflicting Objectives*:

Balancing Trade-offs: Many decision scenarios require trade-offs between competing objectives, where improving one criterion may come at the expense of another. Identifying and managing these trade-offs is challenging and requires careful consideration of the relative importance of each criterion.

Resolving Conflicts: Conflicting objectives among stakeholders can complicate decision-making processes, as different parties may prioritize different criteria or outcomes. Achieving consensus and reconciling conflicting interests is often a complex and time-consuming endeavour [9].

4. *Complexity and Dimensionality*:

High Dimensionality: Multicriteria decision problems often involve a large number of criteria and alternatives, resulting in high-dimensional decision spaces. Exploring this vast solution space efficiently and effectively to identify optimal or satisfactory solutions can be computationally demanding.

Model Complexity: Developing and implementing MCDM models capable of handling complex decision scenarios requires sophisticated mathematical and computational techniques. Managing the complexity of these models and ensuring their tractability pose significant challenges for decision-makers [10].

5. *Lack of Transparency and Interpretability*:

Opaque Decision Models: Some MCDM techniques produce results that are difficult to interpret or explain, limiting the transparency of decision-making processes [11].

Decision-makers may struggle to understand the underlying rationale behind the recommended solutions, leading to skepticism and distrust.

Interpretability Concerns: Lack of interpretability can hinder stakeholders' ability to assess the validity and robustness of MCDM outcomes, as well as their willingness to accept and act upon the recommendations provided by the models.

6. *Data Quality and Availability*:

Data Completeness: Obtaining comprehensive and reliable data for all relevant criteria and alternatives can be challenging in many decision contexts. Incomplete or inconsistent data may lead to biased assessments and unreliable decision outcomes, undermining the credibility of MCDM analyses [12].

Data Accessibility: Accessing relevant data sources and ensuring data accessibility can pose logistical and practical challenges for decision-makers. Limited availability of data or restrictions on data access may constrain the scope and accuracy of MCDM analyses.

7. *Dynamic and Evolving Environments*:

Adaptability Requirements: Decision-making environments are often dynamic and subject to change over time due to technological advancements, market shifts, regulatory changes, and other factors. MCDM models must be capable of adapting to evolving conditions and incorporating new information to remain relevant and effective.

Model Updating Challenges: Updating and maintaining MCDM models to reflect changes in the external environment requires continuous monitoring and adjustment. Ensuring the timeliness and accuracy of model updates can be resource-intensive and may pose logistical challenges for decision-makers [13].

3 Integration of Fuzzy Logic in MCDM

Multicriteria decision-making (MCDM) problems often involve uncertainty, imprecision, and subjectivity, making them inherently challenging to address using traditional decision-making approaches. Fuzzy logic provides a powerful framework for handling these complexities by allowing for the representation and manipulation of vague or ambiguous information in a systematic and flexible manner. In the context of MCDM, the integration of fuzzy logic offers several advantages, including the ability to model subjective preferences, capture uncertainty in decision criteria, and facilitate more robust decision-making processes [14].

3.1 Principles of Fuzzy Logic

At the core of fuzzy logic is the concept of fuzzy sets, introduced by Lotfi Zadeh in the 1960s. Unlike classical (crisp) sets, which assign membership values of either 0 or 1 to elements, fuzzy sets allow for partial membership, where elements can belong to a set to a degree between 0 and 1. This notion of gradual membership enables fuzzy logic to represent and reason with imprecise or uncertain information more effectively. Fuzzy logic also incorporates linguistic variables and fuzzy rules, which allow decision-makers to express their preferences in natural language terms and derive conclusions based on fuzzy inference mechanisms.

3.2 Fuzzy Sets in MCDM

In MCDM, fuzzy sets are used to represent the uncertainty and imprecision associated with decision criteria and alternatives. Decision criteria can be defined as fuzzy sets with membership functions that capture the degree to which alternatives satisfy each criterion. Similarly, alternatives can be represented as fuzzy sets with membership functions indicating their performance levels with respect to each criterion. By defining fuzzy sets for decision criteria and alternatives, decision-makers can model the inherent vagueness and ambiguity in their assessments, enabling more nuanced evaluations and comparisons [4] (Fig. 3).

Several MCDM methods have been developed that leverage fuzzy logic principles to address uncertainty and imprecision in decision-making. These methods extend traditional MCDM techniques, such as the Analytic Hierarchy Process (AHP), TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), and ELECTRE (Elimination and Choice Translating Reality), to accommodate fuzzy preferences and fuzzy evaluations. For example, fuzzy AHP replaces crisp pairwise comparisons with fuzzy linguistic terms, allowing decision-makers to express their preferences in linguistic terms (e.g., “strongly preferred,” “moderately preferred”) rather than precise numerical values. Similarly, fuzzy TOPSIS employs fuzzy sets to represent the performance levels of alternatives and criteria, enabling the ranking of alternatives based on their overall fuzzy closeness to the ideal solution.

Fuzzy set-based MCDM methods leverage fuzzy logic principles to address the complexities of decision-making in uncertain and imprecise environments. These methods extend traditional MCDM techniques to accommodate fuzzy preferences, fuzzy evaluations, and linguistic uncertainty. By incorporating fuzzy sets, membership functions, and fuzzy inference mechanisms, fuzzy set-based MCDM methods

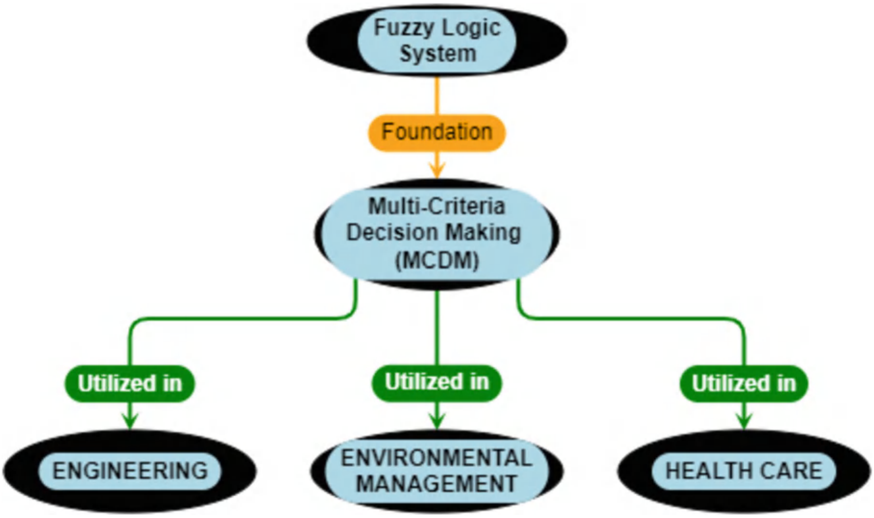

Fig. 3 Fuzzy set in MCDM

provide a flexible and robust framework for decision-makers to evaluate and rank alternatives in multicriteria decision scenarios (Fig. 4).

A. Fuzzy AHP (Analytic Hierarchy Process)

Fuzzy AHP extends the Analytic Hierarchy Process (AHP) by incorporating fuzzy logic to handle imprecise judgments and linguistic uncertainty in pairwise comparisons. In traditional AHP, decision-makers compare alternatives and criteria using crisp numerical values representing their relative importance or preference. In fuzzy AHP, decision-makers use fuzzy linguistic terms (e.g., "strongly preferred," "moderately preferred") to express their judgments, which are then converted into fuzzy membership values using appropriate linguistic variables and membership functions. Fuzzy AHP aggregates these fuzzy preferences to derive overall priority weights

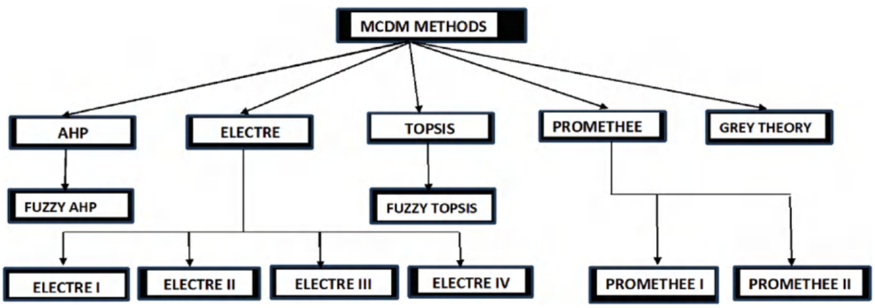

Fig. 4 Classifications of MCDM methods

foralternatives and criteria, allowing decision-makers to make more nuanced and robust decisions in complex decision scenarios.

B. **Fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution)**

Fuzzy TOPSIS extends the TOPSIS method by incorporating fuzzy logic to handle imprecise evaluations of alternatives with respect to multiple criteria. In traditional TOPSIS, alternatives are evaluated based on their distances to the ideal and worst solutions in a decision space defined by the criteria. In fuzzy TOPSIS, these evaluations are performed using fuzzy numbers to represent the performance levels of alternatives and criteria. Decision-makers define fuzzy membership functions for each criterion to capture the uncertainty and imprecision associated with their evaluations. Fuzzy TOPSIS ranks alternatives based on their overall closeness to the ideal and worst solutions, providing a comprehensive ranking that considers both the best and worst-case scenarios.

C. **Fuzzy ELECTRE (Elimination and Choice Translating Reality)**

Fuzzy ELECTRE extends the ELECTRE method by incorporating fuzzy logic to handle imprecise preference information and linguistic uncertainty in decision-making. In traditional ELECTRE, decision-makers compare alternatives based on pairwise outranking relationships defined by concordance and discordance indices. In fuzzy ELECTRE, these comparisons are performed using fuzzy sets and linguistic variables to represent the degree of preference or indifference between alternatives. Decision-makers express their preferences using linguistic terms (e.g., "strongly preferred," "equally preferred") and define fuzzy preference relations based on their subjective judgments. Fuzzy ELECTRE aggregates these fuzzy preference relations to derive overall rankings of alternatives, allowing decision-makers to identify the most preferred alternatives while considering the uncertainty and imprecision inherent in their preferences.

4 Advantages of Fuzzy Set-Based MCDM Methods

Handling Uncertainty and Imprecision: Fuzzy logic addresses the inherent uncertainties and imprecisions in real-world decision-making scenarios. Unlike classical binary logic, which operates in crisp, clear-cut distinctions, fuzzy logic allows for the representation of gradual membership. This means that instead of categorizing elements as strictly belonging or not belonging to a set, fuzzy logic permits the assignment of partial membership values, reflecting the degrees of truth or relevance (Fig. 5).

In MCDM, where decision criteria and assessments are often fraught with uncertainty, fuzzy logic's ability to capture and manipulate vague or ambiguous information is invaluable.

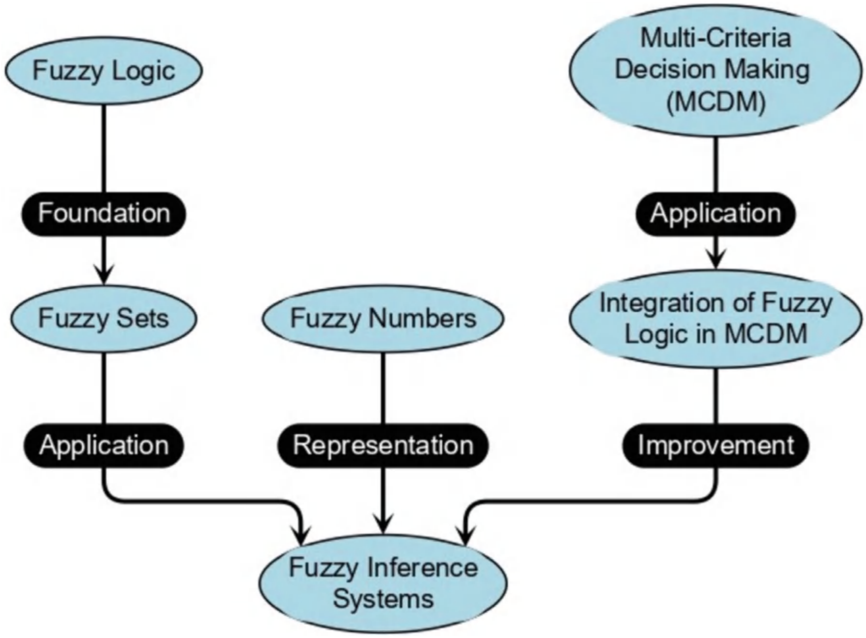

Fig. 5 Fuzzy benefits and applications

Flexibility in Preference Representation: Fuzzy logic provides decision-makers with a flexible and intuitive framework for expressing their preferences. Through linguistic variables and fuzzy sets, decision-makers can describe their preferences in natural language terms, such as “high,” “medium,” or “low,” rather than relying solely on numerical values. This linguistic flexibility enhances the transparency and accessibility of the decision process, allowing stakeholders to understand and contribute to the decision-making process more effectively.

Integration of Subjective Judgments: MCDM frequently involves subjective assessments and expert opinions from multiple stakeholders. Fuzzy logic facilitates the integration of these subjective judgments into the decision-making process by allowing decision-makers to express their preferences in linguistic terms. This integration ensures that decision outcomes reflect the diverse perspectives and domain expertise of stakeholders, leading to more comprehensive and inclusive decision-making.

Robustness to Data Variability: In real-world decision-making scenarios, data can be variable and uncertain, leading to challenges in traditional decision-making approaches. Fuzzy logic-based MCDM methods offer robustness to data variability by accommodating fluctuations in data quality and reliability. Fuzzy sets and linguistic variables enable decision-makers to handle data inconsistency and uncertainty effectively, ensuring that decision outcomes remain stable and reliable across different scenarios and data conditions.

Adaptability to Complex Decision Environments: Fuzzy logic-based MCDM methods are highly adaptable to the complexities of real-world decision environments. Decision-makers can tailor fuzzy sets and membership functions to suit the specific characteristics of the decision problem, allowing for the modeling of complex and multidimensional decision criteria. This adaptability makes fuzzy logic an effective approach for addressing diverse decision challenges across various domains, from engineering and healthcare to finance and environmental management.

Fuzzy set-based MCDM methods offer several advantages:

- **Flexibility**: Fuzzy set-based methods provide a flexible framework for representing and reasoning with imprecise or uncertain information, allowing decision-makers to capture nuances and variations in their assessments.
- **Robustness**: Fuzzy set-based methods are robust to changes in input data and can accommodate different types of uncertainty, including linguistic uncertainty and data uncertainty.
- **Transparency**: Fuzzy set-based methods allow decision-makers to express their preferences in natural language terms, making the decision process more transparent and accessible to stakeholders.
- **Adaptability**: Fuzzy set-based methods can adapt to diverse decision scenarios and problem contexts, making them applicable across a wide range of domains and applications.

5 Applications of Fuzzy Logic in MCDM

Fuzzy logic has found numerous applications in addressing multicriteria decision-making (MCDM) problems across various domains. By providing a flexible and robust framework for handling uncertainty, imprecision, and subjectivity, fuzzy logic-based MCDM methods offer practical solutions to complex decision challenges. Here are some notable applications of fuzzy logic in MCDM:

1. *Engineering Design and Optimization*

In engineering design and optimization, decision-makers often need to evaluate and select the best design alternative considering multiple conflicting objectives such as performance, cost, reliability, and manufacturability. Fuzzy logic-based MCDM methods, such as fuzzy AHP and fuzzy TOPSIS, have been applied to optimize design parameters, select materials, and prioritize design alternatives in engineering projects, such as product design, process optimization, and infrastructure development.

2. *Environmental Management and Sustainability*

Environmental management and sustainability involve making decisions that balance economic, social, and environmental objectives. Fuzzy logic-based MCDM methods have been used to assess the environmental impact of projects, evaluate alternative energy sources, and prioritize sustainable development initiatives. These methods enable decision-makers to account for uncertainties in environmental data, stakeholder preferences, and regulatory requirements when making decisions about resource allocation and policy implementation.

3. *Healthcare Decision-Making*

In healthcare decision-making, clinicians and policymakers must consider multiple criteria, including efficacy, safety, cost-effectiveness, and patient preferences, when selecting treatment options, allocating resources, and designing healthcare policies. Fuzzy logic-based MCDM methods have been applied to prioritize healthcare interventions, optimize resource allocation, and support clinical decision-making processes. These methods enable healthcare professionals to integrate qualitative and quantitative data, as well as expert judgments, in decision-making under uncertainty and ambiguity.

4. *Financial Portfolio Management*

Financial portfolio management involves selecting investment strategies and asset allocations that maximize returns while managing risks and uncertainties. Fuzzy logic-based MCDM methods have been used to construct and optimize investment portfolios, assess risk-adjusted returns, and manage asset allocation decisions. By considering fuzzy preferences, uncertain market conditions, and subjective risk attitudes, these methods help investors and financial managers make informed decisions in dynamic and uncertain financial markets.

5. *Supply Chain Management*

Supply chain management requires optimizing the flow of goods, services, and information across a network of suppliers, manufacturers, distributors, and customers. Fuzzy logic-based MCDM methods have been employed to optimize supplier selection, inventory management, production planning, and distribution scheduling decisions. These methods enable supply chain managers to account for uncertainties in demand forecasts, supplier performance, and logistics constraints when making decisions to improve operational efficiency and customer satisfaction.

6. *Urban Planning and Transportation*

Urban planning and transportation involve making decisions about infrastructure development, land use, and transportation systems to meet the needs of growing urban populations. Fuzzy logic-based MCDM methods have evaluated transportation alternatives, prioritized infrastructure projects, and optimized urban development plans. These methods help urban planners and policymakers consider diverse stakeholder preferences, environmental impacts, and socioeconomic factors in decision-making to promote sustainable urban development and efficient transportation systems.

7. *Risk Management and Decision Support Systems*

Risk management and decision support systems assist decision-makers in identifying, assessing, and mitigating risks associated with various alternatives and scenarios. Fuzzy logic-based MCDM methods have been integrated into risk assessment models, decision support systems, and scenario analysis tools to support uncertainty-free decision-making. These methods enable decision-makers to analyse complex decision landscapes, explore trade-offs, and effectively identify robust solutions that balance risks and rewards.

In summary, fuzzy logic-based MCDM methods have diverse applications across numerous domains, including engineering, environmental management, healthcare, finance, supply chain management, urban planning, and risk management. By providing a systematic and flexible framework for addressing uncertainty, imprecision, and subjectivity in decision-making, fuzzy logic enables decision-makers to make more informed, robust, and effective decisions in complex and uncertain decision environments.

5.1 Case Study: Renewable Energy Investment Decision

Background:

Green Energy Inc., a renewable energy investment firm, is tasked with selecting themost promising renewable energy projects for investment. The firm aims to maximize returns while considering environmental sustainability, technological feasibility, and social impact. However, decision-makers face uncertainty in project performance, imprecisionin sustainability assessments, and subjective preferences regarding investment criteria.

Problem:

Green Energy Inc. needs to prioritize renewable energy projects based on criteria such as energy generation potential, environmental sustainability, cost-effectiveness, and community acceptance. However, the evaluation of projects is complicated by uncertainty in energy yield projections, imprecision in environmental impact assessments, and varying stakeholder perspectives.

Solution:

The firm employs fuzzy logic-based multicriteria decision-making (MCDM) methods to evaluate and rank renewable energy projects. Specifically, they use fuzzy TOPSIS to select projects that balance economic viability with environmental and social considerations.

Implementation Steps:

Criteria Definition: Decision-makers identify key criteria for evaluating renewable energy projects, including energy generation potential, environmental impact, cost-effectiveness, and community acceptance. Each criterion is defined using linguistic terms and fuzzy membership functions to capture uncertainty and imprecision.

Data Collection and Assessment: Project data, including energy yield estimates, environmental impact assessments, and cost projections, are collected and evaluated.

Fuzzy sets are used to represent uncertain and imprecise data, allowing decision-makers to account for variability in project performance and sustainability metrics.

Preference Elicitation: Decision-makers express their preferences for project criteria using linguistic terms (e.g., "high energy yield," "low environmental impact") and fuzzy membership functions. Preferences are elicited through stakeholder consultations, expert opinions, and data analysis, ensuring that decision-makers' subjective judgments are captured effectively.

Fuzzy TOPSIS Analysis: Fuzzy TOPSIS is applied to rank renewable energy projects based on their overall closeness to the ideal investment opportunity. Fuzzy numbers representing project performance on each criterion are aggregated using fuzzy arithmetic operations. The resulting rankings identify projects that offer the best balance of economic, environmental, and social benefits, considering decision-makers' fuzzy preferences and uncertainty in project evaluations.

Green Energy Inc. successfully prioritizes renewable energy projects for investment based on fuzzy logic-based MCDM analysis. The firm selects projects that offer the highest potential for returns while demonstrating environmental sustainability, technological feasibility, and community acceptance. By leveraging fuzzy logic principles to address uncertainty and imprecision in decision-making, Green Energy Inc. makes informed and robust investment decisions that align with its mission of promoting renewable energy adoption and sustainability.

This case study illustrates the practical application of fuzzy logic in multicriteria decision-making within the context of renewable energy investment. By integrating fuzzy logic principles into the decision-making process, Green Energy Inc. effectively addresses uncertainty, imprecision, and subjective preferences, enabling more informed and sustainable investment decisions in the renewable energy sector.

5.2 Case Study: Sustainable Transportation Infrastructure Planning

Background:

The Department of Transportation (DOT) of a rapidly growing city is tasked with prioritizing transportation infrastructure projects to improve mobility, reduce congestion, and enhance sustainability. The DOT aims to select projects that balance economic feasibility with environmental impact, social equity, and community preferences. However, decision-makers face uncertainty in traffic projections, imprecision in environmental assessments, and varying stakeholder priorities.

Problem:

The DOT needs to prioritize transportation infrastructure projects, including road expansions, public transit upgrades, and cycling infrastructure, based on criteria such as traffic congestion relief, environmental sustainability, cost-effectiveness, and public satisfaction. However, the evaluation of projects is complicated by uncertainty in future traffic patterns, imprecision in environmental impact assessments, and subjective preferences among stakeholders.

Solution:

The DOT employs fuzzy logic-based multicriteria decision-making (MCDM) methods to evaluate and rank transportation infrastructure projects. Specifically, they use fuzzy ELECTRE to select projects that address mobility needs while minimizing environmental impact and maximizing social equity.

Implementation Steps:

Criteria Definition: Decision-makers identify key criteria for evaluating transportation infrastructure projects, including traffic congestion relief, environmental impact, cost-effectiveness, and community satisfaction. Each criterion is defined using linguistic terms and fuzzy membership functions to capture uncertainty and imprecision.

Data Collection and Assessment: Project data, including traffic flow data, environmental impact assessments, and cost estimates, are collected and evaluated. Fuzzy sets are used to represent uncertain and imprecise data, allowing decision-makers to account for variability in traffic projections and environmental assessments.

Preference Elicitation: Decision-makers express their preferences for project criteria using linguistic terms (e.g., "high congestion relief," "low environmental impact") and fuzzy membership functions. Preferences are elicited through stakeholder consultations, public surveys, and expert opinions, ensuring that decision-makers' subjective judgments are captured effectively.

Fuzzy ELECTRE Analysis: Fuzzy ELECTRE is applied to rank transportation infrastructure projects based on their overall performance with respect to the defined criteria. Fuzzy preference relations representing decision-makers' preferences are aggregated using fuzzy arithmetic operations. The resulting rankings identify projects that offer the best balance of mobility improvements, environmental sustainability, cost-effectiveness, and community satisfaction, considering uncertainty and imprecision in project evaluations.

The DOT successfully prioritizes transportation infrastructure projects for implementation based on fuzzy logic-based MCDM analysis. The selected projects address critical mobility needs while minimizing environmental impact and maximizing social equity. By leveraging fuzzy logic principles to address uncertainty and imprecision in decision-making, the DOT makes informed and sustainable investments in transportation infrastructure that benefit the city's residents and businesses.

This case study demonstrates the practical application of fuzzy logic in multicriteria decision-making within the context of sustainable transportation infrastructure planning. By integrating fuzzy logic principles into the decision-making process, the DOT effectively addresses uncertainty, imprecision, and subjective preferences, enabling more informed and equitable investment decisions in transportation infrastructure that support the city's long-term sustainability and liability goals.

6 Limitations of Fuzzy Logic in Multicriteria Decision Making (MCDM)

Subjectivity in Linguistic Terms: While linguistic terms in fuzzy logic-based MCDM methods allow for intuitive preference representation, they can introduce subjectivity and ambiguity into the decision process. Different decision-makers may interpret linguistic terms differently, leading to potential inconsistencies in preference representation and decision outcomes. Resolving these differences and ensuring consistency in the interpretation of linguistic terms can be challenging, particularly in large and diverse decision-making contexts.

- Computational Complexity: Fuzzy logic-based MCDM methods can be computationally demanding, especially for decision problems with numerous criteria and alternatives. The aggregation of fuzzy preferences and the computation of fuzzy rankings may require significant computational resources and time, limiting the scalability of these methods. Managing computational complexity becomes crucial, particularly in real-time decision-making applications requiring rapid responses.
- Difficulty in Parameter Tuning: Fuzzy logic-based MCDM methods often involve selecting and tuning parameters such as fuzzy membership functions and aggregation operators. Determining appropriate parameter values can be challenging and may require extensive trial-and-error experimentation or domain expertise. Inadequate parameter tuning can lead to suboptimal decision outcomes or even model instability, highlighting the importance of careful parameter selection and sensitivity analysis.
- Interpretability of Results: Interpretation fuzzy logic-based MCDM results can be challenging, particularly when dealing with complex fuzzy inference mechanisms and aggregation techniques. Decision-makers may struggle to understand the underlying reasoning processes and the implications of fuzzy rankings, potentially hindering the acceptance and adoption of fuzzy logic-based approaches. Ensuring the interpretability and transparency of fuzzy logic-based decision outcomes is essential for building stakeholder trust and confidence.
- Validation and Validation: Assessing the validity and reliability of fuzzy logic-based MCDM methods can be difficult, particularly in the absence of ground truth or benchmark data. Validation techniques for fuzzy logic-based models may be less well-established compared to traditional quantitative methods, raising questions about the robustness and generalizability of fuzzy logic-based decision outcomes. Establishing rigorous validation procedures and conducting sensitivity analyses are essential steps in ensuring the credibility and robustness of fuzzy logic-based MCDM methods.

7 Advancements in Artificial Intelligence (AI) and Machine Learning (ML):

Integration of fuzzy logic with AI and ML algorithms: Researchers are exploring ways to combine fuzzy logic with AI and ML techniques to address complex decision-making problems in uncertain and dynamic environments. By integrating fuzzy logic's ability to handle uncertainty and imprecision with the learning capabilities of AI and ML algorithms, these hybrid systems can provide more robust and adaptive solutions.

Development of hybrid systems: Hybrid systems that combine fuzzy logic with deep learning techniques are being developed to enhance pattern recognition, data analysis, and decision-making. By leveraging the strengths of both fuzzy logic and deep learning, these hybrid systems can capture complex relationships in data while incorporating human-like reasoning and interpretation.

a. **Internet of Things (IoT) and Cyber-Physical Systems (CPS)**:

Use of fuzzy logic in managing IoT and CPS: Fuzzy logic is being applied to manage and control IoT devices and cyber-physical systems, enabling adaptive and autonomous decision-making in interconnected environments. Fuzzy logic-based controllers can adapt to changing conditions and uncertainties in real-time, making them suitable for applications such as smart cities, intelligent transportation systems, and industrial automation [15].

Application in smart cities and transportation: In smart cities, fuzzy logic can optimize energy usage, traffic flow, and resource allocation to improve efficiency and sustainability. In intelligent transportation systems, fuzzy logic-based algorithms can optimize route planning, traffic signal control, and vehicle coordination to reduce congestion and enhance safety [16].

b. **Explainable AI and Interpretable Models**:

Development of interpretable fuzzy logic models: There is a growing emphasis on developing fuzzy logic models that are interpretable and transparent to end-users. By using linguistic variables and fuzzy rules that are easy to understand, these models provide human-understandable explanations for their decisions, enhancing trust and acceptance.

Integration with explainable AI techniques: Fuzzy logic is being integrated with explainable AI techniques to provide transparent and interpretable explanations for complex decisions made by AI systems. By combining fuzzy logic's linguistic representations with techniques such as rule extraction and visualization, these systems can provide insights into their decision-making process [17, 18].

c. **Fuzzy Control in Autonomous Vehicles and Robotics**:

Deployment in autonomous systems: Fuzzy logic-based control systems are being deployed in autonomous vehicles and robots to make real-time decisions in dynamic and uncertain environments. Fuzzy controllers can handle imprecise sensor data, environmental variability, and unpredictable scenarios, making them suitable for applications such as autonomous driving, robotic navigation, and drone control [19].

Research on navigation and obstacle avoidance: Researchers are developing fuzzy logic-based algorithms for navigation, obstacle avoidance, and path planning in autonomous systems. These algorithms enable vehicles and robots to navigate complex environments, avoid obstacles, and plan efficient routes while considering safety constraints and mission objectives [20].

d. **Fuzzy Systems for Personalized Healthcare**:

Utilization in personalized medicine: Fuzzy logic is being utilized in personalized medicine and healthcare decision support systems to tailor treatments and interventions to individual patient needs. By considering patient-specific factors such as genetic makeup, medical history, and lifestyle preferences, fuzzy logic-based models can recommend personalized treatment plans that optimize efficacy and minimize adverse effects [21].

Development of predictive models: Researchers are developing fuzzy logic-based predictive models for disease prognosis, risk assessment, and treatment optimization. These models integrate fuzzy reasoning with medical knowledge and patient data to predict disease outcomes, assess treatment effectiveness, and identify personalized interventions [22].

e. **Explainable Recommender Systems and Personalized Services**:

Integration in recommender systems: Fuzzy logic is being integrated into recommender systems and personalized services to provide transparent and interpretable recommendations based on user preferences and feedback. By using fuzzy logic to model user preferences and item attributes, these systems can generate personalized recommendations that are easy to understand and explain [23].

Application in e-commerce and digital marketing: In e-commerce platforms and digital marketing campaigns, fuzzy logic-based models are used to analyze customer preferences, predict purchase behavior, and tailor product recommendations. These models can improve user satisfaction and engagement by considering fuzzy factors such as product ratings, price sensitivity, and brand affinity.

f. **Fuzzy Logic in Environmental Modeling and Sustainability**:

Use in environmental monitoring and climate modeling: Fuzzy logic-based modeling techniques are used in environmental monitoring, climate modeling, and natural resource management to assess and mitigate environmental risks. By integrating fuzzy reasoning with environmental data and scientific models, these techniques can provide insights into complex environmental systems and support decision-making for sustainable resource management [24].

Integration with sustainability assessment: Fuzzy logic is integrated with sustainability assessment frameworks to evaluate the environmental, social, and economic impacts of policies, projects, and interventions. By considering fuzzy factors such as stakeholder preferences, uncertainty, and trade-offs, these frameworks provide holistic assessments of sustainability and inform decision-making for sustainable development [25].

g. **Ethical and Responsible AI Development**:

Exploration of ethical considerations: Researchers and practitioners are exploring ethical considerations and societal implications of fuzzy logic-based AI systems. This includes addressing fairness, accountability, transparency, and privacy issues to ensure that fuzzy logic-based AI technologies are developed and deployed responsibly.

Adoption of ethical guidelines: Ethical guidelines and regulatory frameworks are being adopted to ensure responsible deployment and use of fuzzy logic-based AI technologies. These guidelines promote ethical practices, transparency, and accountability in the development and deployment of fuzzy logic-based AI systems across various domains.

h. **Collaborative and Adaptive Systems**:

Development of collaborative systems: Collaborative and adaptive fuzzy logic-based systems are being developed to learn and evolve over time through interactions with users and the environment. These systems incorporate feedback mechanisms, learning algorithms, and adaptation strategies to improve their performance and adaptability in dynamicenvironments [26].

Integration with reinforcement learning and evolutionary algorithms: Fuzzy logic is integrated with reinforcement learning and evolutionary algorithms to enable autonomous adaptation and self-improvement in intelligent systems. By combining fuzzy reasoning with learning and optimization techniques, these systems can continuously adapt to changing conditions and learn from experience [27].

i. **Education and Training in Fuzzy Logic and AI**:

Expansion of education programs: Education and training programs in fuzzy logic and AI are expanding to equip professionals with the knowledge and skills needed to develop and deploy fuzzy logic-based systems. This includes courses, workshops, and certifications that cover fuzzy logic principles, algorithms, and applications across various domains [28].

Integration into academic curricula: Fuzzy logic and AI concepts are being integrated into academic curricula at universities and colleges to prepare students for careers in fields such as data science, robotics, and intelligent systems. This integration ensures that students have the necessary foundation in fuzzy logic and AI to address complex decision challenges in their respective fields [29].

8 Conclusion

In this chapter, the profound impact of fuzzy logic on multicriteria decision making (MCDM) has been explored, examining its applications, advantages, limitations, and future trends. Fuzzy logic has emerged as a powerful tool for addressing the complexities of decision-making processes in uncertain, imprecise, and dynamic environments across various domains.

The chapter began by delving into the foundations of multicriteria decision making, highlighting the challenges posed by uncertainty, subjectivity, and conflicting objectives. Fuzzy logic offers a flexible and robust framework for handling these challenges, allowing decision-makers to express preferences in linguistic terms, integrate subjective judgments, and reason with uncertain information.

A detailed examination of fuzzy logic's integration into MCDM methods showcased its practical applications in diverse fields such as engineering, healthcare, finance, and environmental management. From engineering design optimization to personalized healthcare decision support, fuzzy logic-based MCDM methods have demonstrated their effectiveness in addressing real-world decision challenges and improving decision outcomes.

While fuzzy logic offers numerous advantages in MCDM, such as handling uncertainty, flexibility in preference representation, and integration of subjective judgments, it also comes with limitations, including subjectivity in linguistic terms, computational complexity, and challenges in model interpretability. However, ongoing research and developments in explainable AI, collaborative systems, and ethical AI are addressing these limitations and paving the way for more transparent, interpretable, and responsible fuzzy logic-based decision-making systems.

Looking ahead, the future of fuzzy logic in MCDM is promising, with advancements in AI, IoT, healthcare, sustainability, and education driving innovation and progress. As fuzzy logic continues to evolve and integrate with emerging technologies and methodologies, it will play an increasingly pivotal role in addressing complex decision challenges and shaping the future of intelligent systems.

Fuzzy logic is a cornerstone of modern decision-making, providing decision-makers with the tools and techniques to navigate uncertainty, manage complexity, and make informed decisions in an ever-changing world. By embracing fuzzy logic principles and leveraging its capabilities, new possibilities can be unlocked, ushering in a future where decision-making is more intuitive, adaptive, and inclusive.

As the journey of exploration and discovery continues, the power of fuzzy logic must be harnessed to chart a course towards a more resilient, sustainable, and equitable future for all.

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The Assessment of Customer Satisfaction of a Product in a Manufacturing System Using Adaptive Neuro Fuzzy Interference System

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Abstract Customer satisfaction is an asset that should be protected and retained just like any other asset in an organization. The management needs practices that promote customer satisfaction in an organization's service provision. This work presents a descriptive overview that undertook a cross-sectional survey design to establish customer satisfaction in several mechanical and allied industries. Population taken from employees from various divisions and positions of similar organizations and their opinions are recorded and analyzed for a better understanding of the effect of the quality management on customer satisfaction. The objective of the present work can be summarized as the assessment of various factors affecting customer satisfaction. To do so a set of questionnaire is set as per the full factorial design of experiments to access the opinion of customers of various organizations. Qualitative research methodology has been applied due to the large number of respondents who participated. Data was captured through the interview process and research questionnaire to evaluate the respondents focusing on quality control activities and customer satisfaction in the mechanical and allied industries. This method is used to describe the relationship between organizational culture, organizational commitment and person organization fit. The sampling area is the urban area of Mumbai with a sample size of 243. Convenient sampling technique has been adopted to conduct the sample survey. Adaptive Neuro-Fuzzy Interference System (ANFIS) is used to analyze the data. The results of the analysis is fed for mapping a new set of data which is convenient and simple to predict the level of customer satisfaction on the basis of various influencing factors. This work finds its applications in the area of industrial management that uses modern manufacturing environment.

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1 Introduction

Achieving the highest levels of customer satisfaction is one of the key concerns of the many manufacturing and allied industries, as this directly influences the companys' long term competitiveness in the market and determines their survival and success. Manufacturers must constantly improve and enhance their goods and services in order to satisfy the changing demands and expectations of their customers in the fiercely competitive market of today. Due to the complexity of the factors that affect customers' perceptions and behaviors, assessing customer satisfaction is a challenging but utterly necessary undertaking. In this study, we suggest evaluating customer satisfaction in a manufacturing industry using the Adaptive Neuro-Fuzzy Inference System (ANFIS).

ANFIS is a hybrid intelligent system, which is an amalgamation offuzzy inference systems (FISs) and artificial neural networks (ANNs. ANFIS models are used in scenarios where the input data is ambiguous. So, in this context can be used to assess customer feedbackson various aspects of a manufacturer and analyze the data to get the relations and influences of various factors on customer satisfaction.

In this study, an ANFIS model is used to evaluate the complex factors affecting customer satisfaction. The model is used on the data obtained from customer surveys in a manufacturing industry to gauge the most influential factors, which affects customer feedbacks. The ANFIS modelis trained with the answers from customer surveys. The vast amount of ambiguous data obtained from the customer feedback is analyzed using fuzzy logic.

This way of evaluating the customer feedback has various advantages over the conventional methods. First of all, these models can handle huge amount of data that is not possible to be analyzed by conventional statistical techniques. Secondly, this can more efficiently detect patterns and draw conclusions from the data which may not be visible to the naked eye. Andlastly, the customers can get customized solutions based on their needs and wants.

2 Literature Review

The fuzzy logic introduced by Lofti A. Zadeh [1] in 1965, and from then the concept is being used widely in the various fields of engineering and management for decision making, simulation and modeling. In contrast to conventional decision making techniques, which depend upon exact and quantitative data, fuzzy logic offers the capacity to deal with the uncertain data which are ambiguous [2].

Fuzzy logic utilizes fuzzy set theory, which incorporates degrees of membership rather than binary membership. Fuzzy sets are the extended version of classical set theory and the foundation of fuzzy logic [1]. The use of fuzzy set theory makes a system decide on the basis of quantitative approach and an optimum solution can be achieved by the portrayal of ambiguous and uncertain information [3]. The fuzzy logic framework can help for decision making after attaining consensus and revolving conflicts [4]. Organizations use fuzzy logic also to determine the suitability of the decision making style on the basis of a given set of strategic management procedures [5]. The integration of known and measurable criteria can be done with ambiguous and uncertain criteria with the help of fuzzy logic [6]. The collection of data along with the storage and knowledge transfer from the datasets to fuzzy sets can be done and managed by software agents [7]. In contemporary times, fuzzy logic software agents are being used to manage mobile technology [7].

There are several advantageous aspects of using fuzzy logic in managerial decision making with ambiguous data [2]. Fuzzy logic can provide more thorough analysis which helps in managerial decision making [2]. Fuzzy logic is also a flexible technique which can be used in various decision making situations [4]

Fuzzy logic can handle various issues which require involving specific knowledge and skills [8], large amounts of data [7], and experience [9]. Novel fuzzy logic applications include financial applications & supply chain management [10]. The easy interpretability of fuzzy logic makes the technique become popular to non-experts [11]. Contemporary works of integration of machine learning and artificial intelligence with fuzzy logic makes the technique more practical oriented [12].

Fuzzy logic is becoming popular as an important tool in managerial decision making addressing a wide range of advantages. In modern era fuzzy logic must be combined with other techniques to provide more complete and practical oriented management decision making [13].

3 Application of Fuzzy Logic in Manufacturing System

Fuzzy logic has been increasingly applied to different scenarios of production decisions in an attempt to cope with the uncertainty and imprecision embedded in them. Much research on industrial system domains, such as those for parametric optimization, control systems, decision-making systems, and process selection, has recently been conducted regarding fuzzy logic.

In process selection, fuzzy logic has been applied in selecting the best procedure for a given manufacturing activity. For example, some researchers have used fuzzy logic in choosing the best machining procedure concerning the requirements of the final product quality and material characteristics. The fuzzy logic has also been applied by parametric optimization in the optimization characteristics of a manufacturing process. For example, fuzzy logic has been utilized to optimize machining process parameters to obtain products with the desired quality. Today, fuzzy logic is a potent tool within manufacturing systems since it furnishes solutions to highly complex problems that cannot be solved by binary logic. Dealing with vague data and mimicking processes of human decision-making, the ability has brought about radical improvements in quite a few manufacturing areas.

Another important application area is quality control. The fuzzy logic systems have successfully handled the intrinsic uncertainties in industrial processes, thus providing better accuracy in defect detection and classification. For example, Azadeh et al. (2015) showed how a fuzzy-logic-based system improved the quality control of the manufacturing process for printed circuit boards and reduced the failure rate by 15% [14].

Fuzzy logic helps overcome uncertainties in processing times, breakdowns of equipment, and demand fluctuations in production planning and scheduling. According to Sharma and Agrawal, a multi-product assembly line showed an overall effectiveness increase of about 8% in the equipment by applying a fuzzy-logic-based scheduling system (2012) [15].

Another critical component of predictive maintenance is fuzzy logic. Fuzzy systems can better envision the eventual failure of equipment by using several attributes and their complex interactions. Kumar et al. (2019) developed a fuzzy logic-based predictive maintenance system for CNC machines that prevented 23% unscheduled downtime compared to traditional techniques [16]. Fuzzy logic technologies have equally been handy in energy management applications in manufacturing plants. A scheme by Yun et al. (2017) uses a fuzzy logic controller to optimize energy use in an automotive manufacturing plant, saving as much as 12% of the energy expenses without reducing production efficiency [17].

Finally, fuzzy logic has been used in the manufacturing supply-chain management process. In a concurrent consideration of several qualitative and quantitative factors, Amindoust (2018) developed a fuzzy logic-based supplier selection model that increased the accuracy of supplier performance evaluations [18].

These applications show the flexibility and power of fuzzy logic in solving some of the complex problems that are endemic in modern production systems.

4 Adaptive Neuro-Fuzzy Interference System

Adaptive Neuro Fuzzy Interference System (ANFIS) is an advanced AI method, in which neural networks and fuzzy logic come together. In this hybrid approach, neural networks provide plasticity for learning, while fuzzy systems afford reasoning resembling human-like intelligence. Working on the Takagi–Sugeno fuzzy model, ANFIS links input and output through if–then rules. The system's main advantage is its learning capability from data to tune the fuzzy parameters, hence being highly effective for modeling complex and nonlinear relationships.

Therefore, ANFIS can be applied in domains where uncertainty and imprecision handling of information is paramount; examples include control systems related to decision support and predictive modeling. ANFIS synergizes neural networks and fuzzy logic to offer a flexible solution.

Moreover, fuzzy logic has also found applications in industrial decision-making. It is applied in ranking product mixtures according to the contribution or proportion that each component contributes to the perfect or desired product mix. Control systems have employed fuzzy logic to regulate production processes, such as maintaining kiln temperatures during production to ensure product quality. Manufacturing processes benefit from fuzzy logic by monitoring each product's membership level in the target quality, thereby regulating overall product quality. Supply chain management leverages fuzzy logic to handle uncertainties and imprecision in decision-making. Maintenance tasks in manufacturing systems are organized using fuzzy logic, considering the degree of participation of each maintenance activity in the intended plan.

5 Customer Satisfaction Survey

A sample survey is carefully created using the data that has been gathered from the population, The total output of thesetresponsesesis what will yield the interference about the entire population. Data is obtained through individual and group interviews. Data is sometimes gathered using mail surveys. Customer happiness has a direct impact on the quality of a product or service. A set of the customer population is used to assess customer satisfaction. Customer satisfaction is evaluated based on the terms of the purchase, the quality of the product, any flaws in the goods or services, the product pricing, and the level of customer service. Customers' responses are logged in three categories, which usually correspond to worst, better, very good, or low, medium, and high.

A total of 243 clients were contacted by email or in-person interviews. The sample survey is conducted throughout India in order to have a comprehensive insight. From the country's eastern and western coasts, two cities have been chosen. For the sample survey, one organization is chosen from Kolkata (in the east of India) and the other from Mumbai (in the west of India). The purpose of area selection is to ensure that the data set produced by a survey of a sizable population accurately reflects the entire population. To conduct the sample survey, a convenient sampling technique has been chosen. Using this method of qualitative sampling, researchers will contact participants and note their responses based on their availability.

The questionnaire that was distributed to customers is provided below, along with the responses and ratings that followed. Survey respondents' experiences were categorized into 1, 2, and 3 in order to convert human reasoning into crisp data.

5.1 The Customer Questionnaire

The survey was launched on the basis of a set of questions that were used to assess the customer satisfaction about a product being manufactured in an industry. These questions are important as they capture the human experience on the basis of human logic and intelligence into a system that uses artificial intelligence which is designed by the combination of fuzzy logic and artificial neural network. The artificial intelligence system that is being used to predict and create universal dataset is called Adaptive Neuro Fuzzy Intelligence System (ANFIS). Following are the questions that were used to capture human inputs in terms of crisp data. The answers have been utilized to generate crisp data of human logic ranged between good, better and best and so on. The options were framed in the range between 1 and 3, which enables the system to understand human logic or human inputs as crisp data.

Q.1 How do you rate the flexibility in our purchase terms.

Ans. (i) Worst (1 out of 3) (ii) Better (2 out of 3) (iii) Very Good (3 out of 3)

Q.2 How do you rate the quality of our service and goods?

Ans. (i) Worst (1 out of 3) (ii) Better (2 out of 3) (iii) Very Good (3 out of 3)

Q.3 How do you rate our prices of the goods and services?

Ans. (i) Low (1 out of 3) (ii) Medium (2 out of 3) (iii) High (3 out of 3)

Q.4 How do you rate of our after sales service?

Ans. (i) Worst (1 out of 3) (ii) Better (2 out of 3) (iii) Very Good (3 out of 3)

Q.5 What is your experience about the detected defects in our goods and services after sales?

Ans. (i) Very Good (1 out of 3) (ii) Average (2 out of 3) (iii) Bad (3 out of 3)

6 Normalization of Survey Results

An ANFIS network was created and trained with 143 points of normalized training data obtained from the survey. Next, 50 data are used for verification and 50 data for testing. The ANFIS network is trained using a script file that makes use of ANFIS codes. Here, the Gaussian-bell function is utilized as the membership function.

Because there is a significant fluctuation in the numerical values of the input data range, it is beneficial to bring the data to a consistent scale for input to any soft computing methods. This is accomplished by normalizing, using Eq. 1 for a range of 0.1 to 0.9.

$$y = 0.1 + 0.8\left(\frac{x - x_{min}}{x_{max} - x_{min}}\right) \tag{1}$$

where, x = actual value

x_{max} = maximum value of x.

x_{min} = minimum value of x.

y = normalized value corresponding to x.

7 Results and Discussion

The input–output data space from the survey is fed into the ANFIS network. The total 243 nos. of survey data from customer survey were subdivided into three categories to train the network. Total 143 data are feed to the ANFIS network to train the model for the survey. Next 50 data are feed into the system to test the model and last 50 data are used to validate the model. A comparison is made between the survey data and the predicted data by the ANFIS, so that we can check the error of the prediction. These errors were feed into the backward pass of the hybrid learning algorithm to make the network more efficient. If we look into the errors of validation of ANFIS then we can see that the errors are minimized and at the validation level errors are minimized and numerically zero.

In Table 1 the ANFIS predictions of each experiments is shown with their errors. It is seen that the errors is in the acceptable range. The validation data showing zero error, where test data and check data showing some acceptable errors in results.

From the responses given by the customers, pie charts were prepared to know, what percent of the customers have answered what for each survey question. These pie charts are shown in the Fig. 1.

The customer satisfaction levels predicted by ANFIS are showed in a bar chart (Fig. 2) to represent the distribution of customers in each level.

7.1 Predicted Customer Satisfaction Contour Plots

In the Figs. 3, 4 and 5, variouscontour plots of customer satisfaction levels predicted by ANFIS are shown. These plots showcase the changes in Z-axis (predicted customer satisfaction levels) with various factors, such as purchase terms flexibility, product quality, price etc. in X and Y axes. In Fig. 3a, the changes in predicted customer satisfaction is plotted versus flexibility in purchase terms and quality of the product. The plot shows that with improving purchasing flexibility and product quality, the satisfaction also increases. The increase in customer satisfaction seems to be more dependent on product quality over purchase terms.

In Fig. 3b customer satisfaction is monitored based on flexibility in purchase terms and price of the product. A general trend is noticed where, when product is priced at the highest and also the lowest levels, the customer satisfaction seems to be improving. Whereas the customer satisfaction in the medium level price seems to be lower.

In Fig. 3c effects of flexibility in purchase terms and after sales service on predicted customer satisfaction is shown. Here, we can see that after sales service has a bigger impact on customer satisfaction that purchase terms flexibility. Better after sales service improves customer satisfaction tremendously. And in Fig. 3d effects of flexibility in purchase terms and detected defects after sales is plotted. Here we can see satisfaction increasing if detected defects are less. In Fig. 4a variation of predicted

Table 1 Customer responses

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
1	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.51	0.01	Training data
2	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.13	0.03	Training data
3	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.88	−0.02	Training data
4	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.95	0.05	Training data
5	3	2	2	2	1	2	0.9	0.5	0.5	0.5	0.1	0.5	0.49	−0.01	Training data
6	1	2	1	2	1	2	0.1	0.5	0.1	0.5	0.1	0.5	0.52	0.02	Training data
7	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.05	−0.05	Training data
8	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.83	−0.07	Training data
9	1	2	1	2	3	3	0.1	0.5	0.1	0.5	0.9	0.9	0.91	0.01	Training data
10	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.55	0.05	Training data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
11	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.14	0.04	Training data
12	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.84	−0.06	Training data
13	3	2	2	2	1	2	0.9	0.5	0.5	0.5	0.1	0.5	0.49	−0.01	Training data
14	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.52	0.02	Training data
15	2	2	1	2	2	2	0.5	0.5	0.1	0.5	0.5	0.5	0.54	0.04	Training data
16	1	1	3	2	2	1	0.1	0.1	0.9	0.5	0.5	0.1	0.03	−0.07	Training data
17	2	2	1	3	1	3	0.5	0.5	0.1	0.9	0.1	0.9	0.85	−0.05	Training data
18	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.87	−0.03	Training data
19	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.07	−0.03	Training data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
20	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.52	0.02	Training data
21	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.51	0.01	Training data
22	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.86	−0.04	Training data
23	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.53	0.03	Training data
24	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.14	0.04	Training data
25	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.11	0.01	Training data
26	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.49	−0.01	Training data
27	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.11	0.01	Training data
28	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.54	0.04	Training data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
29	2	2	1	2	2	2	0.5	0.5	0.1	0.5	0.5	0.5	0.42	−0.08	Training data
30	1	1	3	2	2	1	0.1	0.1	0.9	0.5	0.5	0.1	0.13	0.03	Training data
31	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.86	−0.04	Training data
32	1	2	1	2	3	3	0.1	0.5	0.1	0.5	0.9	0.9	0.87	−0.03	Training data
33	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.95	0.05	Training data
34	2	2	1	3	1	3	0.5	0.5	0.1	0.9	0.1	0.9	0.92	0.02	Training data
35	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.92	0.02	Training data
36	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.13	0.03	Training data
37	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.06	−0.04	Training data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
38	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.86	−0.04	Training data
39	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.95	0.05	Training data
40	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.16	0.06	Training data
41	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.57	0.07	Training data
42	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.59	0.09	Training data
43	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.82	−0.08	Training data
44	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.58	0.08	Training data
45	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.09	−0.01	Training data
46	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.08	−0.02	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
47	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.85	−0.05	Training data
48	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.06	−0.04	Training data
49	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.87	−0.03	Training data
50	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.45	−0.05	Training data
51	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.07	−0.03	Training data
52	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.92	0.02	Training data
53	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.96	0.06	Training data
54	3	2	2	2	1	2	0.9	0.5	0.5	0.5	0.1	0.5	0.51	0.01	Training data
55	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.13	0.03	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
56	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.45	−0.05	Training data
57	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.58	0.08	Training data
58	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.91	0.01	Training data
59	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.53	0.03	Training data
60	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.12	0.02	Training data
61	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.01	−0.09	Training data
62	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.84	−0.06	Training data
63	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.05	−0.05	Training data
64	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.89	−0.01	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
65	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.91	0.01	Training data
66	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.12	0.02	Training data
67	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.13	0.03	Training data
68	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.95	0.05	Training data
69	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.12	0.02	Training data
70	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.85	−0.05	Training data
71	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.19	0.09	Training data
72	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.58	0.08	Training data
73	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.52	0.02	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
74	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.49	−0.01	Training data
75	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.89	−0.01	Training data
76	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.45	−0.05	Training data
77	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.18	0.08	Training data
78	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.11	0.01	Training data
79	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.93	0.03	Training data
80	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.12	0.02	Training data
81	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.81	−0.09	Training data
82	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.44	−0.06	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
83	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.05	−0.05	Training data
84	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.85	−0.05	Training data
85	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.86	−0.04	Training data
86	2	2	1	3	1	3	0.5	0.5	0.1	0.9	0.1	0.9	0.87	−0.03	Training data
87	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.85	−0.05	Training data
88	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.07	−0.03	Training data
89	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.12	0.02	Training data
90	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.96	0.06	Training data
91	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.91	0.01	Training data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
92	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.93	0.03	Training data
93	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.05	−0.05	Training data
94	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.98	0.08	Training data
95	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.51	0.01	Training data
96	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.13	0.03	Training data
97	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.92	0.02	Training data
98	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.91	0.01	Training data
99	2	2	1	3	1	3	0.5	0.5	0.1	0.9	0.1	0.9	0.95	0.05	Training data
100	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.94	0.04	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
101	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.44	−0.06	Training data
102	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.09	−0.01	Training data
103	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.12	0.02	Training data
104	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.54	0.04	Training data
105	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.11	0.01	Training data
106	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.55	0.05	Training data
107	2	2	1	2	2	2	0.5	0.5	0.1	0.5	0.5	0.5	0.54	0.04	Training data
108	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.04	−0.06	Training data
109	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.09	−0.01	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
110	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.52	0.02	Training data
111	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.14	0.04	Training data
112	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.92	0.02	Training data
113	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.94	0.04	Training data
114	2	2	1	3	1	3	0.5	0.5	0.1	0.9	0.1	0.9	0.91	0.01	Training data
115	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.95	0.05	Training data
116	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.16	0.06	Training data
117	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.17	0.07	Training data
118	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.99	0.09	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
119	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.82	−0.08	Training data
120	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.95	0.05	Training data
121	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.16	0.06	Training data
122	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.97	0.07	Training data
123	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.59	0.09	Training data
124	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.02	−0.08	Training data
125	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.94	0.04	Training data
126	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.84	−0.06	Training data
127	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.09	−0.01	Training data

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Table 1 (continued)

Customer No.	Survey Data: Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Normalised Data: Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
128	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.12	0.02	Training data
129	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.94	0.04	Training data
130	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.84	−0.06	Training data
131	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.89	−0.01	Training data
132	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.12	0.02	Training data
133	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.91	0.01	Training data
134	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.13	0.03	Training data
135	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.88	−0.02	Training data
136	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.95	0.05	Training data

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Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
137	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.09	−0.01	Training data
138	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.52	0.02	Training data
139	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.45	−0.05	Training data
140	3	2	2	2	1	2	0.9	0.5	0.5	0.5	0.1	0.5	0.43	−0.07	Training data
141	1	2	1	2	1	2	0.1	0.5	0.1	0.5	0.1	0.5	0.51	0.01	Training data
142	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.15	0.05	Training data
143	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.54	0.04	Training data
144	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.44	−0.06	Testing Data
145	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.89	−0.01	Testing Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
146	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.52	0.02	Testing Data
147	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.14	0.04	Testing Data
148	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.83	−0.07	Testing Data
149	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.05	−0.05	Testing Data
150	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.07	−0.03	Testing Data
151	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.47	−0.03	Testing Data
152	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.12	0.02	Testing Data
153	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.91	0.01	Testing Data
154	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.86	−0.04	Testing Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
155	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.93	0.03	Testing Data
156	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.14	0.04	Testing Data
157	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.11	0.01	Testing Data
158	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.89	−0.01	Testing Data
159	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.91	0.01	Testing Data
160	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.94	0.04	Testing Data
161	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.02	−0.08	Testing Data
162	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.93	0.03	Testing Data
163	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.06	−0.04	Testing Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
164	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.87	−0.03	Testing Data
165	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.55	0.05	Testing Data
166	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.12	0.02	Testing Data
167	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.92	0.02	Testing Data
168	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.93	0.03	Testing Data
169	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.46	−0.04	Testing Data
170	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.06	−0.04	Testing Data
171	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.15	0.05	Testing Data
172	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.54	0.04	Testing Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
173	3	3	3	3	1	3	0.9	0.9	0.9	0.9	0.1	0.9	0.84	−0.06	Testing Data
174	2	3	2	3	1	3	0.5	0.9	0.5	0.9	0.1	0.9	0.89	−0.01	Testing Data
175	1	1	3	1	3	1	0.1	0.1	0.9	0.1	0.9	0.1	0.12	0.02	Testing Data
176	2	2	3	2	1	2	0.5	0.5	0.9	0.5	0.1	0.5	0.54	0.04	Testing Data
177	1	2	2	2	2	2	0.1	0.5	0.5	0.5	0.5	0.5	0.43	−0.07	Testing Data
178	3	3	3	3	1	3	0.9	0.9	0.9	0.9	0.1	0.9	0.85	−0.05	Testing Data
179	2	3	2	1	1	2	0.5	0.9	0.5	0.1	0.1	0.5	0.47	−0.03	Testing Data
180	3	1	2	1	2	1	0.9	0.1	0.5	0.1	0.5	0.1	0.07	−0.03	Testing Data
181	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.52	0.02	Testing Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
182	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.51	0.01	Testing Data
183	3	2	2	2	1	2	0.9	0.5	0.5	0.5	0.1	0.5	0.46	−0.04	Testing Data
184	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.93	0.03	Testing Data
185	3	2	3	2	2	2	0.9	0.5	0.9	0.5	0.5	0.5	0.54	0.04	Testing Data
186	2	2	2	1	2	1	0.5	0.5	0.5	0.1	0.5	0.1	0.11	0.01	Testing Data
187	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.09	−0.01	Testing Data
188	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.51	0.01	Testing Data
189	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.14	0.04	Testing Data
190	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.42	−0.08	Testing Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
191	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.13	0.03	Testing Data
192	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.06	−0.04	Testing Data
193	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.87	−0.03	Testing Data
194	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
195	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
196	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.1	0.00	Validation Data
197	1	2	3	3	1	3	0.1	0.5	0.9	0.9	0.1	0.9	0.9	0.00	Validation Data
198	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.5	0.00	Validation Data
199	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.1	0.00	Validation Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
200	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.9	0.00	Validation Data
201	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.9	0.00	Validation Data
202	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.1	0.00	Validation Data
203	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.1	0.00	Validation Data
204	2	3	2	3	1	2	0.5	0.9	0.5	0.9	0.1	0.5	0.5	0.00	Validation Data
205	2	1	3	1	3	1	0.5	0.1	0.9	0.1	0.9	0.1	0.1	0.00	Validation Data
206	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.9	0.00	Validation Data
207	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.9	0.00	Validation Data
208	2	2	1	3	1	3	0.5	0.5	0.1	0.9	0.1	0.9	0.9	0.00	Validation Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
209	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
210	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.1	0.00	Validation Data
211	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.1	0.00	Validation Data
212	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
213	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
214	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
215	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
216	3	3	3	2	1	3	0.9	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
217	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.1	0.00	Validation Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
218	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.5	0.00	Validation Data
219	2	3	2	1	1	2	0.5	0.9	0.5	0.1	0.1	0.5	0.5	0.00	Validation Data
220	3	1	2	1	2	1	0.9	0.1	0.5	0.1	0.5	0.1	0.1	0.00	Validation Data
221	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.5	0.00	Validation Data
222	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.5	0.00	Validation Data
223	2	3	2	1	1	2	0.5	0.9	0.5	0.1	0.1	0.5	0.5	0.00	Validation Data
224	3	1	2	1	2	1	0.9	0.1	0.5	0.1	0.5	0.1	0.1	0.00	Validation Data
225	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.5	0.00	Validation Data
226	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.5	0.00	Validation Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
227	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.1	0.00	Validation Data
228	1	1	2	1	3	1	0.1	0.1	0.5	0.1	0.9	0.1	0.1	0.00	Validation Data
229	2	3	3	2	1	3	0.5	0.9	0.9	0.5	0.1	0.9	0.9	0.00	Validation Data
230	1	3	3	3	1	3	0.1	0.9	0.9	0.9	0.1	0.9	0.9	0.00	Validation Data
231	3	3	1	3	2	3	0.9	0.9	0.1	0.9	0.5	0.9	0.9	0.00	Validation Data
232	2	3	2	3	1	3	0.5	0.9	0.5	0.9	0.1	0.9	0.9	0.00	Validation Data
233	1	2	3	2	2	1	0.1	0.5	0.9	0.5	0.5	0.1	0.1	0.00	Validation Data
234	2	3	3	3	1	3	0.5	0.9	0.9	0.9	0.1	0.9	0.9	0.00	Validation Data
235	1	3	3	2	2	2	0.1	0.9	0.9	0.5	0.5	0.5	0.5	0.00	Validation Data

(continued)

Table 1 (continued)

Customer No.	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	Flexibility in purchase terms	Quality of the service or goods	Price of the service or goods	After sales service	Detected defects after sales	Customer satisfaction	ANFIS Prediction	Error	Type of Data
	Survey Data						Normalised Data								
236	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.5	0.00	Validation Data
237	3	2	2	2	1	2	0.9	0.5	0.5	0.5	0.1	0.5	0.5	0.00	Validation Data
238	1	2	1	2	1	2	0.1	0.5	0.1	0.5	0.1	0.5	0.5	0.00	Validation Data
239	3	1	2	1	3	1	0.9	0.1	0.5	0.1	0.9	0.1	0.1	0.00	Validation Data
240	3	1	2	1	2	1	0.9	0.1	0.5	0.1	0.5	0.1	0.1	0.00	Validation Data
241	1	2	2	3	2	2	0.1	0.5	0.5	0.9	0.5	0.5	0.5	0.00	Validation Data
242	2	1	2	2	3	1	0.5	0.1	0.5	0.5	0.9	0.1	0.1	0.00	Validation Data
243	3	2	3	3	2	2	0.9	0.5	0.9	0.9	0.5	0.5	0.5	0.00	Validation Data

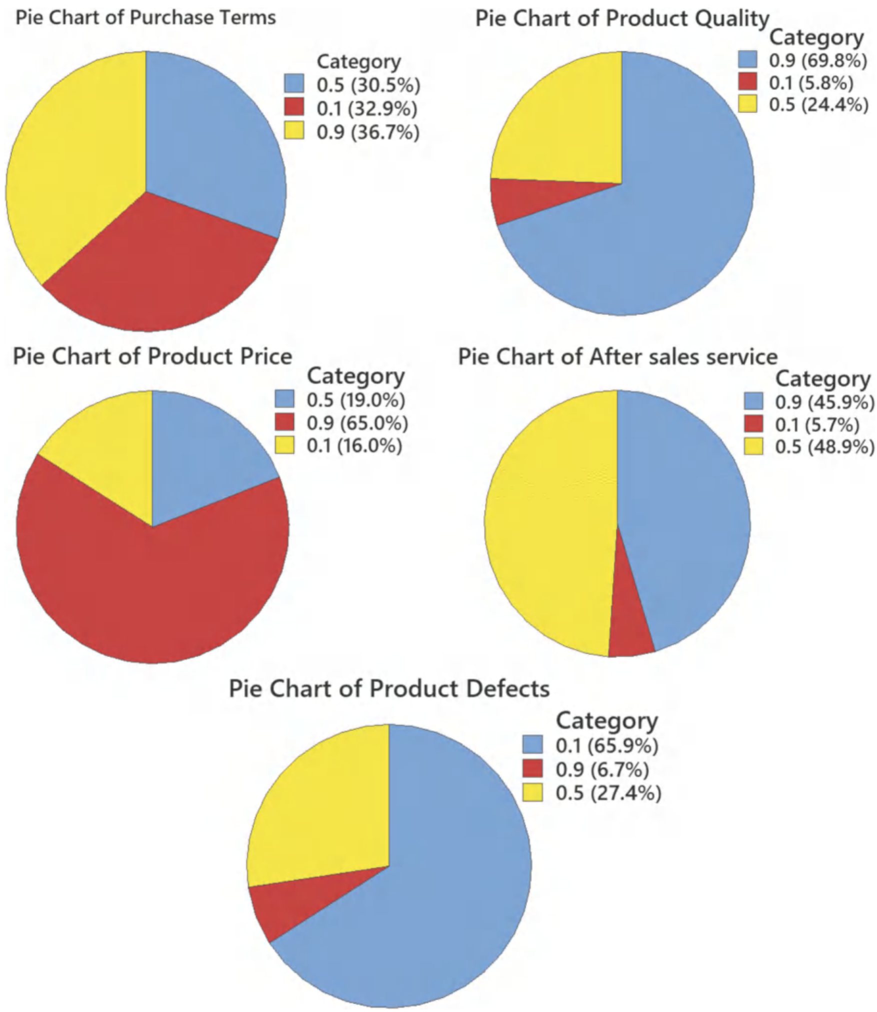

Fig. 1 Tendencies of customer responses on the survey questions

customer satisfaction against the quality and the price of the product is plotted. This shows that customer satisfaction increases with increasing quality, but low and high price both satisfies the customer more that a middling price. From Fig. 4b, we can assess that both product quality and after sales service positively impacts customer satisfaction levels.

Figure 4c effects of quality of the product and detected defects in the product after sales on customer satisfaction are shown. We can see that most customers are willing to deal with a few defects if the products are of the highest quality levels. Figure 4d shows that, customers are willing to pay more for a product if they receive great after sales service.

Fig. 2 Distributions of the levels of customer satisfaction

Figure 5a plots the predicted customer satisfaction against the product price and amount of defects detected in the product after sales. This shows that for highly priced products, the customer satisfaction is highest if the number of defects is the lowest. But, for low priced products, customers are willing to compromise with more defects in their products. In Fig. 5b it is shown that if the after sale service is moderate or high, the customers are happier. But the number of defects in the products should be low as well.

7.2 Predicted Customer Satisfaction Surface Plots

In the Figs. 6, 7 and 8, the Customer Response Surface Plots of Customer Satisfaction (Z Axis) Are Shown.

In Fig. 6a, flexibility in purchase terms versus quality of service/goods are shown. As we can infer from the figure, customer satisfaction increases rapidly when both flexibility in purchase terms and quality of service/goods goes to level 1. In Fig. 6b flexibility in purchase terms versus price of service/goods is shown. Here we can see that satisfaction increases to a level when price of service/goods goes down.

In Fig. 7 flexibility in purchase terms versus after sale service is shown. Here we can see satisfaction gradually increasing when after sale service becomes better. In Fig. 7b flexibility in purchase terms versus detected defects after sales is plotted. Here we can see satisfaction increasing if detected defects are less in Fig. 7c quality

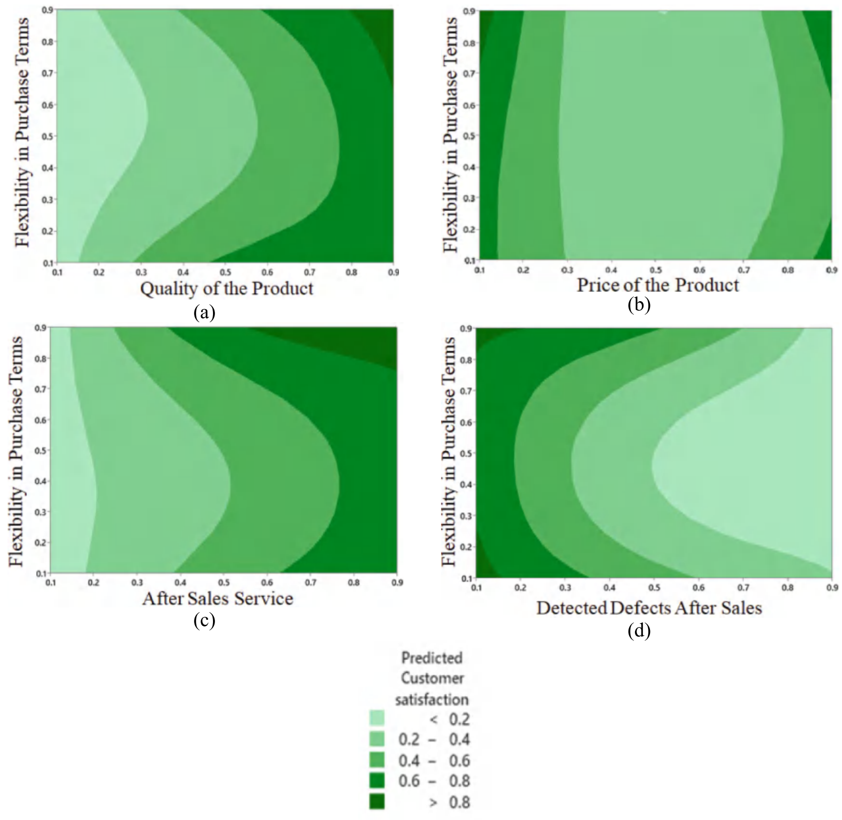

Fig. 3 Contour Plots of Predicted Customer Satisfaction versus **a** Flexibility in Purchase Terms, Quality of the Product; **b** Flexibility in Purchase Terms, Price of the Product; **c** Flexibility in Purchase Terms, After Sales Service; **d** Flexibility in Purchase Terms, Detected Defects after Sales

of service/goods versus price of service/goods. In this the satisfaction increases to a level if price goes down. In Fig. 7d quality of service/goods versus after sale service is plotted, and here with better after sales satisfaction grows rapidly whereas for quality increase it grows relatively slowly.

In Fig. 8a Quality of service/goods versus Detected defects after sales is plotted, and satisfaction goes up if defects are less in number. In Fig. 8b Price of service/goods versus after sale service is plotted and here satisfaction grows more with better after sale service than the price. In Fig. 8c Price of service/goods versus Detected defects after sales is shown, and satisfaction does not change much with the change of these two parameters. In Fig. 8d after sale service versus Detected defects after sales is shown and satisfaction grows with better after sale service but remains the same for detected defects.

The mean responses for each response of the questions asked to customers for customer satisfaction are depicted in the main effects plot as shown in Fig. 9 that

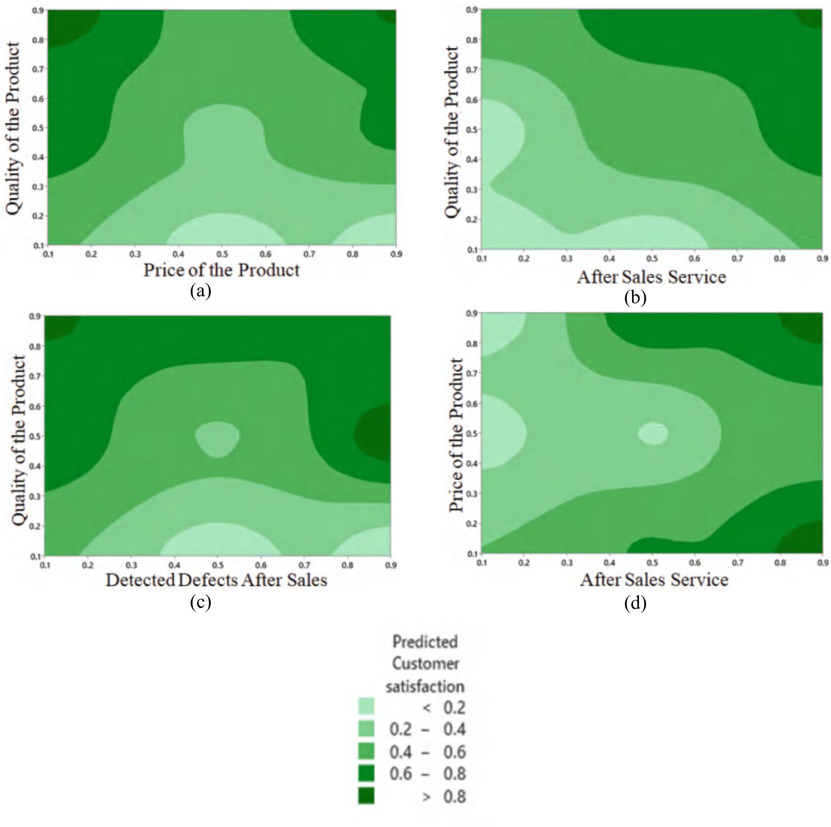

Fig. 4 Contour Plots of Predicted Customer Satisfaction versus **a** Quality of the Product, Price of the Product; **b** Quality of the Product, After Sales Service; **c** Quality of the Product, Detected Defects after Sales; **d** Price of the Product, After Sales Service

shows the nature ofdifference of level means, as the study involves 5 factors and 3 levels. It describes how customer satisfaction changes with the changes in one X variable (purchase terms, product quality etc.).

8 Optimization Using Genetic Algorithm

The objective function of Genetic Algorithm was written in MATLAB R2022b and the program was executed to get the values of population size, number of generation by taking maximum customer satisfaction as the criteria. The graphs of population size versus minimum average response and number of generation versus minimum average response was generated as an output of the executed program.

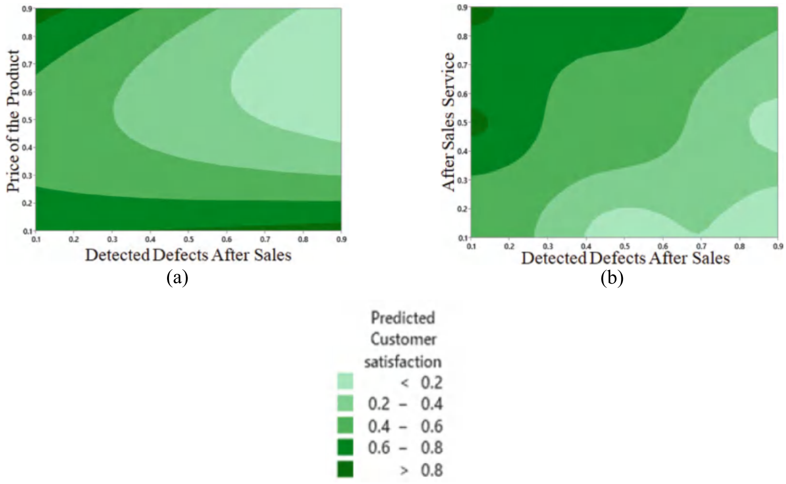

Fig. 5 Contour Plots of Predicted Customer Satisfaction versus **a** Price of the Product, Detected Defects after Sales; **b** After Sales Service, Detected Defects after Sales

Fig. 6 Customer Response surface plot of customer satisfaction (Z-axis) for a Flexibility in purchase terms versus Quality of service/goods, b Flexibility in purchase terms versus Price of service/goods

The best suitable values of population size, number of generation could be found as 65 and 50 respectively where the customer satisfaction is in its maximum values.

After deciding the above factors, the genetic algorithm was simulated. The optimized results are obtained with the following summary of the GA parameters:

- Number of generation = 50
- Population size = 65

The other variables like crossover function, migration fraction, migration interval etc. were kept constant at their default values as in the MATLAB R2022b tool box.

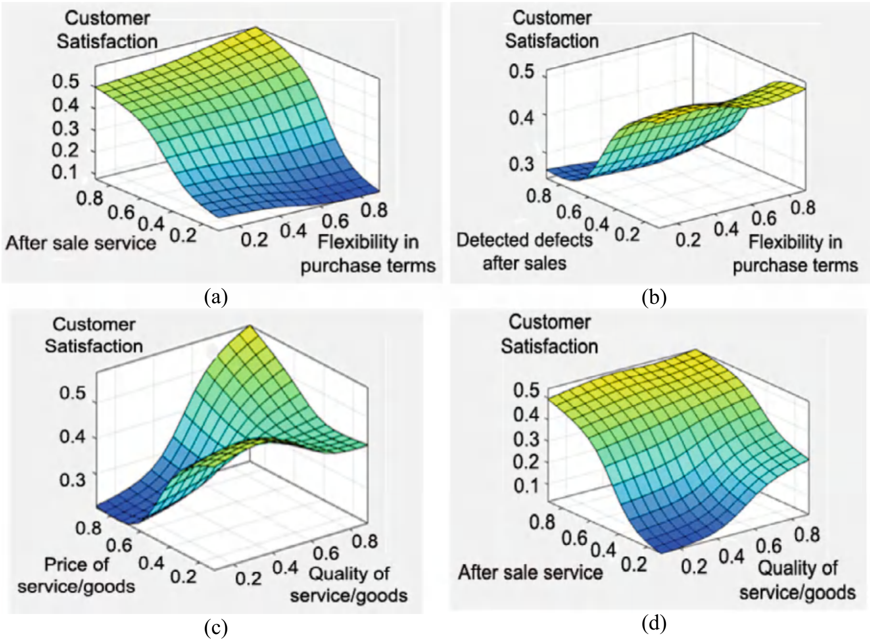

Fig. 7 Customer Response surface plot of customer satisfaction (Z-axis) for a Flexibility in purchase terms versus After sale service, b Flexibility in purchase terms versus Detected defects after sales, c Quality of service/goods versus Price of service/goods, and d Quality of service/goods versus After sale service

The genetic algorithm converges to the best suitable maximum customer satisfaction in the selected generation. The generation was selected as 50. Figure 10 shows how the genetic algorithm is fitted in generation with beat fitness and mean fitness. It can be observed that up to generation no. 13 the mean fitness and best fitness is different. But after the generation no. 13 the two fitness values converge almost and we can observe a steady convergence at generation no. 50.

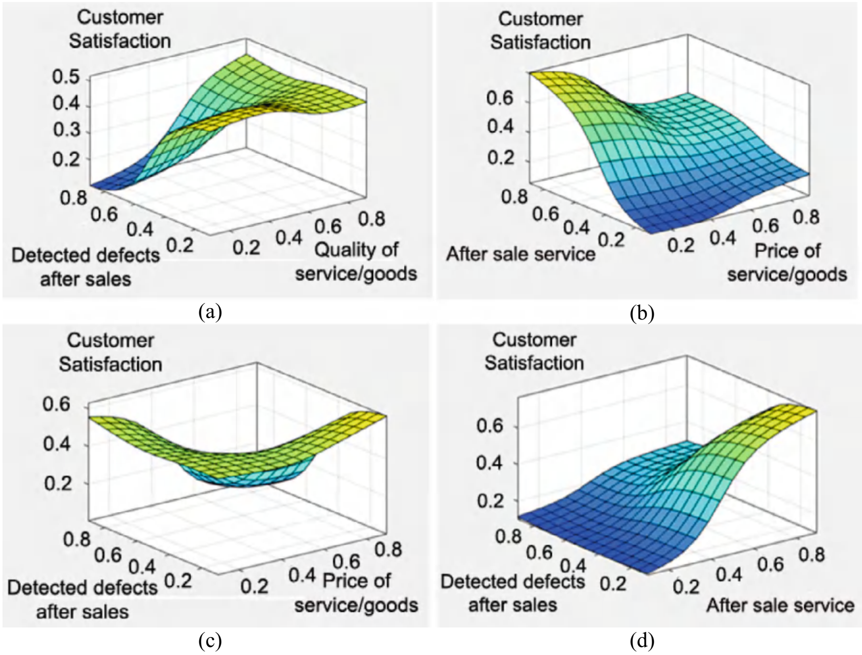

Fig. 8 Customer Response surface plot of customer satisfaction (Z-axis) for **a** Quality of service/goods versus Detected defects after sales, **b** Price of service/goods versus After sale service, **c** Price of service/goods versus Detected defects after sales, and **d** After sale service versus Detected defects after sales

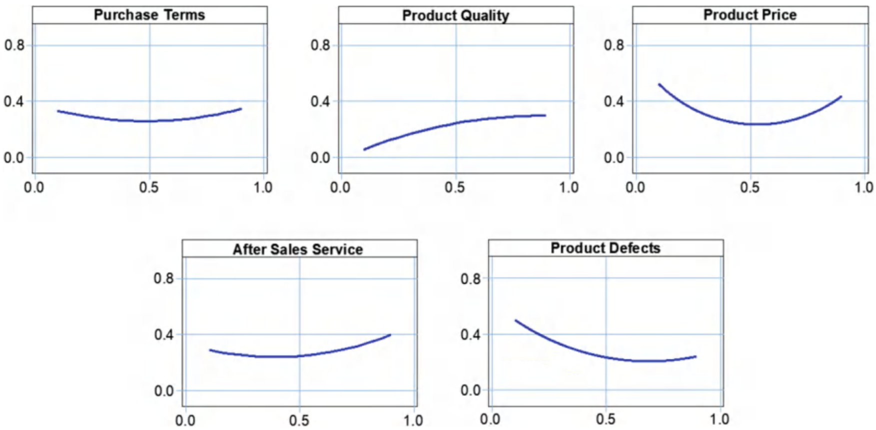

Fig. 9 Main effects plot for customer satisfaction

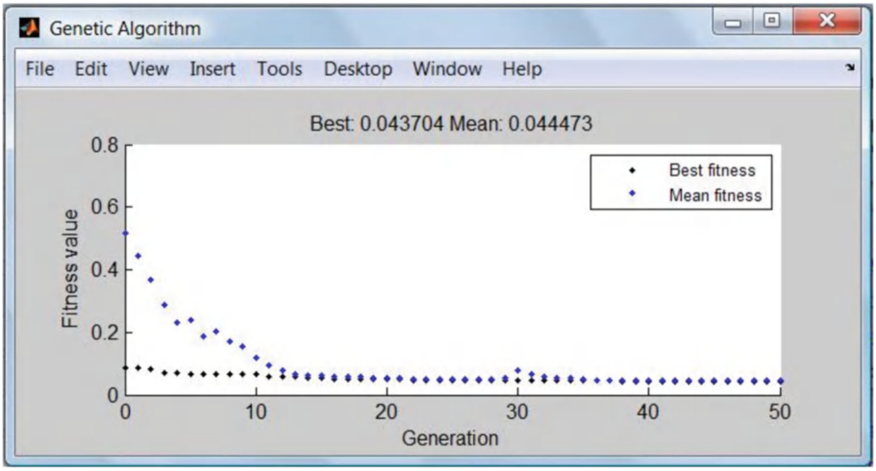

Fig. 10 Fitness values versus generation

9 Conclusion

The present work has established a data mapping, which is accomplished with the help of the combination of fuzzy logic and ANN. The main purpose of the survey done in this study was to capture human logic of rating different factors affecting the satisfaction of a buyer and quantifying the data in an objective way to understand how these factors affect the overall customer satisfaction. The ANFIS model is applied to generate predicted customer satisfaction data to utilize in the genetic algorithm for achieving optimum customer satisfaction with the unique combination of input variables.

Following are the salient observations that may be noted from the study.

- Genetic algorithm results indicate for the maximum level of customer satisfaction, the positively influencing factors (purchase terms, product quality, after sales service) should be maximized and the negatively influencing factors (product price, detected defects) should be minimized.
- Product quality and defects in the products shows notable variation in the mean effect of levels for the customer satisfaction. Other factors remain nominally at their average values.
- From the surface plots, it can be inferred that the customers are willing to pay a more premium price for the product if the quality is high and defects in the products are low.
- If the product price is low, customers are willing to compromise with more defects in their products. But, in this scenario the after sales service should be at a high level.

Limitations:

The work only considers selected input parameters to make the prediction system simplified. The work is limited for direct inputs made by customers, employees and managers. There are other indirect influential parameters like competition, demand in the market etc. They are to be considered into the system to make the prediction more practical. These limitations of the present works create the opportunities of the extension of the present work in future to make the system more real-time oriented.

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Data Availability The authors declare that all the data of the work is available in the work.

Conflict of Interest The authors declare there is no conflict of interest in the work.

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Application of Neutrosophic Pythagorean Supra Topological Spaces in Attribute Decision-Making

R. Narmada Devi and Yamini Parthiban

Abstract Inception of Neutrosophic Pythagorean Supra Topological Space ($\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}}$) concept was initiated. Neutrosophic sets and their associated logics are deemed as crucial components within the realm of approximation theory, denoting a growth of intuitionistic fuzzy sets and conventional fuzzy set. In the initial section, a comprehensive discussion was engaged in concerning the fundamentals of neutrosophic sets, neutrosophic Pythagorean sets, and conceptualization of the $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}$. These similar tactical techniques were then applied to the application of multiple attribute decision-making approaches, one of which involved a Neutrosophic Pythagorean set.

Keywords Neutrosophic topological space · Neutrosophic supra topological space · Neutrosophic pythagorean supra topological space

1 Introduction

The information used in real-world issues like engineering, social, economic, computing, decision making, and medical diagnosis is frequently ambiguous and inaccurate. Because of their fuzziness and ambiguity, this type of data is not always clear, precise, and deterministic. For this kind of data handling, professor Zadeh [16] introduced fuzzy set theory, enabling the representation of imprecise and vague data through partial membership. Zadeh [16] proposed the concept of linguistic variables to enhance approximate reasoning in fuzzy logicapplications. Atanassov [4] introduced intuitionistic fuzzy sets, an extension of fuzzy sets that incorporate a degree of membership, non-membership, and hesitation. Atanassov and Gargov [5]

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proposed interval-valued intuitionistic fuzzy sets, further generalizing intuitionistic fuzzy sets by allowing interval-based membership values. Smarandache [14] introduced neutrosophy, a branch of philosophy that studies truth, falsehood, and indeterminacy independently in decision-making. Ye [15] developed a decision-making method using correlation coefficients in a single-valued neutrosophic environment. Jayaparthasarathy et al. [11] applied neutrosophic supra topological structures in data mining to enhance pattern recognition and classification. Chai et al. [7] developed new similarity measures for single-valued neutrosophic sets and applied them to pattern recognition and medical diagnosis.

Abdel-Basset et al. [1] proposed a multi-criteria decision-making technique based on neutrosophic axiomatic design to improve complex decision processes. Al-Sharqi and Al-Quran [2] introduced similarity measures for interval-complex neutrosophic soft sets and applied them in decision-making and medical diagnosis. Bui et al. [6] explored sequences of neutrosophic soft sets and their application in decision-making for medical diagnosis problems. Rahman et al. [13] proposed a neutrosophic hypersoft set-based MADM approach for heart disease diagnosis using a possibility degree-based setting.

Alshikho et al. [3] applied artificial intelligence and neutrosophic machine learning techniques for diagnosing and detecting COVID-19. Devi and Parthiban [8] introduced decision-making over neutrosophic Pythagorean soft sets using a correlation-based approach. Kumaravel et al. [12] applied fuzzy and neutrosophic cognitive maps to analyze dengue fever patterns and risk factors. Devi and Parthiban [8] proposed decision-making using neutrosophic over soft topological space to handle uncertainty in complex systems. Devi and Parthiban [9] explored neutrosophic over supra exterior modal topological structures for healthcare decision-making applications. In the scope of this chapter, the primary focus revolved around the introduction and detailed analysis of the intriguing mathematical framework known as neutrosophic pythagorean supra topological space. This concept was presented with the intent of facilitating a better understanding of its theoretical foundations and essential principles. The chapter also delved into the intricate world of mappings within this specialized mathematical domain. An exhaustive review of the relationships and associations that mappings establish within the context of neutrosophic pythagorean supra topological space was provided.

To conclude the discourse, a practical numerical illustration was incorporated, offering a tangible example of how this mathematical theory can be applied in a real-world context. Specifically, it was used to classify different types of diseases, illustrating its potential significance in healthcare and disease analysis.

2 Preliminary

This section includes an overview of neutrosophic set$(\mathcal{N}_{\mathfrak{s}})$, neutrosophic pythagorean set$\left(\mathcal{N}_{\mathfrak{P}} - set\right)$.

Definition 2.1 If β be a universal set then $\mathfrak{A}$ is said to be a $\mathcal{N}_{\mathfrak{s}}$ on β is

$$\mathfrak{A} = \{\langle x, \Upsilon_{\mathfrak{A}}(x), \omega_{\mathfrak{A}}(x), \vartheta_{\mathfrak{A}}(x)\rangle : x \in \beta\}$$

where $0 \leq \Upsilon_{\mathfrak{A}}(x) + \omega_{\mathfrak{A}}(x) + \vartheta_{\mathfrak{A}}(x) \leq 3$, forall $x \in \beta$, $\Upsilon_{\mathfrak{A}}(x), \omega_{\mathfrak{A}}(x), \vartheta_A(x) \in [0, 1]$. Here $\Upsilon_{\mathfrak{A}}(x)$ be a degree of membership, $\omega_{\mathfrak{A}}$(x) be a degree of indeterminacy and $\vartheta_{\mathfrak{A}}$(x) be a degree of non-membership of each x $\in \beta$ to the set $\mathfrak{A}$ respectively.

Definition 2.2 If β be a universal set then $\mathfrak{A}$ is said to be a $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ on β with dependent components $\Upsilon_{\mathfrak{A}}$ and $\vartheta_{\mathfrak{A}}$ and independent component $\omega_{\mathfrak{A}}$ such that,

$$\mathfrak{A} = \{\langle x, \Upsilon_{\mathfrak{A}}(x), \omega_{\mathfrak{A}}(x), \vartheta_{\mathfrak{A}}(x)\rangle : x \in \beta\}$$

where $0 \leq (\Upsilon_{\mathfrak{A}}(x))^2 + (\omega_{\mathfrak{A}}(x))^2 + (\vartheta_{\mathfrak{A}}(x))^2 \leq 2$, forall $x \in \beta$, $\Upsilon_{\mathfrak{A}}(x), \omega_{\mathfrak{A}}(x)$ and ϑ_A(x) $\in [0, 1]$.

Note Let $N(\beta)$ is called power set of β.

Definition 2.3 Let $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}$ be a $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ on β then

(i) $\Upsilon_{\mathfrak{A}}(x) \leq \Upsilon_{\mathfrak{B}}(x), \omega_{\mathfrak{A}}(x) \leq \omega_{\mathfrak{B}}(x), \vartheta_{\mathfrak{A}}(x) \geq \vartheta_{\mathfrak{B}}(x)$ for all x $\in \beta$ iff $\mathfrak{A} \subseteq \mathfrak{B}$.
(ii) $\mathfrak{A}\mho\mathfrak{B} = \{x, \max\{\Upsilon_{\mathfrak{A}}(x), \Upsilon_{\mathfrak{B}}(x)\}, \max\{\omega_{\mathfrak{A}}(x), \omega_{\mathfrak{B}}(x)\}, \min\{\vartheta_{\mathfrak{A}}(x), \vartheta_{\mathfrak{B}}(x)\} : x \in \beta\}$.
(iii) $\mathfrak{A}\Omega\mathfrak{B} = \{x, min\{\Upsilon_{\mathfrak{A}}(x), \Upsilon_{\mathfrak{B}}(x)\}, min\{\omega_{\mathfrak{A}}(x), \omega_{\mathfrak{B}}(x)\}, max\{\vartheta_{\mathfrak{A}}(x), \vartheta_{\mathfrak{B}}(x)\} : x \in \beta\}$
(iv) $\mathfrak{A}^{\mathtt{C}} = \{x, \vartheta_{\mathfrak{A}}(x), 1 - \omega_{\mathfrak{A}}(x), \Upsilon_{\mathfrak{A}}(x) : x \in \beta\}$

Definition 2.4 Let $\otimes$ and $\odot$ are called universe $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ and empty $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ on β are defined as $\otimes = \{\langle x, 1, 1, 0\rangle$: x $\in \beta\}$ and $\odot = \{\langle x, 0, 0, 1\rangle$: x $\in \beta\}$.

Remark

(i) $\otimes^{\mathtt{C}} = \odot$.
(ii) $\odot^{\mathtt{C}} = \otimes$.

Definition 2.5 Let $\mathfrak{A}$ be $\mathcal{N}_{\mathfrak{P}}$ score function. If a $\mathcal{N}_{\mathfrak{P}}$ score function is denoted by $\mathfrak{A}_{SVNPSF}$ and defined by

$$\mathfrak{A}_{SVNPSF} = \frac{1}{3m}\left[\sum_{i=1}^{m}[2 + \Upsilon_i - \omega_i - \vartheta_i]\right]$$

Definition 2.6 A $\mathcal{N}_{\mathfrak{P}}$ Topology (NPT) τ is an assortment of subgroups of β then it satisfies the following condition,

(i) $\otimes, \odot \in \tau$.
(ii) Union of the sets of any sub-collection of τ is in τ.
(iii) Intersection of the elements of any finite sub-collection τ is in τ.

Then (β, τ) pair is known as $\mathcal{N}_{\mathfrak{P}}$ topological space $\left(\mathcal{N}_{\mathfrak{P}\mathfrak{T}}\right)$. Where a collection of τ is called a $\mathcal{N}_{\mathfrak{P}}$ open set. Compliment of τ is known as $\mathcal{N}_{\mathfrak{P}}$ closed set.

Definition 2.7 Let two $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}$as $\mathfrak{A} = \{\langle x, \Upsilon_{\mathfrak{A}}(x), \omega_{\mathfrak{A}}(x), \vartheta_{\mathfrak{A}}(x)\rangle : x \in \beta\}$ and $\mathfrak{B} = \{y, \Upsilon_{\mathfrak{B}}(y), \omega_{\mathfrak{B}}(y), \vartheta_{\mathfrak{B}}(y) : y \in \mathfrak{Y}\}$ and $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ be a function,

(i) $\mathfrak{f}^{\rightarrow}(A) = \{\langle \mathrm{y}, \mathfrak{f}^{\rightarrow}(\Upsilon_{\mathfrak{A}})(y), \mathfrak{f}^{\rightarrow}(\omega_{\mathfrak{A}})(y), (1 - \mathfrak{f}^{\rightarrow}(1 - \vartheta_{\mathfrak{A}}))(y)\rangle : y \in \mathfrak{Y}\}$ is a $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}$ on $\mathfrak{Y}$ called the image of $\mathfrak{A}$ under $\mathfrak{f}^{\rightarrow}$
(ii) $\mathfrak{f}^{\leftarrow}(B) = \{x, \mathfrak{f}^{\leftarrow}(\Upsilon_{\mathfrak{B}})(x), \mathfrak{f}^{\leftarrow}(\omega_{\mathfrak{B}})(x), \mathfrak{f}^{\leftarrow}(\vartheta_{\mathfrak{B}})(x) : x \in \beta\}$ is a $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}$ on $\mathfrak{Y}$ is called the pre-image of $\mathfrak{A}$ under. $\mathfrak{f}^{\rightarrow}$

where,

$$\mathfrak{f}^{\rightarrow}(\Upsilon_{\mathfrak{A}})(y) = \begin{cases} \sup\limits_{x\in\mathfrak{f}^{\leftarrow}}(y)\Upsilon_{\mathfrak{A}}(x), & \textit{if } \mathfrak{f}^{\leftarrow}(y) \neq \emptyset \\ 0, & \textit{otherwise} \end{cases}$$

$$\mathfrak{f}^{\rightarrow}(\omega_{\mathfrak{A}})(y) = \begin{cases} \sup\limits_{x\in\mathfrak{f}^{\leftarrow}}(y)\omega_{\mathfrak{A}}(x), & \textit{if } \mathfrak{f}^{\leftarrow}(y) \neq \emptyset \\ 0, & \textit{otherwise} \end{cases}$$

$$(1 - \mathfrak{f}^{\rightarrow}(1 - \vartheta_{\mathfrak{A}}))(y) = \begin{cases} \inf_{x\in\mathfrak{f}^{\leftarrow}}(y)\vartheta_{\mathfrak{A}}(x), & \textit{if and only if } \mathfrak{f}^{\leftarrow}(y) \neq \emptyset \\ 1, & \textit{otherwise} \end{cases}$$

Let us introduce the symbol $\mathfrak{f}_{-}^{\rightarrow}(\vartheta_{\mathfrak{A}})$ for $(1 - \mathfrak{f}^{\rightarrow}(1 - \vartheta_{\mathfrak{A}}))$

Note (i) $\otimes^{\mathbb{C}} = \otimes\backslash\otimes = \{\langle x, 0, 0, 1\rangle : x \in \beta\} = \odot$.

(ii) $\odot^{®} = \otimes\backslash\odot = \{\langle x, 1, 1, 0\rangle : x \in \beta\} = \otimes$

Corollary 2.1 Let $\{\mathfrak{A}\}_{i\in\alpha}^{\infty}$, $\mathfrak{A}$ and $\mathfrak{B}$ be $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}$ on β, then the following are true

(i) $(\Omega_{i\in\alpha}\mathfrak{A}_i)^{\mathtt{C}} = \mho_{i\in\alpha}\mathfrak{A}_i^{\mathtt{C}}$, $(\mho_{i\in\alpha}\mathfrak{A}_i)^{\mathtt{C}} = \Omega_{i\in\alpha}\mathfrak{A}_i^{\mathtt{C}}$
(ii) $\left(\mathfrak{A}^{\mathtt{C}}\right)^{\mathtt{C}} = \mathfrak{A} \cdot \mathfrak{B}^{\mathtt{C}} \subseteq \mathfrak{A}^{\mathtt{C}}$, if $\mathfrak{B} \subseteq \mathfrak{A}$.

Proof (i)

$$\begin{aligned}(\Omega_{i\in\alpha}\mathfrak{H}_i)^C &= \{\langle x|1-, \inf_{i\in\alpha}\left\{Y_{\mathfrak{Y}}(x)\right\}\big|, \big|1 - \inf f_{i\in\alpha}\left\{\omega_{\mathfrak{Y}}(x)\right\}\big|, \big|1 - \sup_{i\in\alpha}\{\vartheta_2(x)|\}\rangle : \mathrm{x} \in \beta\} \\ &= \{x, sup_{i\in\alpha}(|1 - \Upsilon_{\mathfrak{A}i}(x)|), sup_{i\in\alpha}(|1 - \omega_{\mathfrak{A}i}(x)|)..,\ i \in \alpha(|1 - \vartheta_{\mathfrak{A}i}(x)|) : x \in \beta\} \\ &= \mho_{i\in\alpha}\mathfrak{A}_i^{\mathtt{C}}\end{aligned}$$

Similarly, $(\mho_{i\in\alpha}\mathfrak{A}_i)^{\mathtt{C}} = \Omega_{i\in\alpha}\mathfrak{A}_i^{\mathtt{C}}$ is obvious
(ii) we can prove it from (i)

Definition 2.8 Let $\mathfrak{A}$ be a $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ of $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}(\beta, \tau_{\mathfrak{N}\mathfrak{P}})$, then the collection of $\tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}} = \{\mathfrak{A}\Omega\mathfrak{O} : \mathfrak{O} \in \tau_{\mathfrak{N}\mathfrak{P}}\}$ is a $\mathcal{N}_{\mathfrak{P}}$ topology on $\mathfrak{A}$. Where $\tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}}$ is knowns as induced neutrosophi pythagorean topology and $(\mathfrak{A}, \tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}})$ is a $\mathcal{N}_{\mathfrak{P}}$ topological subspace of $\mathcal{N}_{\mathfrak{P}\mathfrak{T}}(\beta, \tau_{\mathfrak{N}\mathfrak{P}})$. The elements of $\tau_{\mathfrak{P}\mathfrak{P}\mathfrak{A}}$ are call the $\mathcal{N}_{\mathfrak{P}}$ open sets and their compliment $\tau^{\mathbb{G}}_{\mathfrak{N}\mathfrak{P}\mathfrak{A}}$ is known as $\mathcal{N}_{\mathfrak{P}}$ closed sets.

Lemma 2.1 Let $(\mathfrak{A}, \tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}})$be a $\mathcal{N}_{\mathfrak{P}}$ subspace of $\mathcal{N}_{\mathfrak{P}\mathfrak{T}}(\beta, \tau_{\mathfrak{N}\mathfrak{P}})$ and $\mathfrak{B} \subseteq \mathfrak{A}$. If $\mathfrak{B}$ is an open in $(\mathfrak{A}, \tau_{\mathfrak{H}\mathfrak{P}\mathfrak{A}})$ and $\mathfrak{A}$ is a $\mathcal{N}_{\mathfrak{P}}$ open set in $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}(\beta, \tau_{\mathfrak{P}\mathfrak{P}})$ then $\mathfrak{B}$ is a $\mathcal{N}_{\mathfrak{P}}$ open in $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}(\beta, \tau_{\mathfrak{P}\mathfrak{P}})$.

Proof Let us consider some $\mathcal{N}_{\mathfrak{P}}$ open set $\mathfrak{O}$ in $(\beta, \tau_{\mathfrak{N}\mathfrak{P}})$ then

$$\mathfrak{B} = \mathfrak{A}\Omega\mathfrak{O}\left[\because \mathfrak{B} \textit{ is open in } (\mathfrak{A}, \tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}})\right]$$

$\Rightarrow$ $\mathfrak{B}$ is a $\mathcal{N}_{\mathfrak{P}}$ open set $(\beta, \tau_{\mathfrak{N}\mathfrak{P}})$.

Remark 2.1 In general, classical topology w.k.t if $(\mathfrak{A}, \tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}})$ is a subspace of $(\beta, \tau_{\mathfrak{N}\mathfrak{P}})$ and $\mathfrak{B} \subseteq \mathfrak{A}$ then

(i) $\mathfrak{B} = \mathfrak{A}\Omega\mathfrak{F}$, where $\mathfrak{F}$ is closed in β if and only if $\mathfrak{B}$ is closed in $\mathfrak{A}$.

(i) $\mathfrak{B}$ is closed in β, if $\mathfrak{B}$ is closed in $\mathfrak{A}$ and $\mathfrak{A}$ is closed in β.

the above are not true for $\mathcal{N}_{\mathfrak{P}}$ topological space.

3 Neutrosophic Pythagorean Supra Topological Space

Definition 3.1 A $\mathcal{N}_{\mathfrak{P}}$ S-Topology $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$ over an $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ $\mathfrak{A}$ is an collection of subsets of β such that

(i) $\phi, \beta \in \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$
(ii) Union of the sets of any sub-collection $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$ is in $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$.

Then $(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}})$ is called a $\mathcal{N}_{\mathfrak{P}}$ S-Topological Space $(\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}})$. An eof $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$ is called a $\mathcal{N}_{\mathfrak{P}}$ S-open set $(\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{D}\mathfrak{S}})$. Its compliment $\tau^{\varsigma\mathbb{S}}_{\mathfrak{N}\mathfrak{P}}$ is called as $\mathcal{N}_{\mathfrak{P}}$ S-closed *set* $(\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{C}\mathfrak{S}})$.

All the $\mathcal{N}_{\mathfrak{P}\mathfrak{T}}$ is a $\mathcal{N}_{\mathfrak{P}}$ S-Topological Space but converse not true.

Definition 3.2 A $\mathcal{N}_{\mathfrak{P}}$ S-topology $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$ is on β is known as an Associate $\mathcal{N}_{\mathfrak{P}}$ S-Topology (ANPST) with $\mathcal{N}_{\mathfrak{P}}$ topology $\tau_{\mathfrak{N}\mathfrak{P}}$ if $\tau_{\mathfrak{N}\mathfrak{P}} \subseteq \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}$.

Definition 3.3 An operator of $\mathcal{N}_{\mathfrak{P}_\mathfrak{s}}$ $\mathfrak{A}$, the $\mathcal{N}_{\mathfrak{P}}$ S-topological interior $\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}\mathfrak{T}}$ and closure $\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}\mathfrak{C}}$ are $\text{int}_{\tau^{\varsigma}_{\mathfrak{P}\mathfrak{H}}}(\mathfrak{A})$ and $c\tau_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}(\mathfrak{A})$ is defined as: int int

$\text{int}_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}(\mathfrak{A}) = \mho\{\mathfrak{G} : \mathfrak{G} \subseteq \mathfrak{A}.\text{ and }\cdot\mathfrak{G} \in \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}\Big\}$ and $c\tau_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}(\mathfrak{A}) = \Omega\{\mathfrak{F} : \mathfrak{A} \subseteq \mathfrak{F}$ and $\mathfrak{F}^{\mathsf{C}} \in \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}\Big\}$.

Corollary 3.4 Assume that $\mathfrak{A}_u$ and $\mathfrak{B}_v (\forall u, v = 1, 2, 3, \ldots)$ are $\mathcal{N}_{\mathfrak{P}_\mathfrak{s}}$ in $\left(\beta, \tau_{\mathfrak{P}\mathfrak{P}_1}\right)$ and $\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)$ and the function $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$.

(i) $\mathfrak{A} \subseteq \mathfrak{f}^{\leftarrow}(\mathfrak{f}^{\rightarrow}(\mathfrak{A}))$
(ii) $\mathfrak{f}^{\rightarrow}(\mathfrak{f}^{\leftarrow}(\mathfrak{B})) \subseteq \mathfrak{B}$
(iii) $(\mathfrak{f}^{\leftarrow}(\mho\mathfrak{B}_v)) = \mho\mathfrak{f}^{\leftarrow}(\mathfrak{B}_v)$
(iv) $\mathfrak{f}^{\leftarrow}\Omega\mathfrak{B}_v = \Omega\mathfrak{f}^{\leftarrow}(\mathfrak{B}_v)$.
(v) $\mathfrak{f}^{\leftarrow}(\otimes) = \otimes$
(vi) $\mathfrak{f}^{\leftarrow}(\odot) = \odot$
(vii) $\mathfrak{f}^{\leftarrow}\left(\overline{\mathfrak{B}}\right) = \overline{\mathfrak{f}^{\leftarrow}(\mathfrak{B})}$

Proposition 3.1 Let $\mathfrak{A}$ be any $\mathcal{N}_{\mathfrak{P}_\mathfrak{s}}$ in $\left(\beta, \tau_{\mathfrak{N}\mathfrak{P}_1}\right)$, we have

(i) $cl_{\tau_{\mathfrak{P}\mathfrak{P}_1}}\left(\overline{\mathfrak{A}}\right) = \overline{int_{\tau_{\mathfrak{N}\mathfrak{P}_1}}(\mathfrak{A})}$
(ii) $\text{int}_{\tau_{\mathfrak{N}\mathfrak{P}_1}}\left(\overline{\mathfrak{A}}\right) = \overline{cl_{\tau_{\mathfrak{P}\mathfrak{P}_1}}(\mathfrak{A})}$.

Theorem 3.1 Let $\mathfrak{A}$ and $\mathfrak{B}$ are two $\mathcal{N}_{\mathfrak{P}_\mathfrak{s}}$ of $\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}\mathfrak{S}}\left(\beta, \tau^{\varsigma}_{\mathfrak{P}\mathfrak{P}}\right)$ then,

(ii) $\mathfrak{A} = \text{int}_{\tau^{\varsigma}_{\mathfrak{T}\mathfrak{P}}}(\mathfrak{A})$ if and only if $\mathfrak{A}$ is a $\left(\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{D}\mathfrak{S}}\right)$.
(iii) $cl_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{H}}}(\mathfrak{A}) \subseteq cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{H}}}(\mathfrak{B})$ if $\mathfrak{A} \subseteq \mathfrak{B}$.
(iv) $\text{int}_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}(\mathfrak{A}) \subseteq \text{int}_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}(\mathfrak{B})$ if $\mathfrak{A} \subseteq \mathfrak{B}$
(v) $cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}(\mathfrak{A})\mho cl_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{H}}}(\mathfrak{B}) \subseteq cl_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{H}}}(\mathfrak{A}\vartheta\mathfrak{B})$.
(vi) $\text{int}_{\tau^{c}_{\mathfrak{P}\mathfrak{S}\mathfrak{j}}}(\mathfrak{A})\text{int}_{\tau^{\varsigma}_{\mathfrak{P}\mathfrak{M}}}(\mathfrak{B}) \subseteq \text{int}_{\tau^{c}_{\mathfrak{W}\mathfrak{S}}}(\mathfrak{H}v\mathfrak{B})$.
(vii) $cl_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{A})\Omega cl_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{B}) \subseteq cl_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{A}\Omega\mathfrak{B})$
(viii) $\text{int}_{\tau^{\varsigma}_{\mathfrak{W}\mathfrak{P}}}(\mathfrak{A})\Omega\text{int}_{\tau^{\varsigma}_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{B}) \subseteq \text{int}_{\tau^{\varsigma}_{\mathfrak{P}\mathfrak{P}}}(\mathfrak{A}\Omega\mathfrak{B})$.
(ix) $\text{int}_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}\left(\mathfrak{A}^{\mathsf{C}}\right) = \left(cl_{\tau^{\varsigma}_{\mathfrak{T}\mathfrak{P}}}(\mathfrak{A})\right)^{\mathsf{C}}$.

Proof (i) and (ii) are obvious by the definition of $\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}}$ and $\mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{T}}$.

(iii) $cl^{\varsigma}_{\tau_{\mathfrak{Z}\mathfrak{P}}}(\mathfrak{A}) = \Omega\left\{\mathfrak{G} : \mathfrak{G}^{\mathsf{C}} \in \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}, \mathfrak{A} \subseteq \mathfrak{G}\right\}$

If $\mathfrak{A} \subseteq \mathfrak{B}$, then

$$cl^{\varsigma}_{\tau_{\mathfrak{Z}\mathfrak{P}}}(\mathfrak{A}) \subseteq \Omega\left\{\mathfrak{G} : \mathfrak{G}^{\mathsf{C}} \in \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}, \mathfrak{B} \subseteq \mathfrak{G}\right\} = cl^{\varsigma}_{\tau_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{B})$$

(iv) Similar to (iii)

(v) Already know if $\mathfrak{A}$ and $\mathfrak{B}$ are two $\mathcal{N}_{\mathfrak{P}_{\mathfrak{s}}}$ then

$$\mathfrak{A}, \mathfrak{B} \subseteq \mathfrak{A}\mathfrak{G}\mathfrak{B}$$

$$\therefore cl^{\varsigma}_{\tau_{\mathfrak{H}\mathfrak{P}}}(\mathfrak{A})\mathcal{S}\Big)l^{\varsigma}_{\tau_{\mathfrak{H}\mathfrak{P}}}(\mathfrak{B}) \subseteq cl^{\varsigma}_{\tau_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{A}\vartheta S\mathfrak{B})$$

By the above result, (vi), (vii) and (viii) are true.
(ix) By the definition, we can write

$$\left(cl^{\mathfrak{S}}_{\tau_{\mathfrak{M}\mathfrak{P}}}(\mathfrak{A})\right)_C = \mathfrak{S}\{\mathfrak{G}^{\mathtt{C}} : \mathfrak{G}^{c'} \in \beta, \mathfrak{G}^{\mathtt{C}} \subseteq \mathfrak{A}^{\mathtt{C}}) \, where, \mathfrak{G}^{c'} \, is\, a\, \mathcal{N}_{\mathfrak{P}\mathfrak{S}\mathfrak{O}} \, in\, \beta = \mathrm{int}\left(\mathfrak{A}^{\mathtt{C}}\right)$$

Thus, $\mathrm{int}_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}}\left(\mathfrak{A}^{\mathtt{C}}\right) = \left(cl_{\tau^{\varsigma}_{\mathfrak{T}\mathfrak{P}}}(\mathfrak{A})\right)^{\mathtt{C}}$.

4 Mapping of Neutrosophic Pythagorean Supra Topological Space

In this part, we established properties of mapping in $\mathcal{N}_{\mathfrak{P}}$ S-topological spaces and $\mathcal{N}_{\mathfrak{P}}$ subspaces.

Definition 4.12 Let us consider two ANPST $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}$ and $\tau^{\varsigma}_{\mathfrak{Y}\mathfrak{P}_2}$ with respect to $\tau_{\mathfrak{N}\mathfrak{P}_1}$ and $\tau_{\mathfrak{N}\mathfrak{P}_2}$. If $\mathfrak{f}^{\rightarrow}$ is a mapping from $\left(\beta, \tau_{\mathfrak{P}\mathfrak{P}_1}\right)$ into $\left(\mathfrak{Y}, \tau_{\mathfrak{H}\mathfrak{P}_2}\right)$ is $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ open if the image of each and every $\mathcal{N}_{\mathfrak{P}}$ open set $\left(\beta, \tau_{\mathfrak{N}\mathfrak{P}_1}\right)$ is $\mathcal{N}_{\mathfrak{P}}$ S-open set $\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right)$ and $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ be $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ continuous then its preimage of every $\mathcal{N}_{\mathfrak{P}}$ open set in $\left(\mathfrak{Y}, \tau_{\mathfrak{P}\mathfrak{P}_2}\right)$ is $\mathcal{N}_{\mathfrak{P}}$ S-open set$\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right)$.

Definition 4.13 Let us consider two ANPST $\tau^{\varsigma}_{\mathfrak{P}\mathfrak{P}_1}$ and $\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}$ with respect to $\mathcal{N}_{\mathfrak{P}\mathfrak{T}S}\tau_{\mathfrak{N}\mathfrak{P}_1}$ and $\tau_{\mathfrak{N}\mathfrak{P}_2}$. If $\mathfrak{f}^{\rightarrow}$ is a mapping from $\mathcal{N}_{\mathfrak{P}\mathfrak{T}}\left(\beta, \tau_{\mathfrak{P}\mathfrak{P}_1}\right)$ into a $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}\left(\mathfrak{Y}, \tau_{\mathfrak{P}\mathfrak{P}_2}\right)$ is said to be $\mathcal{N}_{\mathfrak{P}}$ S-open if the image of every $\mathcal{N}_{\mathfrak{P}}$ S-open set $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}\right)$ is $\mathcal{N}_{\mathfrak{P}}$ S-open set$\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right)$ and $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ is said to be $\mathcal{N}_{\mathfrak{P}}$ S- $\mathcal{N}_{\mathfrak{P}}$ continuous then its preimage of every $\mathcal{N}_{\mathfrak{P}}$ S-open set in $\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right)$ is $\mathcal{N}_{\mathfrak{P}}$ S-open set $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}\right)$.

Definition 4.14 Let $\mathfrak{f}^{\rightarrow}$ is a mapping of $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}\left(\beta, \tau_{\mathfrak{N}\mathfrak{P}}\right)$ into a $\mathcal{N}_{\mathfrak{P}\mathfrak{T}\mathfrak{S}}\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}}\right)$ is said to be a mapping of a $\mathcal{N}_{\mathfrak{P}}$ subspace $\left(\mathfrak{A}, \tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}}\right)$ into a subspace $\left(\mathfrak{B}, \tau_{\mathfrak{N}_2\mathfrak{B}}\right)$ if $\mathfrak{f}^{\rightarrow}(\mathfrak{A}) \subset \mathfrak{B}$.

Definition 4.15 Let $\mathfrak{f}^{\rightarrow}$ be a mapping of a $\mathcal{N}_{\mathfrak{P}}$ subspace $\left(\mathfrak{A}, \tau_{\mathfrak{N}_1\mathfrak{A}}\right)$ of $\mathcal{N}_{\mathfrak{P}}$ topological space $\left(\beta, \tau_{\mathfrak{N}\mathfrak{P}_1}\right)$ into a $\mathcal{N}_{\mathfrak{P}}$ subspace $\left(\mathfrak{B}, \tau_{\mathfrak{N}\mathfrak{P}_2\mathfrak{B}}\right)$ of $\mathcal{N}_{\mathfrak{P}}$ topological space $\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)$ is said to a Relative $\mathcal{N}_{\mathfrak{P}}$ Continuous (RNPC).

$$\mathfrak{f}^{\leftarrow} - 1(\mathfrak{O})\Omega\mathfrak{A} \in \tau_{\mathfrak{N}\mathfrak{P}1\mathfrak{A}} \forall \mathfrak{O} \in \tau_{\mathfrak{N}\mathfrak{P}2\mathfrak{B}}$$

If $\mathfrak{f}^{\rightarrow}\left(\mathfrak{O}'\right) \in \tau_{\mathfrak{N}}\mathfrak{P}_1\mathfrak{B}$ for every $\mathfrak{O}' \in \tau_{\mathfrak{N}\mathfrak{P}_1\mathfrak{A}}$, then $\mathfrak{f}^{\rightarrow}$ is known as Relative $\mathcal{N}_{\mathfrak{P}}$ Open (RNPO).

Theorem 4.1 Let $\mathfrak{f}^{\rightarrow}$ is a $\mathcal{N}_{\mathfrak{P}}$ continuous from $\left(\beta, \tau_{\mathfrak{N}\mathfrak{P}_1}\right)$ into $\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)$ and $\mathfrak{f}^{\rightarrow}(\mathfrak{A}) \subset \mathfrak{B}$. Then, RNPO $\mathfrak{f}^{\rightarrow}$ from $\mathcal{N}_{\mathfrak{P}}$ subspace $\left(\mathfrak{A}, \tau_{\mathfrak{N}\mathfrak{P}\mathfrak{A}}\right)$ of neutrosophic pythagoreran topological space $\left(\beta, \tau_{\mathfrak{N}\mathfrak{P}_1}\right)$ into a $\mathcal{N}_{\mathfrak{P}}$ subspace $\left(\mathfrak{B}, \tau_{\mathfrak{N}\mathfrak{P}_2\mathfrak{B}}\right)$ of neutrosophic pythagoreran topological space $\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)$.

Proof

$$Let \mathfrak{O} \in \tau_{\mathfrak{N})_2\mathfrak{B}}, \exists \mathfrak{G} \in \tau_{\mathfrak{N}\mathfrak{P}}\ such\ that\ \mathfrak{O} = \mathfrak{B}\Omega_{\mathfrak{G}}\ and\ \mathfrak{f}^{\leftarrow}(\mathfrak{G}) \in \tau_{\mathfrak{N}\mathfrak{P}_1}$$
$$\therefore \mathfrak{f}^{\leftarrow}(\mathfrak{O})\Omega\mathfrak{A} = \mathfrak{f}^{\leftarrow}(\mathfrak{B})\Omega\mathfrak{f}^{\leftarrow}(\mathfrak{G})\Omega\mathfrak{A} = \mathfrak{f}^{\leftarrow}(\mathfrak{G})\Omega\mathfrak{A} \in \tau_{\mathfrak{N}\mathfrak{P}_1\mathfrak{A}}$$

Remark 4.3 (i) For every mapping of $\mathcal{N}_{\mathfrak{P}}$ continuous is $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ continuous.

(ii) For every mapping of $\mathcal{N}_{\mathfrak{P}}$ S-continuous is $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ continuous.
(iii) Mappings of all $\mathcal{N}_{\mathfrak{P}}$ continuous and the $\mathcal{N}_{\mathfrak{P}}$ S-continuous are independent to one other.
(iv) Mappings of all the $\mathcal{N}_{\mathfrak{P}}$ S-open and $\mathcal{N}_{\mathfrak{P}}$ open are independent.

Note Convers part of the above remark is not true.

Theorem 4.2 Let $\mathfrak{f}^{\rightarrow}$ is a mapping from $\left(\beta, \tau_{\mathfrak{H}\mathfrak{P}_1}\right)$ into $\left(\mathfrak{Y}, \tau_{\mathfrak{H}\mathfrak{P}_2}\right)$ then the subsequent claims are equivalent,

(i) $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.
(ii) Preimage of every $\mathcal{N}_{\mathfrak{P}}$ closed set in $\left(\mathfrak{Y}, \tau_{\mathfrak{P}\mathfrak{P}_2}\right)$ is a $\mathcal{N}_{\mathfrak{P}}$ S-closed in.
(iii) For all $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}\mathfrak{A}$ in $\mathfrak{Y}$, $cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{H}_1}}(\mathfrak{f}^{\leftarrow}(\mathfrak{A})) \subseteq \mathfrak{f}^{\leftarrow}\left(cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{H}\mathfrak{P}}}(\mathfrak{A})\right)$.
(iv) For all $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}\mathfrak{B}$ in β, $\mathfrak{f}^{\rightarrow}\left(cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{H}\mathfrak{P}}}(\mathfrak{B})\right) \subseteq cl_{\tau_{\mathfrak{N}\mathfrak{F}_2}}(\mathfrak{f}^{\rightarrow}(\mathfrak{B}))$.
(v) For all $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}\mathfrak{A}$ in $\mathfrak{Y}$, $\mathrm{int}\tau_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}}(\mathfrak{f}^{\leftarrow}(\mathfrak{A})) \supseteq \mathfrak{f}^{\leftarrow}\left(\mathrm{int}_{\tau_{\mathfrak{M}\mathfrak{P}_2}}(\mathfrak{A})\right)$.

Proof (i) $\Rightarrow$ (ii): Let $\mathfrak{f}^{\rightarrow}$ is a mapping and $\mathfrak{A}$ be a $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous and $\mathcal{N}_{\mathfrak{P}}$ closed set in $\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)$,

$$\mathfrak{f}^{\leftarrow}\left(\otimes_{\mathfrak{Y}} - \mathfrak{A}\right) = \otimes_{\beta} - \mathfrak{f}^{\leftarrow}(\mathfrak{A}), \forall \mathfrak{f}^{\leftarrow}(\mathfrak{A}) \in \left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}\right).$$

(ii) $\Rightarrow$ (iii): $\mathcal{N}_{\mathfrak{P}}$ closed $cl_{\mathfrak{N}\mathfrak{H}_{\mathfrak{P}_2}}(\mathfrak{A})$ in $\left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)$, for each $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}$ $\mathfrak{A}$ in $\mathfrak{Y}$ then by remark w.k.t,

$$\mathfrak{f}^{\leftarrow}\left(cl_{\tau_{\mathfrak{N}\mathfrak{P}_2}}(\mathfrak{A})\right) = cl_{\tau_{\mathfrak{N}_{\mathfrak{P}_1}}}\left(\mathfrak{f}^{\leftarrow}\left(cl_{\tau_{\mathfrak{N}_2}}(\mathfrak{A})\right)\right) \supseteq cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{H}_1}}(\mathfrak{f}^{\leftarrow}(\mathfrak{A}))$$

$(iii) \Rightarrow (iv) : \mathfrak{f}^{\leftarrow}\left(cl_{\tau_{\mathfrak{N}_{\mathfrak{P}_2}}}(\mathfrak{f}^{\rightarrow}(\mathfrak{B}))\right) \supseteq cl_{\tau^{\varsigma}_{\mathfrak{N}_1}}(\mathfrak{f}^{\leftarrow}(\mathfrak{f}^{\rightarrow}(\mathfrak{B}))) \supseteq cl_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}}(\mathfrak{B}), \forall \mathfrak{B} \subseteq \mathfrak{f}^{\leftarrow}(\mathfrak{A}) \in \beta$

$\mathfrak{f}^{\rightarrow}\left(cl_{\tau^{\varsigma}_{\mathfrak{P}'1}}(\mathfrak{B})\right) \subseteq cl_{\tau_{\mathfrak{P}_2}}(\mathfrak{f}^{\rightarrow}(\mathfrak{B}))$

(iv) $\Rightarrow$ (ii): Let $\mathfrak{B} = \mathfrak{f}^{\leftarrow}(\mathfrak{A}), \forall \mathfrak{A} \in \left(\mathfrak{Y}, \tau_{\mathfrak{N}_2}\right)$ then,

$$\mathfrak{f}^{\rightarrow}\left(cl_{\tau^{\varsigma}_{\mathfrak{P}_1}}(\mathfrak{B})\right) \subseteq cl_{\mathcal{N}\mathcal{H}_{\mathfrak{P}2}}(\mathfrak{f}^{\rightarrow}(\mathfrak{B})) \subseteq \mathfrak{f}^{\leftarrow}(\mathfrak{A}) = \mathfrak{B} \therefore \mathfrak{B} = \mathfrak{f}^{\leftarrow}(\mathfrak{A})$$

$(v) \Rightarrow (i) : \mathfrak{f}^{\leftarrow}(\mathfrak{A}) = \mathfrak{f}^{\leftarrow}\left(\mathrm{int}_{\tau_{\mathfrak{M}\mathfrak{P}_2}}(\mathfrak{A})\right) \subseteq \mathrm{int}_{\tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}}\left(\mathfrak{f}^{\leftarrow}(\mathfrak{A}), \forall \mathfrak{A} \in \left(\mathfrak{Y}, \tau_{\mathfrak{N}\mathfrak{P}_2}\right)\right.$

$\Rightarrow \mathfrak{f}^{\leftarrow}(\mathfrak{A}) \in \left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}\right)$

Theorem 4.3 Let $\mathfrak{f}^{\rightarrow}$ is a mapping from $\left(\beta, \tau_{\mathfrak{P}\mathfrak{P}_1}\right)$ into $\left(\mathfrak{Y}, \tau_{\mathfrak{P}\mathfrak{P}_2}\right)$ then the following statements are equivalent,

(i) The mapping $\mathfrak{f}^{\rightarrow} : \left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}\right) \rightarrow \left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right)$ is $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ S-continuous.

(ii) Preimage of every $\mathcal{N}_{\mathfrak{P}}$ S-closed set in $\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{F}\mathfrak{P}_2}\right)$ is a $\mathcal{N}_{\mathfrak{P}}$ S-closed in $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}\right)$.

Proof From the previous theorem, the proof is simple.

Theorem 4.4 Let $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ and $\mathfrak{g}^{\rightarrow} : \mathfrak{Y} \rightarrow \mathfrak{X}$ are $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ continuous and $\mathcal{N}_{\mathfrak{P}}$ continuous, then $\mathfrak{f}^{\rightarrow} \circ \mathfrak{g}^{\rightarrow} : \beta \rightarrow \mathfrak{X}$ is $\mathcal{S}^{\varsigma}$-$\mathcal{N}_{\mathfrak{P}}$ continuous.

Proof Let $\mathfrak{A}$ be a $\mathcal{N}_{\mathfrak{P}}$ open set in $\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{F}\mathfrak{P}_2 2}\right)$, then
$\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.

$$\Rightarrow \mathfrak{f}^{\leftarrow}\left(\otimes_{\mathfrak{Y}} - A\right) = \mathfrak{f}^{\leftarrow}(\otimes\mathfrak{Y}) - \mathfrak{f}^{\leftarrow}(\mathfrak{A}) = \otimes_{\beta} - \mathfrak{f}^{\leftarrow}(\mathfrak{A})$$

$\therefore$ $\mathfrak{f}^{\leftarrow}(\mathfrak{A})$ be a $\mathcal{N}_{\mathfrak{P}}$ S-open in $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}\right)$. $\mathfrak{g}^{\rightarrow} : \mathfrak{Y} \rightarrow \mathfrak{X}$ is neutrosophic pythagorean continuous.
$\mathfrak{g}^{\rightarrow} : \mathfrak{Y} \rightarrow \mathfrak{X}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous. ($\because$ *By remark* 2.6)

$$\Rightarrow \mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}} - A) = \mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}}) - \mathfrak{g}^{\leftarrow}(\mathfrak{A}) = \mathfrak{g}^{\leftarrow}\left(\otimes_{\mathfrak{Y}}\right) - \mathfrak{g}^{\leftarrow}(A)$$

Let $\mathfrak{g}^{\leftarrow}(\mathfrak{A})$ be a $\mathcal{N}_{\mathfrak{P}}$ S-open in $\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right).\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_2}\right)$.

Then the composition of $\mathfrak{f}^{\leftarrow}$ and $\mathfrak{g}^{\leftarrow}$ is

$$\begin{aligned}\mathfrak{f}^{\leftarrow} \circ \mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}} - .A) = \mathfrak{f}^{\leftarrow}(\mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}} - A)) \\ = \quad \mathfrak{f}^{\leftarrow}\left(\otimes_{\mathfrak{Y}} - \mathfrak{g}^{\leftarrow}(\mathfrak{A})\right) \\ = \quad \otimes\beta - \mathfrak{f}^{\leftarrow}(\mathfrak{g}^{\leftarrow}(\mathfrak{A})) \\ = \quad \otimes\beta - \mathfrak{h}^{\rightarrow}(\mathfrak{A})\end{aligned}$$

where $\mathfrak{h}^{\rightarrow}(\mathfrak{A}) = \mathfrak{f}^{\leftarrow}(\mathfrak{g}^{\leftarrow}(\mathfrak{A}))$ be a $\mathcal{N}_{\mathfrak{P}}$ S-open in $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}1}\right)$.
$\therefore \mathfrak{f}^{\rightarrow} \circ \mathfrak{g}^{\rightarrow} : \beta \rightarrow \mathfrak{X}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.

Theorem 4.5 Let $\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ and $\mathfrak{g}^{\rightarrow} : \mathfrak{Y} \rightarrow \mathfrak{X}$ are $\mathcal{N}_{\mathfrak{P}}$ S-continuous and $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous, then $\mathfrak{f}^{\rightarrow} \circ \mathfrak{g}^{\rightarrow} : \beta \rightarrow \mathfrak{X}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.

Proof Let $\mathfrak{A}$ be a $\mathcal{N}_{\mathfrak{P}\mathfrak{s}}$ Given that,
$\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ is $\mathcal{N}_{\mathfrak{P}}$ S-continuous.
$\mathfrak{f}^{\rightarrow} : \beta \rightarrow \mathfrak{Y}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.

$$(\because \textit{By} \text{ remark } 3.3)$$

$\Rightarrow \mathfrak{f}^{\leftarrow}\left(\otimes_{\mathfrak{Y}} - A\right) = \mathfrak{f}^{\leftarrow}(\otimes\mathfrak{Y}) - \mathfrak{f}^{\leftarrow}(\mathfrak{A}) = \otimes\beta - \mathfrak{f}^{\leftarrow}(\mathfrak{A})$.
$\therefore \mathfrak{f}^{\leftarrow}(\mathfrak{A})$ be a $\mathcal{N}_{\mathfrak{P}}$ S-open in $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}_1}\right)$.
$\mathfrak{g}^{\rightarrow} : \mathfrak{Y} \rightarrow \mathfrak{X}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.

$$\Rightarrow \mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}} - A) = \mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}}) - \mathfrak{g}^{\leftarrow}(\mathfrak{A}) = \mathfrak{g}^{\leftarrow}\left(\otimes_{\mathfrak{Y}}\right) - \mathfrak{g}^{\leftarrow}(A)$$

Let $\mathfrak{g}^{\leftarrow}(\mathfrak{A})$ be a $\mathcal{N}_{\mathfrak{P}}$ S-open in $\left(\mathfrak{Y}, \tau^{\varsigma}_{\mathfrak{P}\mathfrak{P}_2}\right)$.
Then the composition of $\mathfrak{f}^{\leftarrow}$ and $\mathfrak{g}^{\leftarrow}$ is

$$\begin{aligned}\mathfrak{f}^{\leftarrow} \circ \mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}} - A) &= \mathfrak{f}^{\leftarrow}(\mathfrak{g}^{\leftarrow}(\otimes_{\mathfrak{X}} - A)) \\ &= \mathfrak{f}^{\leftarrow}\left(\otimes_{\mathfrak{Y}} - \mathfrak{g}^{\leftarrow}(\mathfrak{A})\right) \\ &= \otimes_{\beta} - \mathfrak{f}^{\leftarrow}(\mathfrak{g}^{\leftarrow}(\mathfrak{A})) \\ &= \otimes\, \beta - \mathfrak{h}^{\leftarrow}(\mathfrak{A})\end{aligned}$$

where $\mathfrak{h}^{\rightarrow}(\mathfrak{A}) = \mathfrak{f}^{\leftarrow}(\mathfrak{g}^{\leftarrow}(\mathfrak{A}))$ be a a $\mathcal{N}_{\mathfrak{P}}$ S-open in $\left(\beta, \tau^{\varsigma}_{\mathfrak{N}\mathfrak{P}}\right)$.
$\therefore \mathfrak{f}^{\rightarrow} \circ \mathfrak{g}^{\rightarrow} : \beta \rightarrow \mathfrak{X}$ is $\mathcal{S}^{\varsigma} - \mathcal{N}_{\mathfrak{P}}$ continuous.

5 Algorithm

Step 1: Collection of Data

Consider $\mathfrak{m}$ attributes $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \ldots, \mathcal{A}_{\mathfrak{m}}$, $\mathfrak{n}$ attributes $\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \ldots, \mathfrak{C}_{\mathfrak{n}}$ and $\mathfrak{p}$ attributes $M_1, M_2, M_3, \ldots, M_{\mathfrak{p}}(\mathfrak{n} \leq \mathfrak{p})$ for multi-attributes decision making problem (MADMP).

$\mathcal{D}_{\mathfrak{p}} \backslash \mathcal{A}_{\mathfrak{i}}$	$\mathcal{A}_1$	$\mathcal{A}_2$	·	·	·	$\mathcal{A}_m$	$\mathcal{A}_{\mathfrak{i}} \backslash \mathcal{C}_{\mathfrak{j}}$	$\mathcal{C}_1$	$\mathcal{C}_2$	·	·	·	$\mathcal{C}_n$
$\mathcal{D}_1$	$\mathfrak{d}_{11}$	$\mathfrak{d}_{12}$	·	·	·	$\mathfrak{d}_{1m}$	$\mathcal{A}_1$	$\mathfrak{a}_{11}$	$\mathfrak{a}_{12}$	·	·	·	$\mathfrak{a}_{1n}$
$\mathcal{D}_2$	$\mathfrak{d}_{21}$	$\mathfrak{d}_{22}$	·	·	·	$\mathfrak{d}_{2m}$	$\mathcal{A}_2$	$\mathfrak{a}_{21}$	$\mathfrak{a}_{22}$	·	·	·	$\mathfrak{a}_{2n}$
$\mathcal{D}_3$	$\mathfrak{d}_{31}$	$\mathfrak{d}_{32}$	·	·	·	$\mathfrak{d}_{3m}$	$\mathcal{A}_3$	$\mathfrak{a}_{31}$	$\mathfrak{a}_{32}$	·	·	·	$\mathfrak{a}_{3n}$
$\mathcal{D}_p$	$\mathfrak{d}_{p1}$	$\mathfrak{d}_{p2}$	·	·	·	$\mathfrak{d}_{pm}$	$\mathcal{A}_m$	$\mathfrak{a}_{m1}$	$\mathfrak{a}_{m2}$	·	·	·	$\mathfrak{a}_{mn}$

Step 4: Final Decision

Establish the single-valued neutrosophic pythagorean score values for decisions $\mathfrak{C}_1 \leq \mathfrak{C}_2 \leq \cdots \leq \mathfrak{C}_{\mathfrak{n}}$ and also for the attributes $M_1 \leq M_2 \leq \cdots \leq M_{\mathfrak{p}}$. Choose the attribute $M_{\mathfrak{p}}$ for $\mathfrak{C}_1$, $M_{\mathfrak{p}-1}$ for $\mathfrak{C}_2$ etc. If $\mathfrak{p} > \mathfrak{n}$ then leave out $M_{\mathfrak{k}}$, where $\mathfrak{k} = 1, 2, \ldots, \mathfrak{n} - \mathfrak{p}$.

6 Numerical Example

The prevalence of infectious diseases is increasing in the modern era, primarily attributed to the proliferation of viruses, bacteria, fungi, and other pathogens. The task of categorizing diverse sets of symptoms under a singular disease classification presents a formidable challenge within the realm of medical diagnosis. In this context, we use an example problem of disease identification to highlight the usefulness and relevance of the previously discussed technique.

As infectious diseases continue to evolve and diversify, healthcare professionals are confronted with the intricate task of accurately diagnosing these conditions. This often involves discerning the specific causative agents, considering the various symptoms exhibited by the affected individuals, and establishing a comprehensive understanding of the disease's pathogenesis. Classifying a wide range of symptoms under one general disease category requires a systematic and thoughtful approach.

In the context of our discussion, a disease identification problem serves as a practical demonstration of the aforementioned strategy's efficiency and relevance. This illustrative example showcases how a systematic and holistic approach to diagnosis can aid in addressing the growing challenges posed by infectious diseases, ultimately leading to more effective treatments and better public health outcomes.

Step 1: Collection of Data

The data gathered through consultations with medical professionals, encompassing four categories of microorganisms - viruses, bacteria, fungi, and parasites - known for causing infectious diseases, as well as the associated symptoms like a runny nose, loss of appetite, diarrhea, hair loss, and anemia, has been systematically organized into a (Tables 1 and 2). The aim is to ascertain the pathogens and establish the diseases they induce, which include common cold, stomach flu, E.coli infection, ringworm, and hookworm infestation (Tables 3, 4, 5 and 6).

Step 2: Construction of $\mathcal{N}_{\mathfrak{P}}$ S-topology for $(\mathfrak{E}_{\mathrm{j}})$ and $(M_{\mathfrak{k}})$:

(i) $\tau_{\mathrm{j}}^{\varsigma} = \mathcal{A} \cup \mathcal{B}$, Where $\mathcal{A} = \left\{\otimes, \odot, \mathfrak{a}_{1j}, \mathfrak{a}_{2\mathrm{j}}, \mathfrak{a}_{3j}, \ldots, \mathfrak{a}_{\mathrm{m}j}\right\}$ and $\mathcal{B} = \left\{\mathfrak{a}_{1j} \cup \mathfrak{a}_{2j}, \mathfrak{a}_{1j} \cup \mathfrak{a}_{3\mathrm{j}}, \ldots, \mathfrak{a}_{\mathrm{m}-1j} \cup \mathfrak{a}_{\mathrm{mj}}\right\}$
(ii) $\nu_{\mathfrak{k}}^{\varsigma} = \mathcal{C} \cup \mathcal{D}$, Where $\mathcal{C} = \{\otimes, \odot, \mathfrak{d}_{\mathfrak{k}1}, \mathfrak{d}_{\mathfrak{k}2}, \mathfrak{d}_{\mathfrak{k}3}, \ldots, \mathfrak{d}_{\mathfrak{k}\mathrm{m}}\}$ and $\mathcal{D} = \{\mathfrak{d}_{\mathfrak{k}1} \cup \mathfrak{d}_{\mathfrak{k}2}, \mathfrak{d}_{\mathfrak{k}1} \cup \mathfrak{d}_{\mathfrak{k}3}, \ldots, \mathfrak{d}_{\mathfrak{k}\mathrm{m}-1} \cup \mathfrak{d}_{\mathfrak{e}\mathrm{m}}\}$.

Step 3: Find $\mathcal{N}_{\mathfrak{P}}$ score function:

Table 1 Symptoms for microorganism

Symptoms\Microorganism	Virus$(\mathfrak{E}_1)$	Bacteria $(\mathfrak{E}_2)$	Fungi $(\mathfrak{E}_3)$	Parasite $(\mathfrak{E}_4)$
Runny nose $(\mathcal{A}_1)$	(0.7, 0, 0.3)	(0.4, 0.1, 0.6)	(0.9, 0.2, 0.1)	(0.1, 0.3, 0.9)
Loss of appetite $(\mathcal{A}_2)$	(0.4, 0.3, 0.6)	(0.1, 0.2, 0.9)	(0, 0.3, 1)	(0.8, 0.4, 0.2)
Diarrhea $(\mathcal{A}_3)$	(0.7, 0, 0.3)	(0.2, 0.1, 0.8)	(0.4, 0.1, 0.6)	(0.4, 0.2, 0.6)
Hair loss $(\mathcal{A}_4)$	(0.5, 0.1, 0.5)	(0.3, 0.3, 0.7)	(0.9, 0.1, 0.1)	(0.5, 0.2, 0.5)
Anemia $(\mathcal{A}_5)$	(0.6, 0.3, 0.4)	(0.4, 0, 0.6)	(0.2, 0.1, 0.8)	(0.4, 0.3, 0.6)

Table 2 Symptoms for the diseases

Diseases \ Symptoms	Runnynose $(\mathcal{A}_1)$	Loss of appetite $(\mathcal{A}_2)$	Diarrhea $(\mathcal{A}_3)$	Hair loss $(\mathcal{A}_4)$	Anemia $(\mathcal{A}_5)$
Tuberculosis (M_1)	(0.6, 0.3, 0.4))	(0.7, 0.1, 0.3)	(0, 0.3, 1)	(0.1, 0.2, 0.9)	(0, 0.3, 1)
Malaria (M_2)	(0.1, 0.1, 0.9)	(0.2, 0.3, 0.8)	(0.2, 0.3, 0.8)	(0.4, 0.4, 0.6)	(0.9, 0, 0.1)
Common cold (M_3)	(0.9, 0, 0.1)	(0, 0.2, 1)	(0.7, 0.2, 0.3)	(0.9, 0.3, 0.1)	(0.2, 0, 0.8)
Swin flu (M_4)	(0.2, 0.5, 0.8)	(0.1, 0.4, 0.9)	(0.2, 0.5, 0.8)	(0.2, 0.3, 0.8)	(0.2, 0.4, 0.8)
Ring worm (M_5)	(0.9, 0, 0.1)	(0.3, 0.6, 0.7)	(0.3, 0.6, 0.7)	(0.3, 0.1, 0.7)	(0.5, 0.3, 0.5)

Table 3 $\mathfrak{E}_{\mathrm{i}}$

	$\mathcal{A} \cup \mathcal{B}$
τ_1^{ς}	$\mathcal{A} = \{(1, 1, 0), (0, 0, 1), (0.7, 0, 0.3), (0.4, 0.3, 0.6), (0.5, 0.1, 0.5), (0.6, 0.3, 0.4)\}$ $\mathcal{B} = \{(0.7, 0.3, 0.3)\}$
τ_2^{ς}	$\mathcal{A} =$ $\{(1, 1, 0), (0, 0, 1), (0.4, 0.1, 0.6), (0.1, 0.2, 0.9), (0.2, 0.1, 0.8), (0.3, 0.3, 0.7), (0.4, 0, 0.6)\}$ $\mathcal{B} = \{(0.4, 0.2, 0.6), (0.4, 0.1, 0.6), (0.4, 0.3, 0.6)\}$
τ_3^{ς}	$\mathcal{A} =$ $\{(1, 1, 0), (0, 0, 1), (0.9, 0.2, 0.1), (0, 0.3, 1), (0.4, 0.1, 0.6), (0.9, 0.1, 0.1), (0.2, 0.1, 0.8)\}$ $\mathcal{B} = \{(0.9, 0.3, 0.1), (0.9, 0.2, 0.1)\}$
τ_4^{ς}	$\mathcal{A} =$ $\{(1, 1, 0), (0, 0, 1), (0.1, 0.3, 0.9), (0.8, 0.4, 0.2), (0.4, 0.2, 0.6), (0.5, 0.2, 0.5), (0.4, 0.3, 0.6)\}$ $\mathcal{B} = \{(0.8, 0.4, 0.2), (0.4, 0.3, 0.6), (0.5, 0.3, 0.5)\}$

Table 4 $M_{\mathfrak{k}}$

	$\mathcal{C} \cup \mathcal{D}$
ν_1^{ς}	$\mathcal{C} =$ $\{(1, 1, 0), (0, 0, 1), (0.6, 0.3, 0.4), (0.1, 0.1, 0.9), (0.9, 0, 0.1), (0.2, 0.5, 0.8), (0.9, 0, 0.1)\}$ $\mathcal{D} = \{(0.6, 0.3, 0.4), (0.9, 0.3, 0.1), (0.6, 0.5, 0.4)\}$
ν_2^{ς}	$\mathcal{C} =$ $\{(1, 1, 0), (0, 0, 1), (0.7, 0.1, 0.3), (0.2, 0.3, 0.8), (0, 0.2, 1), (0.1, 0.4, 0.9), (0.3, 0.6, 0.7)\}$ $\mathcal{D} = \{(0.7, 0.3, 0.3), (0.7, 0.2, 0.3), (0.7, 0.4, 0.3), (0.7, 0.6, 0.3)\}$
ν_3^{ς}	$\mathcal{C} =$ $\{(1, 1, 0), (0, 0, 1), (0, 0.3, 1), (0.2, 0.3, 0.8), (0.7, 0.2, 0.3), (0.2, 0.5, 0.8), (0.3, 0.6, 0.7)\}$ $\mathcal{D} = \{(0.2, 0.3, 0.8), (0.7, 0.3, 0.3), (0.2, 0.5, 0.8), (0.3, 0.6, 0.7)\}$
ν_4^{ς}	$\mathcal{C} =$ $\{(1, 1, 0), (0, 0, 1), (0.1, 0.2, 0.9), (0.4, 0.6, 0.4), (0.9, 0.3, 0.1), (0.2, 0.3, 0.8), (0.3, 0.1, 0.7)\}$ $\mathcal{D} = \{(0.4, 0.4, 0.6), (0.9, 0.3, 0.1), (0.2, 0.3, 0.8), (0.3, 0.2, 0.7)\}$
ν_5^{ς}	$\mathcal{C} =$ $\{(1, 1, 0), (0, 0, 1), (0.1, 0.2, 0.9), (0.4, 0.6, 0.4), (0.9, 0.3, 0.1), (0.2, 0.3, 0.8), (0.3, 0.1, 0.7)\}$ $\mathcal{D} = \{(0.9, 0.3, 0.1), (0.2, 0.3, 0.8), (0.2, 0.4, 0.8), (0.5, 0.3, 0.5)\}$

$\mathcal{N}_{\mathfrak{P}}$ score functions $\left(\mathfrak{S}_{\mathfrak{F}}\right)$ of $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathfrak{E}_j$ and $M_{\mathfrak{k}}$

(i) $\mathfrak{S}_{\mathfrak{F}}(\mathfrak{E}_{\mathrm{j}}) = \begin{cases} \mathfrak{S}_{\mathfrak{F}}(\mathcal{A}) & \textit{if}\, \mathfrak{S}_{\mathfrak{F}}(\mathcal{B}) = 0 \\ \frac{1}{2}\left[\mathfrak{S}_{\mathfrak{F}}(\mathcal{A}) + \mathfrak{S}_{\mathfrak{F}}(\mathcal{B})\right] & \textit{otherwise} \end{cases}$

where, (1) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{A}) = \frac{1}{3(\mathfrak{m}+2)}\left(\sum_{\mathrm{i}=1}^{\mathfrak{m}+2}\left[2 + \mu_{\mathrm{i}} - \sigma_{\mathrm{i}} - \gamma_{\mathrm{i}}\right]\right)$

(2) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{B}) = \frac{1}{3\mathfrak{q}}\left(\sum_{\mathrm{i}=1}^{\mathfrak{q}}\left[2 + \mu_{\mathrm{i}} - \sigma_{\mathrm{i}} - \gamma_{\mathrm{i}}\right]\right)$

$\forall \mathfrak{q}$ denotes the number of elements in $\mathcal{B}$ and $\mathrm{j} = 1, 2, \ldots, n$

(ii) $\mathfrak{S}_{\mathfrak{F}}(M_{\mathfrak{k}}) = \begin{cases} \mathfrak{S}_{\mathfrak{F}}(\mathcal{C}) & \textit{if}\, \mathfrak{S}_{\mathfrak{F}}(\mathcal{D}) = 0 \\ \frac{1}{2}\left[\mathfrak{S}_{\mathfrak{F}}(\mathcal{C}) + \mathfrak{S}_{\mathfrak{F}}(\mathcal{D})\right] & \textit{otherwise} \end{cases}$

where, (1) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{C}) = \frac{1}{3(\mathfrak{m}+2)}\left(\sum_{\mathrm{i}=1}^{\mathfrak{m}+2}\left[2 + \mu_{\mathrm{i}} - \sigma_{\mathrm{i}} - \gamma_{\mathrm{i}}\right]\right)$

(2) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{D}) = \frac{1}{3\mathfrak{r}}\left(\sum_{\mathrm{i}=1}^{\mathfrak{r}}\left[2 + \mu_{\mathrm{i}} - \sigma_{\mathrm{i}} - \gamma_{\mathrm{i}}\right]\right)$

$\forall \mathfrak{r}$ denotes the number of elements in $\mathcal{D}$ and $\mathfrak{k} = 1, 2, \ldots, n$.

Table 5 $\mathfrak{S}_{\mathfrak{F}}\mathfrak{E}_{\mathrm{j}}$

$\mathcal{A}$	$\mathcal{B}$	$\mathfrak{E}_{\mathrm{j}}$
0.5944	0.7000	$\mathfrak{E}_1 = 0.6472$
0.4809	0.5333	$\mathfrak{E}_2 = 0.5071$
0.5714	0.8500	$\mathfrak{E}_3 = 0.7107$
0.4761	0.6000	$\mathfrak{E}_4 = 0.5380$

Table 6 $\mathfrak{S}_{\mathfrak{F}}M_{\mathfrak{k}}$

$\mathcal{C}$	$\mathcal{D}$	$M_{\mathfrak{k}}$
0.5952	0.6770	$M_1 = 0.6361$
0.4285	0.6750	$M_2 = 0.5517$
0.4238	0.4083	$M_3 = 0.4160$
0.5095	0.5333	$M_4 = 0.5214$
0.5047	0.5250	$M_5 = 0.5148$

Step 4: Final Decision

Arrange $\mathfrak{S}_{\mathfrak{F}}$ for the attributes M_1, M_2, M_3, M_4, M_5 and alternatives $\mathfrak{E}_1$, $\mathfrak{E}_2$, $\mathfrak{E}_3$, $\mathfrak{E}_4$ in progression is result to $M_3 < M_5 < M_4 < M_2 < M_1$ and $\mathfrak{E}_2 < \mathfrak{E}_4 < \mathfrak{E}_1 < \mathfrak{E}_3$.

7 Conclusion

This work shows neutrosophic pythagorean set operation and mapping in tendering with neutrosophic pythagorean supra topological space. Final decision conclude that tuberculosis is a bacterial disease, malaria is a parasitic disease, swine flu is causes by virus and Ring worm is a fungal disease. $\mathcal{N}_{\mathfrak{P}}$ plays an essential role in many different kinds of application areas, including information technology, support systems for decision-making, electronic database platforms, detection of diseases, multi-criteria higher-level cognitive issues, etc.

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